The discovery of high $T_c$ superconductivity in hole doped CuO$_2$ planes has focused strong interest on the spin $S = 1/2$ two-dimensional (2D) Heisenberg antiferromagnet, $H = J \sum \mathbf{S}_i \cdot \mathbf{S}_j$, where $J (\approx 1500$ K) is the exchange interaction [1]. Previous experimental studies of the 2D Heisenberg model by $^{63}$Cu NMR [2] and neutron scattering [3] have concentrated on the sharp peak of the dynamical structure factor $S(q, \omega)$ located at the corners of the first Brillouin zone, $\mathbf{Q} = (\pm \pi/a, \pm \pi/a)$, where $\mathbf{q}$ is the wave vector, $\omega$ is the frequency, and $a$ is the lattice constant. This antiferromagnetic peak has a momentum width $\sim 1/\xi$, the inverse of the spin correlation length. Accordingly, these experiments have probed the long wavelength properties ($\gg \xi$) near $\mathbf{q} = \mathbf{Q}$.

On the other hand, virtually nothing is known experimentally at short wavelengths ($\ll \xi$) in the paramagnetic state, except for the dispersion of the high energy spin excitations [4]. In 2D, the high energy spin waves can exist even without long range magnetic order because of the strong short range spin correlations for $T \ll J$ [1]. Theoretically, the renormalization group analysis of the nonlinear sigma model by Chakravarty, Halperin, and Nelson predicted that the spin waves in the 2D Heisenberg antiferromagnet behave as noninteracting free particles at short distance scales even in the paramagnetic state [5]. This intriguing prediction for 2D has a close analogy in QCD known as the asymptotic freedom of quarks [6] and is in remarkable contrast to the 3D system, in which the spin wave excitations are highly damped above the bulk Néel ordering temperature $T_N^{2D}$ [7]. Unfortunately, regardless of the strong theoretical interest, many believed that probing the nature of the short wavelength elementary excitations was beyond the current experimental technology [1,8(a)]. Among the unresolved questions are how these spin waves are damped by thermal effects [8] and how mobile holes doped in the CuO$_2$ planes contribute to the elementary excitations [9].

To shed light on these issues from experiments, we report in this Letter an $^{17}$O NMR study of magnetic excitations in undoped and lightly hole doped (weakly metallic) CuO$_2$ planes of high $T_c$ cuprates in an extremely broad range of temperature ($0.2 \leq T/J \leq 0.5$). We deduce, for the first time, the magnon damping, $\Gamma$, of the $S = 1/2$ 2D Heisenberg antiferromagnet at short wavelengths, and test theoretical predictions [5,8]. Moreover, we demonstrate the progressive evolution of the low-energy quasiparticle excitation spectrum across the insulator-metal transition with hole doping.

As an experimental system for the undoped 2D Heisenberg model, either $^{17}$O isotope enriched single crystals, uniaxially aligned powder, or partially aligned powder samples of Sr$_2$CuO$_2$Cl$_2$ [3] were used depending on experimental necessities. We determined the bulk Néel temperature as $T_N^{3D} = 257$ K based on $^{13}$Cl NMR measurements, in agreement with Suh et al. [10]. The major advantage of Sr$_2$CuO$_2$Cl$_2$ is that the magnetic anisotropy is so weak ($J_{xy}/J \sim 10^{-4}$ [3]) that the isotropic 2D Heisenberg behavior is robust even down to $\approx 280$ K (see Fig. 1 and [3,10]). For the study of lightly hole doped CuO$_2$ planes, we used $^{17}$O isotope enriched single crystals of La$_{2-x}$Sr$_x$CuO$_4$ with $x = 0.025$ and 0.035 [11]. All $^{17}$O NMR measurements were conducted for clearly resolved NMR transitions with the typical line width 2 to 15 KHz.

Using $^{17}$O NMR, we probed the short wavelength excitations in the CuO$_2$ planes of insulating and weakly metallic high $T_c$ cuprates. We measured the spin wave damping for an $S = 1/2$ 2D quantum Heisenberg antiferromagnet for the first time. The results establish the nearly free behavior (asymptotic freedom) of the high energy spin waves, even without long range magnetic order. Light hole doping dramatically enhances the low energy excitation spectrum below 300 K.

PACS numbers: 76.60.–k, 74.25.Nf, 74.72.–h

**References**

1. K. R. Thurber, A. W. Hunt, T. Imai, F. C. Chou, and Y. S. Lee
2. Center for Materials Science and Engineering, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
3. (Received 31 July 1996; revised manuscript received 17 March 1997)
4. Using $^{17}$O NMR, we probed the short wavelength excitations in the CuO$_2$ planes of insulating and weakly metallic high $T_c$ cuprates. We measured the spin wave damping for an $S = 1/2$ 2D quantum Heisenberg antiferromagnet for the first time. The results establish the nearly free behavior (asymptotic freedom) of the high energy spin waves, even without long range magnetic order. Light hole doping dramatically enhances the low energy excitation spectrum below 300 K. [S0031-9007(97)03483-2]
5. The discovery of high $T_c$ superconductivity in hole doped CuO$_2$ planes has focused strong interest on the spin $S = 1/2$ two-dimensional (2D) Heisenberg antiferromagnet, $H = J \sum \mathbf{S}_i \cdot \mathbf{S}_j$, where $J (\approx 1500$ K) is the exchange interaction [1]. Previous experimental studies of the 2D Heisenberg model by $^{63}$Cu NMR [2] and neutron scattering [3] have concentrated on the sharp peak of the dynamical structure factor $S(q, \omega)$ located at the corners of the first Brillouin zone, $\mathbf{Q} = (\pm \pi/a, \pm \pi/a)$, where $\mathbf{q}$ is the wave vector, $\omega$ is the frequency, and $a$ is the lattice constant. This antiferromagnetic peak has a momentum width $\sim 1/\xi$, the inverse of the spin correlation length. Accordingly, these experiments have probed the long wavelength properties ($\gg \xi$) near $\mathbf{q} = \mathbf{Q}$.

On the other hand, virtually nothing is known experimentally at short wavelengths ($\ll \xi$) in the paramagnetic state, except for the dispersion of the high energy spin excitations [4]. In 2D, the high energy spin waves can exist even without long range magnetic order because of the strong short range spin correlations for $T \ll J$ [1]. Theoretically, the renormalization group analysis of the nonlinear sigma model by Chakravarty, Halperin, and Nelson predicted that the spin waves in the 2D Heisenberg antiferromagnet behave as noninteracting free particles at short distance scales even in the paramagnetic state [5]. This intriguing prediction for 2D has a close analogy in QCD known as the asymptotic freedom of quarks [6] and is in remarkable contrast to the 3D system, in which the spin wave excitations are highly damped above the bulk Néel ordering temperature $T_N^{2D}$ [7]. Unfortunately, regardless of the strong theoretical interest, many believed that probing the nature of the short wavelength elementary excitations was beyond the current experimental technology [1,8(a)]. Among the unresolved questions are how these spin waves are damped by thermal effects [8] and how mobile holes doped in the CuO$_2$ planes contribute to the elementary excitations [9].

To shed light on these issues from experiments, we report in this Letter an $^{17}$O NMR study of magnetic excitations in undoped and lightly hole doped (weakly metallic) CuO$_2$ planes of high $T_c$ cuprates in an extremely broad range of temperature ($0.2 \leq T/J \leq 0.5$). We deduce, for the first time, the magnon damping, $\Gamma$, of the $S = 1/2$ 2D Heisenberg antiferromagnet at short wavelengths, and test theoretical predictions [5,8]. Moreover, we demonstrate the progressive evolution of the low-energy quasiparticle excitation spectrum across the insulator-metal transition with hole doping.

As an experimental system for the undoped 2D Heisenberg model, either $^{17}$O isotope enriched single crystals, uniaxially aligned powder, or partially aligned powder samples of Sr$_2$CuO$_2$Cl$_2$ [3] were used depending on experimental necessities. We determined the bulk Néel temperature as $T_N^{3D} = 257$ K based on $^{13}$Cl NMR measurements, in agreement with Suh et al. [10]. The major advantage of Sr$_2$CuO$_2$Cl$_2$ is that the magnetic anisotropy is so weak ($J_{xy}/J \sim 10^{-4}$ [3]) that the isotropic 2D Heisenberg behavior is robust even down to $\approx 280$ K (see Fig. 1 and [3,10]). For the study of lightly hole doped CuO$_2$ planes, we used $^{17}$O isotope enriched single crystals of La$_{2-x}$Sr$_x$CuO$_4$ with $x = 0.025$ and 0.035 [11]. All $^{17}$O NMR measurements were conducted for clearly resolved NMR transitions with the typical line width 2 to 15 KHz.

Using $^{17}$O NMR, we probed the short wavelength excitations in the CuO$_2$ planes of insulating and weakly metallic high $T_c$ cuprates. We measured the spin wave damping for an $S = 1/2$ 2D quantum Heisenberg antiferromagnet for the first time. The results establish the nearly free behavior (asymptotic freedom) of the high energy spin waves, even without long range magnetic order. Light hole doping dramatically enhances the low energy excitation spectrum below 300 K. [S0031-9007(97)03483-2]

**References**

1. K. R. Thurber, A. W. Hunt, T. Imai, F. C. Chou, and Y. S. Lee
2. Center for Materials Science and Engineering, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
3. (Received 31 July 1996; revised manuscript received 17 March 1997)
4. Using $^{17}$O NMR, we probed the short wavelength excitations in the CuO$_2$ planes of insulating and weakly metallic high $T_c$ cuprates. We measured the spin wave damping for an $S = 1/2$ 2D quantum Heisenberg antiferromagnet for the first time. The results establish the nearly free behavior (asymptotic freedom) of the high energy spin waves, even without long range magnetic order. Light hole doping dramatically enhances the low energy excitation spectrum below 300 K. [S0031-9007(97)03483-2]
In what follows, we will deduce information regarding spin waves and electron-hole pair excitations based on the measurements of the nuclear spin-lattice relaxation rate $1/T_1$. $1/T_1$ is related to $S(q, \omega) = \chi''(q, \omega)/(1 - \exp(-\hbar \omega/k_BT))$, where $\chi''(q, \omega)$ is the imaginary part of the dynamical electron spin susceptibility, as [12]

$$\frac{1}{T_1} = \frac{\gamma_n^2}{\mu_B^2 \hbar} \sum_q \chi''(q, \omega_n),$$

where $\omega_n$ is the NMR frequency, $\gamma_n$ is the nuclear gyromagnetic ratio. The quantization axis of our measurements of $1/T_1$ is along the crystal $c$ axis. $^{17}F(q)$ is the wave vector dependent hyperfine form factor [13]

$$^{17}F(q)^2 = 4(C_{\text{bond}}^2 + C_{\text{perp}}^2) \cos^2(q_a a/2),$$

$$^{63}F(q)^2 = 2[A_{ab} + 2B[\cos(q_x a) + \cos(q_y a)]^2$$

for the planar $^{17}$O [Eq. (2a)] and $^{63}$Cu [Eq. (2b)] sites. The hyperfine interaction tensors, $A_{\alpha}, B$, and $C_{\alpha}$, were determined from measurements of the NMR shift and uniform susceptibility as described below. The subscript, $\alpha$ = bond, $c$, or perp, refers to the direction of the magnetic field, along the Cu-O bond axis, along the crystal $c$ axis, or perpendicular to both the bond and $c$ axis, respectively. Since $S(q, \omega)$ is the space-time Fourier transform of the spin-spin correlation function, and the NMR frequency is very low $(\hbar \omega_n/k_B = 1 \text{ mK})$, $1/T_1$ probes the $q$ summation of the low energy part of the elementary excitation spectrum, or slow spin dynamics. In Fig. 1, we present the uniform spin susceptibility $\chi'(q = 0)$ deduced from the measurements of bulk susceptibility and $^{17}$O NMR shifts [14],

$$^{17}K_{a,\text{chem}} = \frac{2C_a}{N_A \mu_B} \chi'(q = 0) + ^{17}K_{a,\text{chem}}.$$  

$^{17}K_{a,\text{chem}}$ is the temperature independent chemical shift, and determined in YBa$_2$Cu$_3$O$_6$ to be 0.04% for the Cu-O bond axis ($\alpha$ = bond) [15]. At $T/J > 0.35$, we also deduced $\chi'(q = 0)$ from $^{63}$Cu NMR shifts $^{63}K_{ab}$ in the $ab$ plane by matching the results with $^{17}K_{\text{bond}}$ between 425 and 550 K. We also found that the anisotropy of $^{17}K$ in Sr$_2$CuO$_2$Cl$_2$ and La$_{2-x}$Sr$_x$CuO$_4$ (not shown) is very close to that in YBa$_2$Cu$_3$O$_6$ [15]. Applying the standard $K_x$ plot analysis [14] to Sr$_2$CuO$_2$Cl$_2$, we determined $C_{\text{bond}} = 83 \pm 7$, $C_c = 40 \pm 4$, $C_{\text{perp}} = 53 \pm 11 \text{ kOe/\mu_B}$. We also obtained $A_{ab} = 31 \pm 45, A_c = -84 \pm 36, B = 37 \pm 8 \text{ kOe/\mu_B}$, and $^{63}K_{ab,\text{chem}} = 0.27\%$.

In Figs. 1 and 2, we present the results of $^{63}1/(T_1 T)$ and $^{17}1/(T_1 T)$ measured for the $^{63}$Cu and $^{17}$O sites. To facilitate the quantitative comparison possible between Sr$_2$CuO$_2$Cl$_2$ ($J_{2122} = 1450 \text{ K}$ [16]) and Sr-doped La$_2$CuO$_4$ ($J_{214} = 1530 \text{ K}$ [4]) independent of slightly different values of $J$, the values of $1/(T_1 T)$ are multiplied by $J$, because at finite temperatures, $1/(T_1 T) \propto 1/J$ [17]. The most striking feature is that $^{17}1/T_1$ and $^{63}1/T_1$ show completely different dependencies on temperature. Because of the critical slowing down of the long wavelength, antiferromagnetic spin fluctuations at $q = Q$, $^{63}1/T_1$ for the undoped 2D Heisenberg antiferromagnet diverges exponentially toward $T = 0$ ($= T_{N}^{(2)}$) following $^{63}1/T_1 = T^{1.5} \exp(1.13J/T)$ [2,18]. Light doping mildly suppresses the divergent behavior [2]. On the other hand, since $^{17}F(q = 0) = 0$, $^{17}1/T_1$ is insensitive to the long wavelength critical dynamics around $q = Q$ [13], and senses the short wavelength mode ($1/\xi < q < Q$) of $S = 1/2$ Cu spin fluctuations. Also, $^{89}$Y $1/T_1$ as reported by Ohno et al. [19] for YBa$_2$Cu$_3$O$_6$ should be sensitive to the short wavelength spin fluctuations and appears to be qualitatively consistent with this picture over their limited temperature range.

According to the renormalization group analysis, the elementary excitations at short wavelengths in 2D Heisenberg antiferromagnets are spin waves with asymptotic freedom [5]. This property allowed Chakravarty et al. to use a spin wave expansion to calculate $^{17}1/T_1$ at $T_N < T \ll J$ as [18,20]

$$^{17}1/T_1 = \frac{T_a}{3h^2c} \left[1 + C_2 \left(\frac{T}{2\pi p_s}\right) + O(T^2)\right],$$

where $c = \sqrt{2}Z_c J_a/\hbar$ is the spin wave velocity with $Z_c = 1.18$, and $C_2 = -1.88$ [20]. This theoretical prediction is shown in Fig. 2 (solid line) together with a high temperature expansion (dashed line) by Singh and Gelfand [17] and Monte Carlo results (open squares) by Sandvik and Scalapino [21]. These parameter-free theoretical predictions agree well with our data.

Our results of $^{17}1/T_1$ indicate that free spin waves are indeed a good description of the quasiparticle excitations.
at short wavelengths even at $T \approx T_N^{bd}$. Then, we can
deduce the effective thermal damping $\Gamma$ of the spin waves
at finite temperatures from $\chi''(q, \omega)$ as follows. For damped
spin waves, we can express $\chi''(q, \omega)$ as

$$\chi''(q, \omega) = \chi'(q) \left( \frac{\omega \Gamma(q)}{[\omega - \omega(q)]^2 + \Gamma^2(q)} + \frac{\omega \Gamma(q)}{[\omega + \omega(q)]^2 + \Gamma^2(q)} \right), \quad (5)$$

where $\omega(q) = 2Jz_{a} \sqrt{1 - \left[\cos(q_x) + \cos(q_y)\right]^2 / 4}$ is
the spin wave dispersion ($\gg \omega_n$), and $\Gamma(q)$ represents
the spin wave damping [4,8]. The $q$ dependence of $\chi'(q)$ is known analytically [22], and we normalize
the absolute value of $\chi'(q)$ using the results in Fig. 1.
Then the only unknown parameter in Eq. (1) is $\Gamma(q)$.

Monte Carlo results by Makivic and Jarrell [8(c)] and analytic results by Kopietz [8(b)] indicate that
$\Gamma(q)$ shows little dependence on the wave vector $q$
in most of the Brillouin zone except near $q = 0$ and $Q$.
Therefore, we let $\Gamma(q) = \Gamma$ be independent of $q$
except for the regions near $q = 0$ and $Q$, and consider $\Gamma$
as the wave vector averaged damping at short wavelengths.
The $q$ dependence of $\Gamma(q)$ used for the calculation is
shown schematically in the inset to Fig. 3. Near $q = Q$
where the dynamical scaling form of the damping
$\gamma(q) [23]$ satisfies $\gamma(q) < \Gamma$, we let $\Gamma(q) = \gamma(q)$. This
guarantees that the same form of $\chi''(q, \omega)$ used to fit
$\chi''(q, \omega)$ to the low temperature behavior ($T \ll J$) of $\chi''(q, \omega)$
as shown in Fig. 1, consistent with the earlier finding for $\text{La}_2\text{CuO}_4$ [2]. For the region near $q = 0$, we
neglect the contribution from $q < 1/\xi$ [24]. Monte Carlo
calculations by Sandvik and Scalapino [21] indicate that
this contribution is $\approx 25\%$ at $T = 0.5J$ and decreases with
lowering temperature ($\approx 10\%$ at $T = 0.4J$). Including
this contribution would lower our calculated value of the
damping $\Gamma$ by the amount of the contribution (but
within the error bars). Essentially, we are measuring the
damping $\Gamma$ by the fact that the size of the tail of $\chi''(q, \omega)$
at $\omega = \omega_n \approx 0$ is determined by the energy width $\Gamma$. For
the short wavelength region, the contribution to $\chi''(q, \omega)$
is $\approx \Gamma$. We note that the deviation of $\chi''(q, \omega)$ from
the Lorentzian form of Eq. (5) can change our estimate of
the magnitude of $\Gamma$ [25], but this does not affect
the conclusions of this paper.

We plot the temperature dependence of $\Gamma$ deduced from
Eq. (5) in Fig. 3. We emphasize two key findings. First, the
damping $\Gamma$ is smaller than the highest excitation energy
$\omega(q) = 0.3$ eV [4] of the short wavelength magnons
in the entire temperature range we studied. This observation
establishes that magnons are well defined (long life-
time) elementary excitations for short wavelengths even
in the paramagnetic state up to $T = 0.4J$ in $S = 1/2$ 2D
quantum Heisenberg antiferromagnets. Second, the value
of $\Gamma$ is in good agreement with theoretical predictions
based on low temperature analytic calculations, $\hbar \Gamma \sim
3J(T/J)^3$ [8(b)], and high temperature Monte Carlo
simulations [8(c)].

Next, we would like to address another fundamental
question: how do the elementary excitations evolve when
we dope holes into the CuO$_2$ planes and transform
the system to metallic behavior? The measurement of
$1/\chi''(q, \omega)$ is best suited to answering this question,
because $^1_7$O NMR is efficient in probing the electron-
hole pair excitations at the Fermi surface, while $63^{1}/_1 T_1$
mon is dominated by the divergently large, long wavelength
collective spin dynamics at $q = Q$. The results of
$1/\chi''(q, \omega)$ in hole doped $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$ are compared with the results of undoped $\text{Sr}_2\text{CuO}_2\text{Cl}_2$ in Fig. 2. At
these doping levels, previous studies established that the
resistivity is linear in temperature down to $T_{\text{loc}} < 300 \text{ K}$
[26,27], where holes begin to localize, followed by the
spin-glass transition at $T_{sg} \approx 10 \text{ K}$ [28]. We identified
$T_{\text{loc}}$ as $250 \text{ K}$ from the onset of the dramatic increase of
$1/\chi''(q, \omega)$ at the apical oxygen sites. At $T_{sg}$, $1/\chi''(q, \omega)$
diverged (see Fig. 2 for the results observed for $x = 0.035$).
In what follows, we will focus our attention on the
temperature dependence of $1/\chi''(q, \omega)$ at the planar
site above $T_{\text{loc}}$, where carriers are mobile and resistivity
is roughly proportional to temperature.

The temperature dependence of $1/\chi''(q, \omega)$ is surpris-
ningly similar for the undoped and lightly hole doped
samples, but with a weakly temperature dependent
increase of $1/\chi''(q, \omega)$ for the doped samples. We estimate
this increase as $1/\chi''(q, \omega) \approx 0.05 - 0.065 \text{ sec}^{-1}\text{K}^{-1}$
and $0.06 - 0.085 \text{ sec}^{-1}\text{K}^{-1}$ for $x = 0.025$ and $0.035$, respectively. It is also interesting to recall that Reven
et al. observed $1/\chi''(q, \omega) \approx 0.4 \text{ sec}^{-1}\text{K}^{-1}$ at $T \leq 300 \text{ K}$
in optimally doped $\text{La}_{1.85}\text{Sr}_{0.15}\text{CuO}_4$ [29]. The increase in
$1/\chi''(q, \omega)$ appears to be proportional to the amount of hole
doping $x$. Angle resolved photo emission experiments for
undoped $\text{Sr}_2\text{CuO}_2\text{Cl}_2$ by Wells et al. show that the highest
occupied band is peaked at $q = (\pm \pi/2a, \pm \pi/2a) = Q/2$
with a bandwidth $(2.2 \pm 0.5)J$ [30]. Therefore, the simplest interpretation is that the electron-hole pair (Stoner) excitations with wave vectors, $(0, 0), (\pm \pi/a, 0), (0, \mp \pi/a), (\pm \pi/a, \pm \pi/a)$, connecting the hole pockets at $\mathbf{Q}/2$ give rise to an additional contribution to $171/(T_1 T)$ without changing the spin wave contribution. However, our calculations based on the rigid band picture showed that the Stoner continuum is very close to the spin wave dispersion over most of the Brillouin zone except near $q = (\pm \pi/a, 0)$ and $(0, \pm \pi/a)$ because the width of the spin wave dispersion $= 2.36J$ is within experimental error equal to the bandwidth. The conventional wisdom for the spin waves in metals is that the electron-hole pair excitations damp the spin waves if the dispersion of the spin wave merges into the Stoner continuum [31]. This suggests that the magnons and the electron-hole pair excitations will interact strongly, perhaps resulting in complete renormalization of the damping, $\Gamma(q)$. We note that the temperature $T$ and doping $x$ dependence of $171/(T_1 T)$ is quite similar to that of $1/\xi(T, x)$ observed by Keimer et al. [27]. Knowing that $171/(T_1 T) \times 8 = \Gamma(T, x)$, this is related to $\xi(T, x)$ as $\Gamma(T, x) \propto c/\xi(T, x)$.

To summarize, we deduced the effective damping $\Gamma$ of the short wavelength magnons of the $S = 1/2$ 2D Heisenberg antiferromagnet in a broad range of temperature $(0.2 \leq T/J \leq 0.5)$, contrary to the prevailing perception in the community that $\Gamma$ was not measurable with current technology. Our results establish the asymptotic freedom of paramagnetic spin waves in 2D, and the temperature dependence of the damping. The low-energy excitations in the hole doped, weakly metallic CuO$_2$ planes show a similar temperature dependence to the undoped sample, but with a weakly temperature dependent increase from the addition of electron-hole pair excitations. We suggest that the spin waves may interact strongly with electron-hole pair excitations.

We are indebted to P. Kopietz and A. Sandvik for communicating unpublished work with us, S. Chakravarty, M. Gelfand, A. Sokol, A. Chubukov, Q.M. Shi, S. Sachdev, P.A. Lee, X.G. Wen, M.A. Kastner, T. Gresytak, R.J. Birgeneau, and M. Greven for their helpful discussions, and D. Cory for loaning us his magnet. This work was supported by NSF DMR 96-23858, NSF DMR 94-00334, and in part by the Mitsui foundation. T. I. is an A.P. Sloan Research Fellow.

[24] In the limit of $q = 0$, Eq. (5) is not valid, instead we expect a diffusive contribution, $171/T_{1, \text{diff}} \propto (1/D(T))\ln(1/\omega_n)$ [18]. However, Chakravarty et al. predicted that this contribution is small because the diffusion constant $D(T)$ diverges exponentially at low temperatures [5]. Experimentally, the value of $171/T_1$ for both Sr$_2$CuO$_2$Cl$_2$ and La$_{2-x}$Sr$_x$CuO$_4$ ($x = 0.035$) at room temperature was the same at 14.1 $T$ ($\omega_n = 81.4$ MHz), and 9 $T$ ($\omega_n = 52.0$ MHz), indicating no diffusive enhancement at 9 $T$.
[25] For example, using a squared Lorentzian form for $\chi^0(q, \omega)$ barely changes the half-width $\Gamma$ of $\chi^0(q, \omega)$ for $T \approx 500$ $K$, though $\Gamma$ increases up to a factor of $\sim 3$ at the lowest temperatures.