SEMICLASSICAL COUPLED WAVE THEORY FOR 1D PHOTONIC CRYSTALS WITH DISSIPATION

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ABSTRACT
We show that a modified form of the semiclassical coupled wave theory gives a reliable description of wave propagation in a 1D photonic crystal with complex refractive index. The key idea of the theory remains the same: to include both variable amplitudes and variable (geometric-optics) phases in the counter-propagating coupled waves within the photonic crystal. Here, however, the phases become complex. As a result, the coupling amplitudes between the counter-propagating waves gain new properties.

1. INTRODUCTION
Photonic crystals are artificial dielectric structures with periodic modulation of refractive index, which have attracted considerable attention in the last two decades. Due to Bragg reflection, electromagnetic (optical) waves cannot propagate through such structures in certain directions, at certain frequencies. Hence, photonic crystals can exhibit band gaps (even omnidirectional band gaps in certain cases) and, as a result, control the propagation of electromagnetic waves in novel ways, with obvious application to dielectric mirrors, dielectric waveguides, and dielectric laser cavities.

While retaining most of useful properties of 2D and 3D photonic crystals, a 1D dielectric periodic structure with high refractive index contrast is much more attractive from a technological point of view. The usual theoretical methods for wave propagation in 1D photonic crystals are the Floquet-Bloch formalism, coupled wave theory, and the transfer matrix method. Among these three, the coupled wave approach [1, 2, 3] offers superior physical insight and gives simple analytical results in limiting cases. Unfortunately, the conventional coupled wave theory of Kogelnik fails in the case of high refractive index contrast, which is essential for a functional 1D photonic crystal.

In this paper, the recently developed semiclassical coupled wave theory [4, 5, 6, 7, 8] has been extended from real to complex dielectric permittivity, characteristic of dissipative media. This version of coupled wave theory includes both variable amplitude and variable (geometric-optics) phases in the counter-propagating waves. While analytically almost as simple as conventional coupled wave theory, our method is essentially exact for any achievable ratio (e.g. 1:5) of the real parts of refractive indices of the materials available to build devices. To demonstrate the influence of absorption in a clear manner we consider only normal (along the axis of periodicity) wave propagation in 1D photonic crystals. One interesting effect can be observed when the real part of the dielectric permittivity modulation depth is less than the absorption modulation depth: at some frequencies the wave penetrates the modulated absorbing medium to a distance greater than the distance in a medium with an equivalent mean absorption on the period. This is the one-dimensional analogue of the Borrmann effect [9, 10].

2. SEMICLASSICAL COUPLED WAVE THEORY
We consider a slab whose normal is the z-axis, occupying the region \(0 < z < L\). The complex index of refraction \(\tilde{n}(z) = \tilde{n}(z + d)\) varies periodically in the z-direction, but does not depend on x or y. The complex index \(\tilde{n}(z)\) is expressed in terms of the real refractive index \(n(z)\) and extinction coefficient \(\kappa\) as \(\tilde{n}(z) = n(z) + i\kappa(z)\). Monochromatic plane waves with angular frequency \(\omega\) and vacuum wave number \(k = \omega/c\) propagate inside the medium in the z-direction. For arbitrary polarization, Maxwell’s equations in the periodic medium reduce to the wave equation for the electric field \(E(z)\)

\[
\frac{d^2 E(z)}{dz^2} + k^2 \tilde{n}^2(z) E(z) = 0.
\]

where \(E(z,t) = E(z)\exp(-i\omega t)\). Equation (1) with an arbitrary complex periodic function \(\tilde{n}(z)\) is the Hill
optics phases $\pm$ slowly varying amplitudes in terms of two counter-propagating waves with resonances. The quantity $\phi$ is a so-called Bloch phase that is generally complex ($\phi = \phi' + i\phi''$). The peculiarities of wave propagation through a periodic structure depend mainly on the dispersion relation $\phi = \phi(k)$, which is conveniently written in the form of the dispersion equation $\Re(\cos(\phi(k)))$ as a function of $k$. There are two physically different regions of parameters for our periodic slab. In the first, called allowed bands, $|\Re(\cos(\phi))| < 1$; in the second, $|\Re(\cos(\phi))| > 1$, and the forward Bloch wave $E_{B,1}(z)$ is exponentially damped, even in the absence of real absorption ($\kappa(z) = 0$). Such regions are called forbidden bands and their centers are Bragg resonances.

The essence of the semiclassical coupled wave theory is as follows. It is based on use of the approximation of geometrical optics which is related to the WKB approach in quantum mechanics. Instead of Bloch waves, we seek a solution of the Hill equation (1) in terms of two counter-propagating waves with slowly varying amplitudes $A^{(\pm)}(z)$ and geometric-optics phases $\pm \psi(z)$, i.e.

$$E(z) = \frac{A^{(+)}(z)}{\sqrt{n(z)}} e^{i\psi(z)} + \frac{A^{(-)}(z)}{\sqrt{n(z)}} e^{-i\psi(z)},$$

(3)

where

$$\psi(z) = k \int_0^z \tilde{n}(z') dz' \equiv k \int_0^z n(z') dz' + i k \int_0^z \kappa(z') dz'.$$

(4)

One can see that $\psi(z + d) = \psi(z) + \psi(d)$. After the substitution of Eq. (3) into Eq. (1) we have an identity if the amplitudes $A^{(\pm)}(z)$ satisfy the system

$$\frac{dA^{(+)}(z)}{dz} = S^{(-)}(z) A^{(-)}(z),$$

$$\frac{dA^{(-)}(z)}{dz} = S^{(+)}(z) A^{(+)}(z),$$

(5)

where

$$S^{(\pm)}(z) = \frac{1}{2\tilde{n}(z)} \frac{d\tilde{n}(z)}{dz} \exp(\pm 2i\psi(z)).$$

(6)

The system (5) is exact. Introducing the phase-averaged complex refractive index

$$\tilde{n}_{av} = \psi(d)/kd = n_{av} + i\kappa_{av}$$

(7)

we find that the quantities $S^{(\pm)}(z) \exp(\mp 2ik\tilde{n}_{av}z)$ are periodic functions that can be Fourier expanded as

$$S^{(\pm)}(z) = e^{\pm 2ik\tilde{n}_{av}z} \sum_{m=-\infty}^{m=+\infty} s_m^{(\pm)} e^{i2mz}.$$  

(8)

The coefficients $s_m^{(\pm)}$ can be expressed in a form which is particularly suitable for layered periodic structures with piecewise continuous $n(z)$

$$s_m^{(\pm)} = \frac{1}{2d} \mathcal{P} \int_0^d \frac{d\tilde{n}(z)}{dz} e^{2i(\pm \psi(z) + k\kappa_{av}z - \frac{2mz}{2})} dz + \frac{1}{2d} \sum_{j} \ln \left( \frac{n(z_j + 0)}{n(z_j - 0)} \right) e^{2i(\pm \psi(z_j) + k\kappa_{av}z_j - \frac{2mz_j}{2})}.$$  

(9)

The $\mathcal{P}$ implies a principal value integral, and the sum over $j = 1, 2, ...$ takes into account the contribution to $s_m^{(\pm)}$ of jumps in the refractive index $n(z)$ at the points of discontinuity $z_j$ within the period. The quantities $n(z_j \pm 0)$ are the limiting values of the refractive index $n(z)$ at the right/left of a point of discontinuity $z_j$.

Physically, the coefficients $s_m^{(\pm)}$ represent the magnitude of coupling between the two counter-propagating waves (6) due to the $m$-th Fourier components of the functions $S^{(\pm)}(z)$. One can see that for periodic structures with real refractive indices the coupling coefficient $s_m^{(\pm)}$ is just the complex conjugate of $s_m^{(\pm)}$ [5, 6]. That relation obviously fails in a general case considered here.

Following the method of averaging [11], we assume that the main contribution to the exact solutions of Eqs. (4) is provided by the slowly varying components of the functions $S^{(\pm)}(z)$. To find those components we use the substitution

$$k\kappa_{av} = \frac{\pi}{d} q + \delta q$$

(10)

and assume that $\delta q d < 1$. It means that we seek solutions in the vicinity of the so-called Bragg resonances $k_q = \pi q/(n_{av} d)$, $q = 1, 2, 3, ...$ of our periodic structure. Then, we can rewrite Eq. (8) in the form

$$S^{(\pm)}(z) = \left[ s_{\mp q}^{(\pm)} + \sum_{m \neq \mp q} s_m^{(\mp)} e^{i2\pi q(m \mp q)z} \right] \exp \left[ \pm 2i(\delta q - k\kappa_{av})z \right].$$

(11)

One can see that the slowly-varying components are $s_{\mp q}^{(\pm)} \exp[\pm 2i(\delta q - k\kappa_{av})z]$. Discarding the fast oscillating terms in Eq. (11), from the exact system (5), we obtain the approximate form

$$\frac{dA^{(+)}(z)}{dz} = e^{2[\kappa_{av} - i\delta q]z} s_q^{(-)} A^{(-)}(z),$$

$$\frac{dA^{(-)}(z)}{dz} = e^{-2[\kappa_{av} - i\delta q]z} s_q^{(+)} A^{(+)}(z).$$

(12)

Solving these we find

$$A^{(+)}(z) = \left[ F(\gamma + k\kappa_{av} - i\delta q)e^{-\gamma z} + G s_q^{(-)} e^{\gamma z} \right] \exp[\pm(k\kappa_{av} - i\delta q)z],$$

$$A^{(-)}(z) = \left[ -F s_q^{(+)} e^{-\gamma z} + G(\gamma + k\kappa_{av} - i\delta q)e^{\gamma z} \right] \exp[\pm(-k\kappa_{av} - i\delta q)z].$$

(13)
where
\[ \gamma^2 = s_{q}^{(-)} s_{q}^{(+)} + (k \kappa - i \delta) s_{q}^{(+)} \].

If we substitute the above functions (13) into Eq. (3) and compare it with Eq. (2), we obtain the Bloch phase in the form
\[ \phi = \pi q + i \gamma d \],
for each q-th zone along the k-axis
\[ \pi \left( \frac{1}{2} + q \right) < kn_{av}d < \pi \left( \frac{1}{2} + q \right) \].

Now we can calculate the reflection and transmission coefficients for a wave incident on a matched periodic structure. By matched, we mean that the refractive index is continuous across the exterior boundaries at \( z = 0 \) and \( z = L \), i.e. there is no Fresnel reflection from them. Eq. (3) represents a solution of the Hill equation (1) in terms of right and left-moving components. Therefore, the amplitude reflection and transmission coefficients for a wave incident on the structure from the left, i.e. from the region \( z < 0 \), can be found from the expressions
\[ r = \frac{A^{(-)}(0)}{\sqrt{n}(0)} e^{-i\psi(0)} \quad \text{and} \quad t = \frac{A^{(+)}(L)}{\sqrt{n}(0)} e^{i\psi(L)} \] under the conditions
\[ A^{(+)}(0) e^{i\psi(0)} = 1, \quad A^{(-)}(L) e^{-i\psi(L)} = 0 \].

The first relation in Eq. 18 is just a normalization condition, while the second expresses the radiation principle: no propagation of a left-moving wave in the region \( z > L \). From these conditions we can find the constants \( F \) and \( G \) to be substituted into expressions (17) for reflection and transmission amplitudes. The final result is
\[ r = -s_{q}^{(+)} \sinh(\gamma L) / \gamma \cosh(\gamma L) + (k \kappa - i \delta) \sinh(\gamma L) \],
\[ t = \gamma e^{i\pi Nq} / \gamma \cosh(\gamma L) + (k \kappa - i \delta) \sinh(\gamma L) \].

3. APPLICATION TO BILAYER PHOTONIC CRYSTAL

To illustrate the modified semiclassical coupled wave theory, we consider a symmetrical two-layered periodic medium with complex refractive indices \( n_{1} \approx 1 + i \kappa_1 \) and \( n_{2} \approx 2 + i \kappa_2 \) and layer thicknesses \( d_1 \) and \( d_2 \) such that \( d = d_1 + d_2 \), as shown in Fig. 1. For such a structure we have
\[ n_{av} = \frac{n_1 d_1 + n_2 d_2}{d}, \quad \kappa_{av} = \frac{\kappa_1 d_1 + \kappa_2 d_2}{d} \].

We consider reflection and transmission for a symmetrical bilayer photonic crystal of \( N = 5 \) periods whose layers have refractive indices \( \tilde{n}_1 = 1.05 \) (no dissipation) and \( \tilde{n}_2 = 2.05 + 0.1 i \); and thicknesses \( d_1 = 150 \) nm and \( d_2 = 50 \) nm. This is the structure with moderate modulation \( (n_2 - n_1 \approx 1) \) of the refractive index. We note that a dense material having a refractive index as low as 1.05 has very recently been prepared in the laboratory [12]. Therefore, the above photonic crystal is almost matched to air, which has refractive index close to one. The exact numerical results for reflection and transmission can be obtained with the aid of the transfer matrix method [13, 14, 15]. Conventional coupled wave theory provides expressions similar to Eq. (19), but with different forms for the cou-
pling coefficients and average refractive indices and extinction coefficients. In particular,
\[ n_{av} = \sqrt{\frac{n_1^2 d_1 + n_2^2 d_2}{d}}, \quad \kappa_{av} = \sqrt{\frac{\kappa_1^2 d_1 + \kappa_2^2 d_2}{d}}, \]
(22)
and
\[ s_m^{(+)} = s_m^{(-)} = (-)^m \frac{n_2^2 - n_1^2}{\pi m} \sin \left( \pi m \frac{d_2}{d} \right). \]
(23)
As a result, the values for \( \gamma \) and \( \delta_q \) would also be different. In Figs. 2, 3 we show the results of all three approaches. One can see that conventional coupled wave theory generally fails in the second zone along the \( k \)-axis, \( \frac{3}{2} \pi/(n_{av} d) < k < \frac{5}{2} \pi/(n_{av} d) \), while the semiclassical coupled wave theory works well for all frequencies of the incoming waves.

![Diagram](image)

Fig. 3. Transmission probability vs frequency for the bilayer photonic crystal with moderate \( (n_2 - n_1 \approx 1) \) refractive index modulation. The parameters of the crystal are as described in the text. The lines are as in Fig. 2.

Finally, we should mention that semiclassical coupled wave theory predicts the one-dimensional analogue of the Borrmann effect [9, 10]. The analysis of Eqs. (19) shows that at Bragg resonances, i.e. at \( k = \pi q/(n_{av} d) \), when the modulation depth of the real part of the dielectric permittivity is less than the absorption modulation depth, the wave penetrates the modulated absorbing medium further than in a medium with an equivalent mean absorption on the period.

4. REFERENCES


