Kirchhoff's Circuit Rules

Text section 28.3

Kirchhoff’s circuit rules

Practice:
Chapter 28, problems 17, 19, 25, 26, 43

Kirchhoff’s Circuit Rules

**Junction Rule:** total current in = total current out at each junction (from conservation of charge).

**Loop Rule:** Sum of emfs and potential differences around any closed loop is zero (from conservation of energy).
**Junction Rule:** conservation of charge.

\[ I_1 = I_2 + I_3 \]

or

\[ I_1' = -I_2' \quad I_2' = -I_2' \quad I_3' = -I_3' \]

\[ I_1' + I_2' + I_3' = 0 \]

**Loop Rule:** conservation of energy.

Follow a test charge \( q \) around a loop:

\[ \sum q \times (\Delta V_i) = 0 \quad \text{around any loop in circuit.} \]

- **changes going from left to right**
  - \( \Delta V = -IR \)
  - \( \Delta V = +IR \)
  - \( \Delta V = \epsilon \)
  - \( \Delta V = Q/C \)
Find the current through each battery.

The junction rule will give:

A) $I_1 + I_2 + I_3 = 0$
B) $-I_1 + I_2 + I_3 = 0$
C) $I_1 - I_2 + I_3 = 0$
D) $I_1 + I_2 - I_3 = 0$
E) none of these
Example

Find the current through each battery.

Quiz

The junction rule will give:

A) $I_1 + I_2 + I_3 = 0$
B) $-I_1 + I_2 + I_3 = 0$
C) $I_1 - I_2 + I_3 = 0$
D) $I_1 + I_2 - I_3 = 0$
E) none of these
**Quiz**

The loop rule applied to loop \textit{abcd} will give:

A) $9A - 18I_1 - 3I_3 = 0$
B) $9A + 18I_1 - 3I_3 = 0$
C) $9A + 18I_1 + 3I_3 = 0$
D) $9A - 18I_1 + 3I_3 = 0$
E) none of these

**Quiz**

The loop rule applied to loop \textit{abda} will give:

A) $12A - 18I_1 + 6I_2 = 0$
B) $12A - 18I_1 - 6I_2 = 0$
C) $6A - 18I_1 - 6I_2 = 0$
D) $6A + 18I_1 + 6I_2 = 0$
E) $6A - 18I_1 + 6I_2 = 0$
Find the current through each battery.

\[ I_1 = 400 \text{ mA}, \quad I_2 = 200 \text{ mA} \]

Solution:

Loop abcda:
\[
9 \text{ V} - I_1 \cdot 18 \Omega + 3\Omega(-I_1 - I_2) = 0 \\
\Rightarrow 21I_1 + 3I_2 = 9 \text{ A} \quad \text{(1)}
\]

Loop dbcd:
\[
3 \text{ V} - 6\Omega \cdot I_2 + 3\Omega(-I_1 - I_2) = 0 \\
\Rightarrow 3I_1 + 9I_2 = 3 \text{ A} \quad \text{(2)}
\]

7×(2) − (1):
\[
60 \cdot I_2 = 12 \text{ A} \quad \Rightarrow I_2 = 200 \text{ mA}
\]

3×(1) − (2):
\[
60 \cdot I_1 = 24 \text{ A} \quad \Rightarrow I_1 = 400 \text{ mA}
\]
Example

Find the current through each battery.

\[ I_3 = -(I_1 + I_2) \]

Find the current through each battery.

**Answers:** \( I_1 = 171 \text{ mA}, \ I_2 = -64.3 \text{ mA} \)
Solution:

Loop abcda: \[ 6 \, \text{V} - I_1 \cdot 10\, \Omega + 40\, \Omega (-I_1 - I_2) = 0 \]
\[ \Rightarrow 50I_1 + 40I_2 = 6 \quad \ldots \quad (1) \]

Loop dbcd: \[ 3 \, \text{V} - 20\, \Omega \cdot I_2 + 40\, \Omega (-I_1 - I_2) = 0 \]
\[ \Rightarrow 40I_1 + 60I_2 = 3 \quad \ldots \quad (2) \]

\[ 4\times(1) - 5\times(2) : -140 \, I_2 = 9 \quad \Rightarrow \quad I_2 = -64 \, \text{mA} \]

\[ 3\times(1) - 2\times(2) : 70 \, I_1 = 12 \quad \Rightarrow \quad I_1 = 171 \, \text{mA} \]

Exercise:

**What is \( V_{ab} \) (i.e., \( V_a - V_b \)) when the switch is open?**

**Exercise for fun:** Find the current through the switch when it is closed.
The loop rule requires that $V_{ab}$ (i.e., $V_a - V_b$) obeys:

A) $V_{ab} = (200\,\Omega)I_1 + (200\,\Omega)I_2$
B) $V_{ab} = (200\,\Omega)I_1 - (200\,\Omega)I_2$
C) $V_{ab} = -(200\,\Omega)I_1 + (200\,\Omega)I_2$
D) $V_{ab} = -(200\,\Omega)I_1 - (200\,\Omega)I_2$

"Series and parallel" rules don't help in this case. You have to go back to the fundamentals—Kirchhoff's Circuit Rules.

(Answer: $R_{\text{eff}} = 1.4\,\Omega$)
1) Use Kirchhoff’s rules to write everything in terms of, e.g., $I_3$.

2) Divide $V_{\text{TOTAL}}/I_{\text{TOTAL}}$.

Show that:

$I_1 = I_5 = 3I_3$

$I_2 = I_4 = 2I_3$

$I_{\text{TOTAL}} = 5I_3$

$V_{\text{TOTAL}} = (7\Omega)I_3$
Solution to part A:

\[ V_{cd} = 12 \text{ V} = I_1 \cdot (250\Omega + 250\Omega) = I_2 \cdot (100\Omega + 400\Omega) \]

\[ V_{ad} = I_1 \cdot 250\Omega \quad \Rightarrow \quad \frac{V_{ad}}{V_{cd}} = \frac{I_1 \cdot 250\Omega}{I_1 \cdot 500\Omega} \]

\[ \Rightarrow \quad V_{ad} = \frac{1}{2} \cdot 12 \text{ V} = 6 \text{ V} \]

\[ V_{bd} = 12 \text{ V} \cdot \frac{400}{500} = 9.6 \text{ V} \]

\[ \Rightarrow \quad V_{ab} = V_{ad} - V_{bd} = -3.6 \text{ V} \quad (3.6 \text{ V}, \quad V_a - V_b) \]

Solution to part b: \( V_d - V_c = 12 \text{ V} \)

Loop adba: \[ +250I_1 - 100I_2 = 0 \]

\[ \Rightarrow \quad 250I_1 = 100I_2 \]

\[ \Rightarrow \quad I_2 = \frac{5}{2}I_1 \quad (1) \]

Loop abca: \[ 250(I_1 + I_2) = 400(I_2 - I_3) \]

\[ \Rightarrow \quad 250I_1 + 250I_3 = 1000I_1 - 400I_3 \]

\[ \Rightarrow \quad I_3 = \frac{750}{650}I_1 = \frac{15}{13}I_1 \quad (2) \]

Then: \( V_d - V_c = 12 \text{ V} = (250)(I_1 + I_3) + 250I_1 \)

\[ 12 \text{ V} = 250 \cdot \left(2 + \frac{15}{13}\right)I_1 \quad \leftarrow \text{Solve...} \]

\[ I_1 = 15.2 \text{ mA} \quad I_2 = 38.1 \text{ mA} \quad I_3 = 17.6 \text{ mA} \]
Notice that "series and parallel" don't help in this case. You have to go back to Kirchhoff’s Rules.

(Answer: $R_{\text{eff}} = \left(\frac{155}{74}\right) \Omega$)