Angular Momentum III

- Examples with conservation of angular momentum
- Collisions involving rotation

Serway 11.4

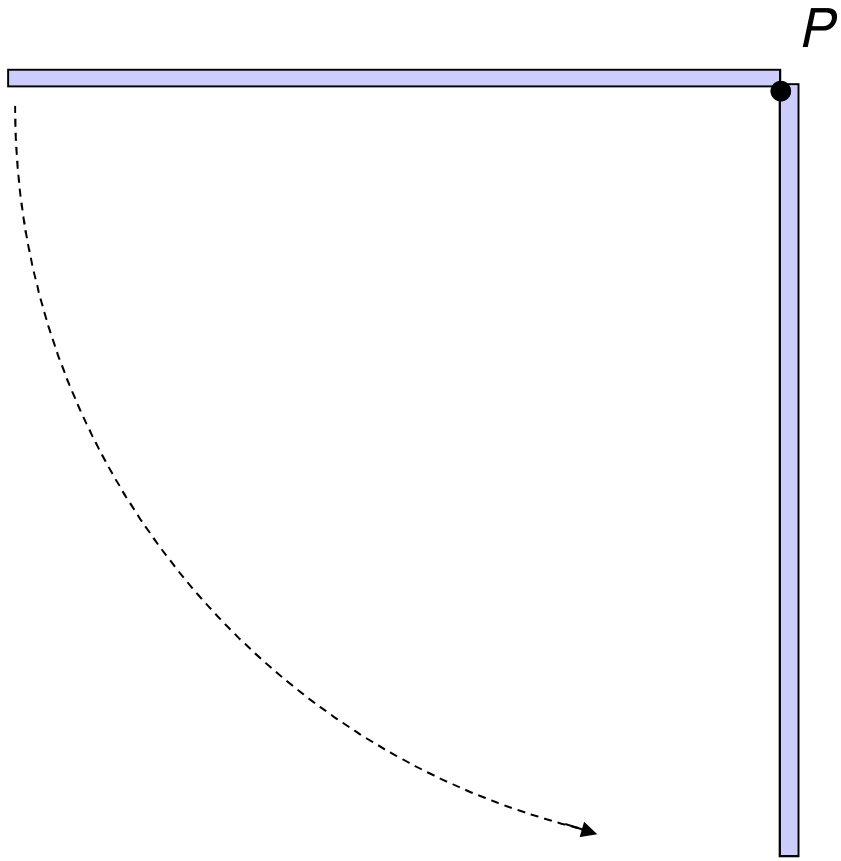
Practice: Chapter 11, problems 30, 33, 36, 39, 41, 51, 53, 63

Collisions: Collisions can conserve angular momentum as well as linear momentum.

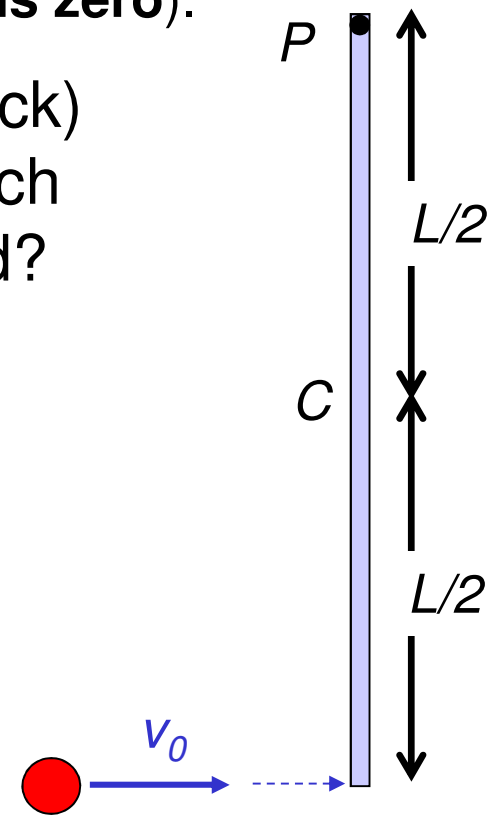
Total *linear* momentum is conserved if there is no *external* force during the collision (or if the external forces are small compared to the forces the colliding bodies exert on each other).

Total *angular* momentum is conserved if there is no *external torque* during the collision (or if the external torques are small).

Example: *Two identical thin sticks are suspended from a pivot at P. One is held horizontal and released. If they stick together when they collide, how high do they swing?*



Example: A metre stick (mass M, length $b = 1\mathrm{m}$, moment of inertia I) is suspended from one end by a frictionless pivot at P. A ball of mass m, velocity v_0, strikes the other end of the (stationary) stick at right angles, and stops (**final velocity of the ball is zero**).



Quiz: Comparing the system (ball plus stick) just before and just after the collision, which of the following are necessarily conserved?

A) Angular momentum about P
B) Angular momentum about *C*
C) linear momentum
D) Kinetic energy
E) All of the above

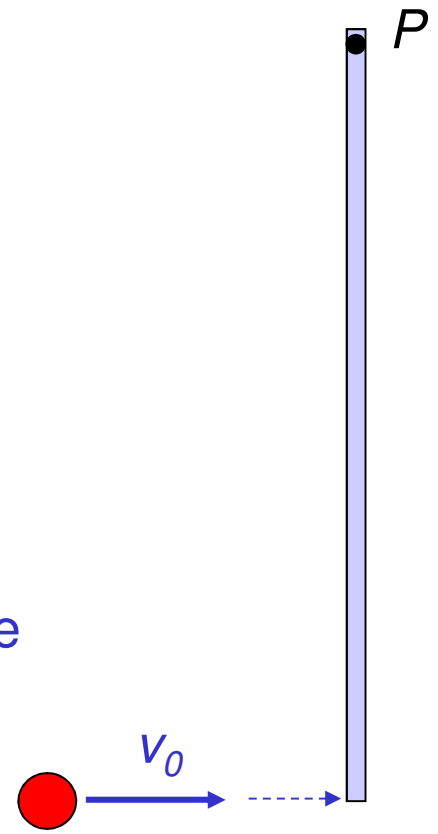
Example Problems:
Stick and ball have equal masses m; frictionless pivot at P.

1) If the ball hits the end and stops, find the final angular velocity of the stick.

2) If the ball hits the end and *sticks*, find the final angular velocity of the stick.

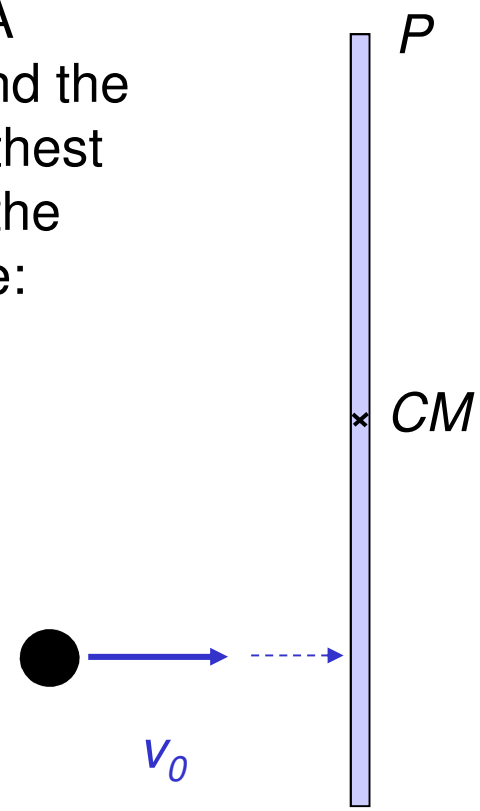
3) Find the speed of the ball which will cause the stick to swing 30° in each case.

4) Do all of the above if the ball hits the centre instead of the end.



Quiz

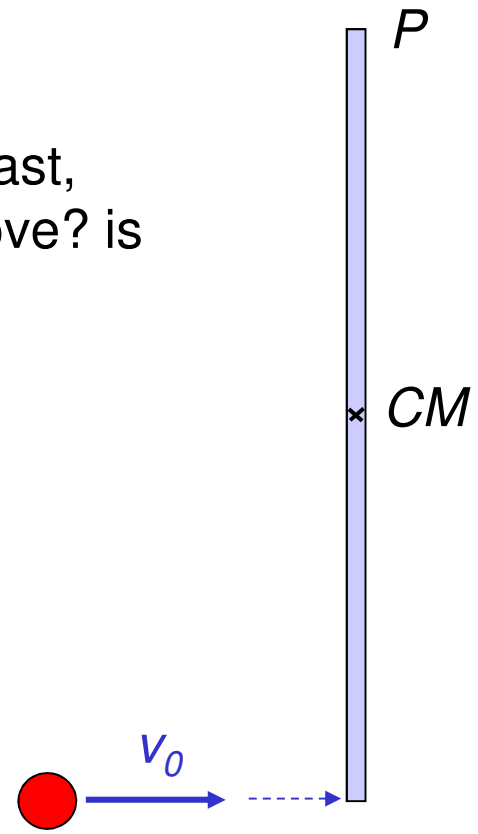
A stick (uniform thin rod) is lying on the ice. A hockey puck hits the stick, at right angles, and the stick starts to slide. Point *P* is on the end farthest from where the puck hits. Immediately after the collision, the end *P* will **initially** start to move:



A) in a direction parallel to v_0
B) in a direction opposite to v_0
C) at an angle (not 0° or 180°) to v_0
D) It depends where the puck hits

Example: The metre stick is resting on a frictionless surface (not attached to anything) before the ball hits the end at right angles. Assuming the ball still stops, what should we write for the stick?

What will the motion of the stick be like? How fast, and in which direction will the end *P* initially move? is the collision elastic?



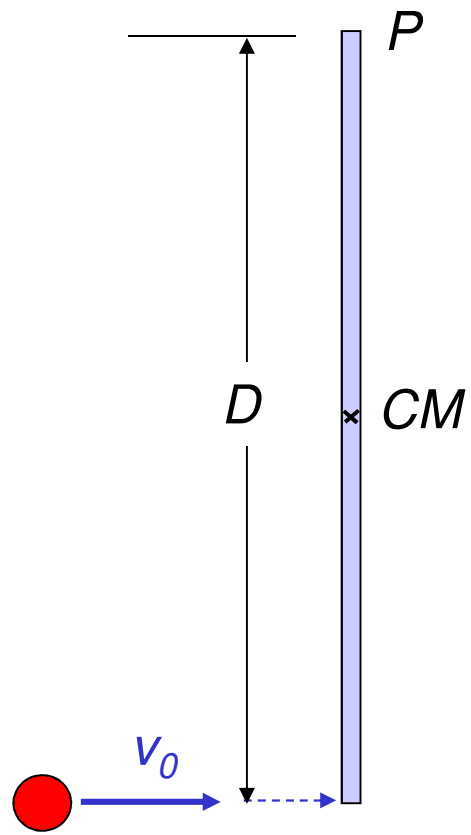
Now there is no external force. So,

Linear momentum is conserved:

Angular momentum (*e.g.*, about the *CM*) is conserved:

If the collision is elastic (it might not be!), kinetic energy is conserved:

(forward) velocity of *P* is



Then solve, depending on the question....
- How does P move?
- Is there a value of m/M such that the collision will be elastic?

Summary

In general, for a rigid body, $\boxed{\mathbf{L} = \mathbf{r} \times (m\mathbf{v}_{CM}) + \mathrm{I}_{CM}\boldsymbol{\omega}}$

In collisions, angular momentum will be conserved it there is no external torque.

Practice: Chapter 11, problems 29, 31, 37, 49, 50, 51