McMaster University's Physics@Mac Online Physics Competition
December 6, 2017

General Statistics:

In this tenth competition, 756 teams from 102 different schools participated.

Eight teams had a perfect score of 10. The average score was 5.15.

Students competed in two categories – Grade 11 or below, and Grade 12. Where there was a tie score, prize winners were determined by elapsed times. Almost every team completed the test in under 75 minutes and most finished in less than 60 minutes.

Cash prizes of $100 per team were awarded to the top three teams in Grade 12 and the top three teams in Grade 11 (or below). Certificates of Honourable Mention were awarded to teams in Grade 12 who achieved a score of at least 8 and teams in Grade 11 (or below) who achieved a score of at least 7.

Answer, success rate, statistics and solution for each question:

Question 1: Answer: D – 54.9% correct (A: 33.6%, B: 3.3%, C: 8.2%)

The slope of a displacement-time graph at any instance in time (slope of a tangent line to the curve) gives the instantaneous velocity at that time. In this plot, the slope of one train's d-t curve is constant, meaning this train travels at a constant velocity for the whole trip. Since the velocity is constant, this train also has no acceleration. The other train's d-t curve does have a changing instantaneous slope, therefore this train's velocity is changing over time (the slope is decreasing, meaning it is slowing down). However, we can see that at some moment in time, the instantaneous slope will be the same as the constant slope of the first train. At this moment in time (approximately at $t = m/2$), the velocity of both trains are the same. The overall plot shows displacement versus time, and both trains start at the same initial position ($x = 0$). Therefore, since both curves have the same x position at point B (and time m), the two trains meet at point B. Using these arguments, we can also eliminate all the other possible answers.

Question 2: Answer: C – 56.8% correct (A: 12.1%, B: 28.3%, D: 2.8%)

The surface is frictionless and both objects are travelling with equal speeds. Therefore, we can say that gravity and normal force are the only forces acting on each object, and that energy is conserved. For each object, its initial kinetic energy is entirely converted to gravitational potential energy at its maximum height up the hill. Therefore, we have:

$$K = U_g = \tfrac{1}{2}mv^2 = mgh \quad \rightarrow \quad h = v^2/2g$$

We can see that the final height h of either object only depends on the object's speed which is the same for both objects. Therefore they both slide the same height up the hill.

Question 3: Answer: B – 51.4% correct (A: 6.3%, C: 31%, D: 11.3%)

Let's guess that there is about 1 mm of distance between each human hair so that there is one hair per every 1 mm x 1 mm: Surface Area $A \approx 0.001$ m * 0.001 m $= 10^{-6}$ m^2. If we approximate a human head as a sphere with radius $r \approx 0.1$ m, and say that about half of one's head is covered with hair, the total hair-containing surface area is
$A_{tot} \approx 0.5*4\pi r^2 = 0.5*4\pi(0.1 \text{ m})^2 \approx 0.1$ m^2.
The total number of hairs on a person's head would therefore be
$A/A_{tot} = 10^{-6}$ m^2/10^{-1} m$^2 = 10^5$.
If we assume each hair to be about 10 cm in length, the total length of all the hairs laid end to end in a straight line is 0.1 m * 10^5 hairs $= 10^4$ m $= 10$ km.

Question 4: Answer C – 62.5% correct (A: 16.6%, B: 16.3%, D: 4.6%)

The focal point of a lens is the point where rays parallel to the axis of the lens meet after reflection or refraction. Therefore, it doesn't matter where a light ray strikes the lens – by definition, the light rays will always be focused at the focal point.

Question 5: Answer B – 51.4% correct (A: 14%, C: 8.4%, D: 26.2%)

First consider the circuit with the switch open. In this case, we have a simple series circuit with two identical bulbs – let's say they each have a resistance R. The battery creates a potential difference in the circuit, ε. To find the current I in this circuit, we combine the two resistors (the two bulbs) to find the effective resistance: $R_T = R_A + R_B = 2R$. Using Ohm's Law ($V = IR$), $\varepsilon = I(2R)$. Therefore $I = \varepsilon/2R$. This is the current that passes through bulb A and bulb B when the switch is open.

When the switch is closed, we have a circuit with three total bulbs and two are connected in parallel. All the bulbs are the same still so again they each have resistance R. We want to use resistor rules to combine the three resistors into one effective resistor. The two bulbs in parallel have total resistance $R_p = (1/R + 1/R)^{-1} = R/2$. Next we combine the resistance R_p and the resistance of bulb A: $R_T = R_A + R_p = R + R/2 = 3R/2$. Using Ohm's Law, $\varepsilon = I(3R/2)$. Therefore $I = 2\varepsilon/3R$. This is the total current in the circuit, and is the current that passes through bulb A. Going back to the original closed circuit (all 3 bulbs), after passing through bulb A, the current must split between bulb B and the unnamed bulb. The current divides evenly between the two bulbs since their resistances are equal. This means that the current through bulb B (and the unnamed bulb) is $I = \varepsilon/3R$.

Comparing the currents through bulbs A and B before and after the switch is closed, we have: Before: $I_A = I_B = \varepsilon/2R$. After: $I_A = 2\varepsilon/3R$; $I_B = \varepsilon/3R$.

The current through bulb A is greater when the switch is closed versus when it is open. The current through bulb B is smaller when the switch is closed versus when it is open. Therefore b) is the only correct answer.

Question 6: Answer D – 27.6% correct (A: 22.6%, B: 28.7%, C: 21.1%)

The two pennies are thrown with different velocities, which means they can each have any speed and any direction. If we couple this with the fact that there is a time delay between the throws, we can come up with a scenario where each of a) to c) does not hold. For example, imagine throwing the first penny straight up and then throwing the second penny straight up just as the first penny reaches its maximum height due to the initial throw. The second penny will be moving up and the first penny will be moving down, so the vertical distance between them is initially decreasing. This scenario shows that a) and b) cannot always be true. We can debunk c) by imagining that we throw the first penny with no horizontal velocity and the second penny with some horizontal velocity.

Question 7: Answer: B – 57.9% correct (A: 31.3%, C: 6.3%, D: 4.5%)

Perfectly inelastic means that energy is not conserved and the two carts stick together after colliding. Perfectly elastic means that energy is conserved and the colliding carts do not stick together. We calculate the initial and final momentum of each collision starting with the first collision, knowing that momentum is always conserved, and $\boldsymbol{p} = m\boldsymbol{v}$.

1st collision: $\boldsymbol{p_i} = 2m\boldsymbol{v_0}$, $\boldsymbol{p_f} = (2m + 2m)\boldsymbol{v_1}$; $\boldsymbol{p_i} = \boldsymbol{p_f}$; $2m\boldsymbol{v_0} = 4m\boldsymbol{v_1}$
➔ $\boldsymbol{v_1} = \boldsymbol{v_0}/2$ This is the velocity that carts 1 and 2 have as they move together.

2nd collision: $\boldsymbol{p_i} = 4m(\boldsymbol{v_0}/2)$, $\boldsymbol{p_f} = (4m)\boldsymbol{v_{cart\ 3}} + 4m\boldsymbol{v_{cart\ 1+2}}$; $\boldsymbol{p_i} = \boldsymbol{p_f}$;
$4m(\boldsymbol{v_0}/2) = (4m)\boldsymbol{v_{cart\ 3}} + 4m\boldsymbol{v_{cart\ 1+2}}$. We can simplify and rearrange this equation to get $\boldsymbol{v_{cart\ 1+2}} = \boldsymbol{v_0}/2 - \boldsymbol{v_{cart\ 3}}$ (Eq. 1). We want to solve for $\boldsymbol{v_{cart\ 3}}$. To do this we need another equation which we can get by knowing that the collision is perfectly elastic: the kinetic energies before and after the collision are equal, and $K = \frac{1}{2}mv^2$, so we have $\frac{1}{2}(4m)(\boldsymbol{v_0}/2)^2 = \frac{1}{2}(4m)(\boldsymbol{v_{cart\ 3}})^2 + \frac{1}{2}(4m)(\boldsymbol{v_{cart\ 1+2}})^2$. Simplifying, $\boldsymbol{v_0}^2/4 = (\boldsymbol{v_{cart\ 3}})^2 + (\boldsymbol{v_{cart\ 1+2}})^2$. Finally, we insert Eq. 1 into this equation. After rearranging and simplifying we find that $\boldsymbol{v_{cart\ 3}} = \boldsymbol{v_0}/2$.

3rd collision: We can use the same argument for this collision, because the collision is also perfectly elastic and involves the same masses and same initial speed. Therefore, $\boldsymbol{v_{cart\ 4}} = \boldsymbol{v_0}/2$.

Question 8: Answer: D – 43.9% correct (A: 17.7%, B: 10%, C: 28.4%)

Initially, Physics Girl and the cart (combined mass M) have gravitational potential energy $U_g = mgh = Mg(2h/3)$. To reach the top of the hill, they would need to have a total energy of Mgh. Therefore, they need to gain an additional $Mgh - Mg(2h/3) = Mgh/3$ of spring energy in order to have the necessary total energy to make it back to the top of the hill. The track is frictionless so energy is conserved.

Question 9: Answer: B – 44% correct (A: 24.6%, C: 2.8%, D: 28.6%)

We first want to find the acceleration of the system. We know that both boxes slide together. Using Newton's 2nd Law ($\boldsymbol{F} = m\boldsymbol{a}$) and only looking at magnitudes, $\boldsymbol{F_L} = (3m)\boldsymbol{a} \rightarrow |\boldsymbol{a}| = F_L/3m$. The same is true when $\boldsymbol{F_R}$ is applied: $|\boldsymbol{a}| = F_R/3m$.

Next, let's draw a free body diagram for the box with mass m when $\boldsymbol{F_L}$ is being applied.

$$F_{spring} \longleftarrow \bullet \longrightarrow F_L$$
$$m$$

Writing the equation of motion from $\boldsymbol{F} = m\boldsymbol{a}$, and noting that $F_{spring} = kx_L$, we get $\boldsymbol{F_L} - kx_L = m\boldsymbol{a}$. Inserting $\boldsymbol{F_L} = (3m)\boldsymbol{a}$ and rearranging for x_L, we find $x_L = 2ma/k$.

We then do the same thing for the box with mass $2m$, but this time with the force from the right side $\boldsymbol{F_R}$ so that we can draw a similar FBD for this $2m$ box.

$$F_R \longleftarrow \bullet \longrightarrow F_{spring}$$
$$2m$$

Now $F_{spring} = kx_R$, and we have $\boldsymbol{F_R} - kx_R = (2m)\boldsymbol{a}$ as our equation of motion. Inserting $\boldsymbol{F_R} = (3m)\boldsymbol{a}$ and rearranging for x_R, we find $x_R = ma/k$.

To complete the question, we want x_L/x_R:
$x_L/x_R = (2ma/k)/(ma/k) = 2/1 = 2$.

Question 10: Answer D – 69.4% correct (A: 28.8%, B: 0.7%, C: 1.1%)

Since the car is not moving, it has zero acceleration. Therefore, all of the forces on the car must be balanced to sum to zero. The rope is applying a tension force to the car but the car is not moving, therefore there must be another force or forces that are counteracting this tension to make the net force on the car zero. This means d) is the only correct answer. The rope is applying an external tension force on the car, so the inertia argument does not hold.