Construction of a Non-Fermi Liquid Ground State

(Condensed Matter Journal Club Presentation)

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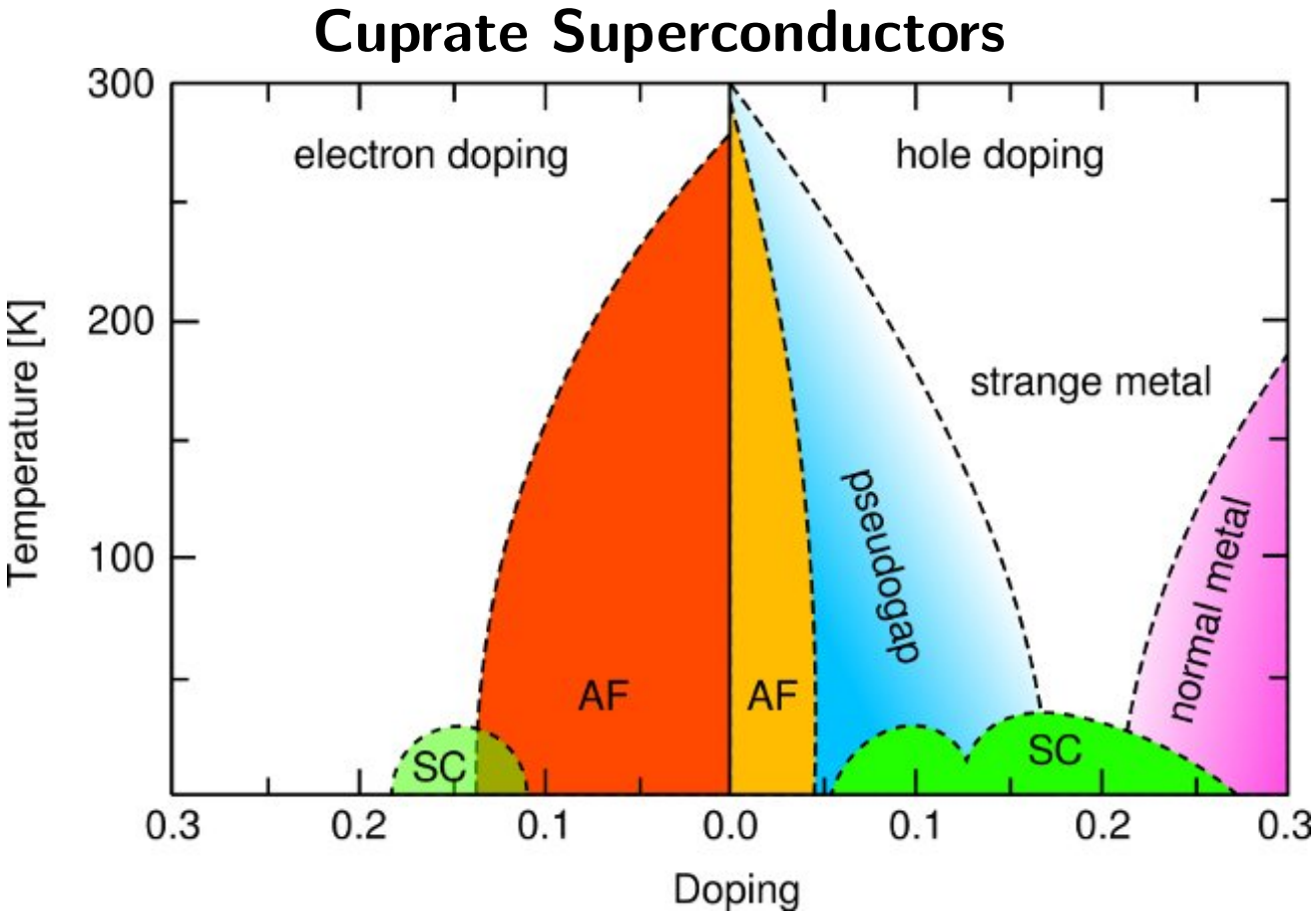
Outline

- **Motivation and Prelimanaries**
- **Non-Fermi liquid (d-wave Metal) of Jiang et. al.**
- **Motrunich and Fisher's d-wave Bose Metal**
- **Numerical signatures in quasi-1D systems**
- **Comments & Discussion**

Motivation

Understanding **Strange Metal Phase (non-Fermi Liquids)**

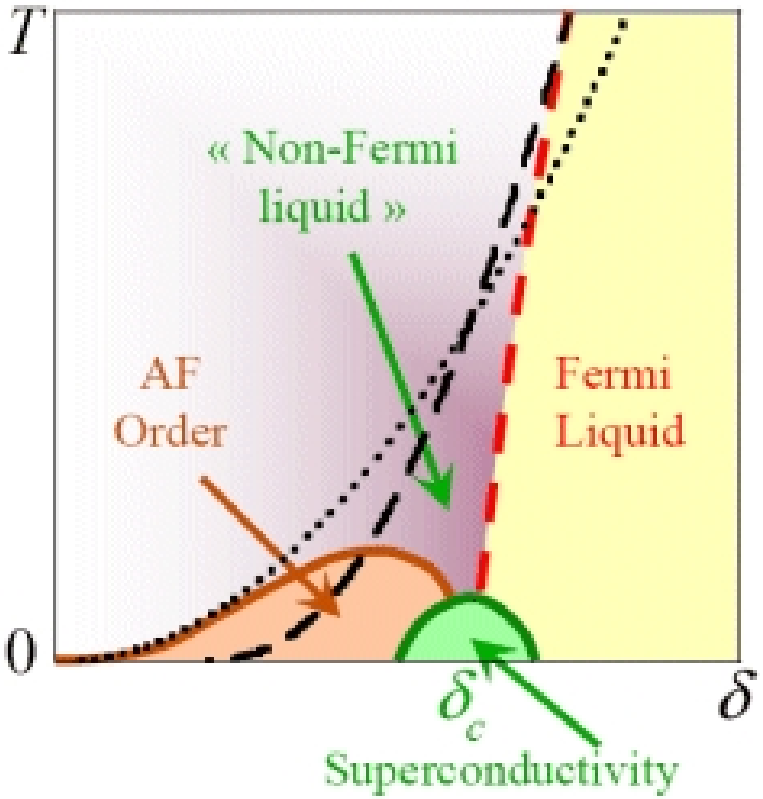


Taken from: **http://www.physik.uni-augsburg.de/∼grasersi/**

Motivation

Understanding **Strange Metal Phase (non-Fermi Liquids)**

Heavy Fermions

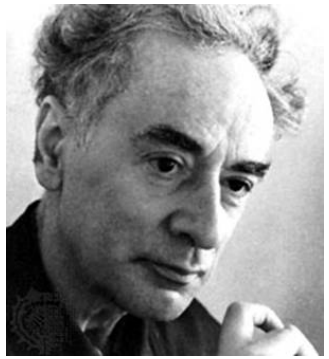


Taken from: **http://www.toulouse.lncmi.cnrs.fr**

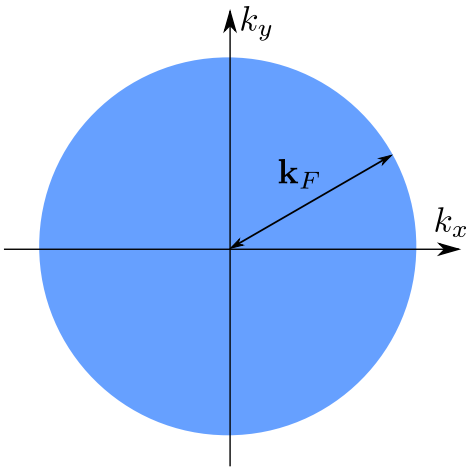
Preliminaries - **Landau-Fermi Liquid Theory**

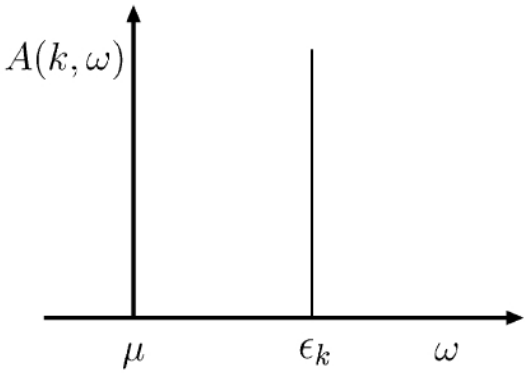
Non-interacting electrons

$$\mathcal{H}_0 = \sum_{\substack{i,j \\ \sigma}} \left[t_{ij}\, c^\dagger_{i,\sigma} c_{j,\sigma} + \text{h.c.} \right] = \sum_{\mathbf{k}} \xi_{\mathbf{k}} c^\dagger_{\mathbf{k},\sigma} c_{\mathbf{k},\sigma}$$



Lev Landau





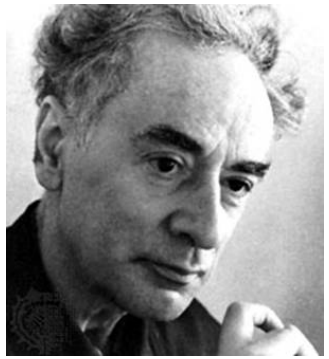
$$\mathcal{G}^0(\mathbf{k},t) = -i\Big[\theta\big(\xi_{\mathbf{k}}\big)\theta(t) - \theta\big(-\xi_{\mathbf{k}}\big)\theta(-t)\Big] e^{-it\xi_{\mathbf{k}}}$$

$$\mathcal{G}^0(\mathbf{k},\omega) = \frac{1}{\omega - \xi_{\mathbf{k}} + i0^+ \text{sgn}(\omega)}$$

Preliminaries - **Landau-Fermi Liquid Theory**

Interacting electrons (D≥2)

$$\mathcal{H} = \sum_{\substack{i,j \\ \sigma}} \left[t_{ij}\, c^{\dagger}_{i,\sigma} c_{j,\sigma} + \text{h.c.} \right] + \mathcal{H}_{\text{int}}$$



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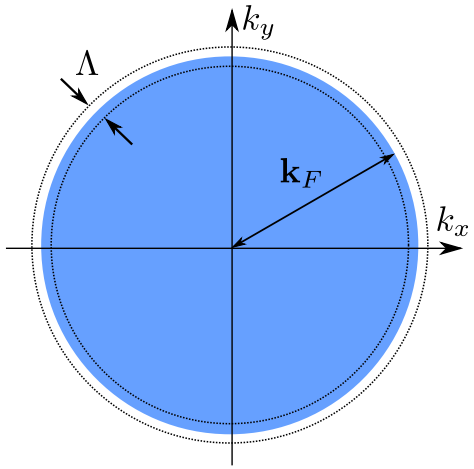
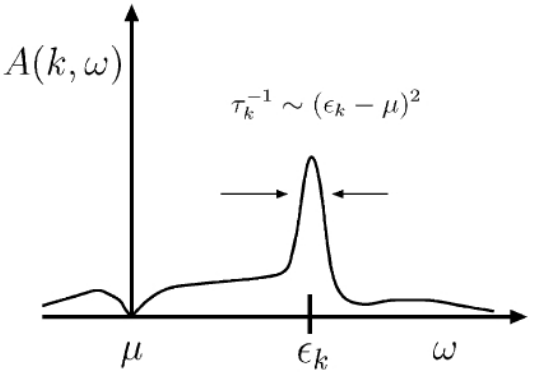




$$\mathcal{G}(\mathbf{k},\omega) = \mathcal{G}_{\text{coh}}(\mathbf{k},\omega) + \mathcal{G}_{\text{incoh}}(\mathbf{k},\omega)$$

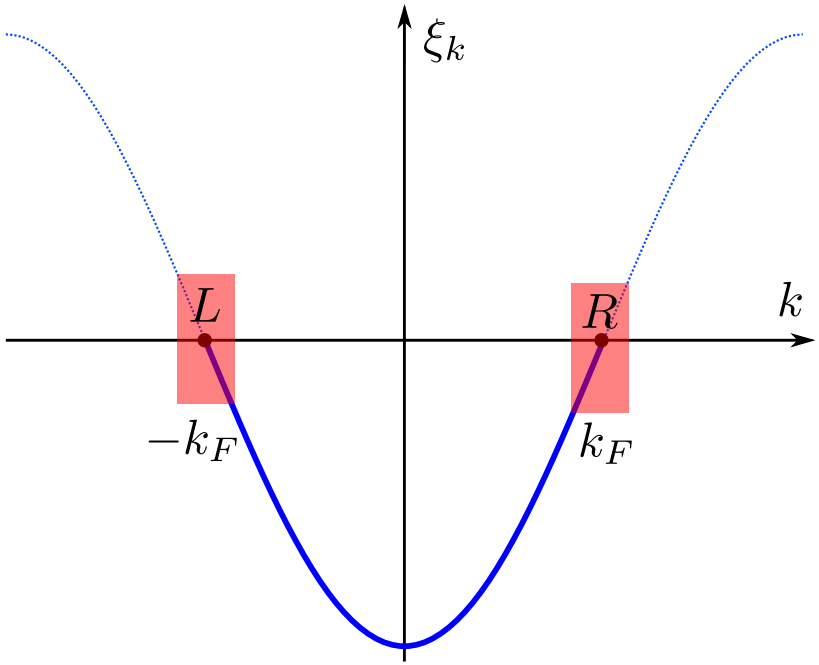
$$\mathcal{G}_{\text{coh}}(\mathbf{k},\omega) = \frac{Z_{\mathbf{k}}}{\omega - \xi_{\mathbf{k}} + \frac{i}{\tau_{\mathbf{k}}}}$$

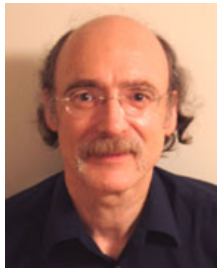
Preliminaries - **Tomonaga-Luttinger Theory**

Hubbard model in 1D

$$\mathcal{H} = t \sum_{i,\sigma} \left[c^{\dagger}_{i,\sigma} c_{i+1,\sigma} + \text{h.c.} \right] + U \sum_{i} c^{\dagger}_{i,\uparrow} c_{i,\uparrow} c^{\dagger}_{i,\downarrow} c_{i,\downarrow}$$

Non-interacting part:





Duncan Haldane

Sin-Itiro Tomonaga

J. M. Luttinger

Preliminaries - **Tomonaga-Luttinger Theory**

Scattering channels:

$$\mathcal{H}_1 = \nu g_1 \left[\psi^\dagger_{R,\sigma} \psi_{L,\sigma} \psi^\dagger_{L,-\sigma} \psi_{R,\sigma} \right]$$

$$\mathcal{H}_2^c = \nu g_2^c \left(J_{R,\uparrow} + J_{R,\downarrow} \right) \left(J_{L,\uparrow} + J_{L,\downarrow} \right)$$

$$\mathcal{H}_2^s = \nu g_2^s \left(J_{R,\uparrow} - J_{R,\downarrow} \right) \left(J_{L,\uparrow} - J_{L,\downarrow} \right)$$

$$\mathcal{H}_3 = \nu \frac{g_3}{2} \left[\psi^\dagger_{R,\sigma} \psi^\dagger_{R,-\sigma} \psi_{L,\sigma} \psi_{L,-\sigma} + \text{h.c.} \right]$$

$$\mathcal{H}_4^c = \nu \frac{g_4^c}{2} \left[\left(J_{R,\uparrow} + J_{R,\downarrow} \right)^2 + \left(J_{L,\uparrow} + J_{L,\downarrow} \right)^2 \right]$$

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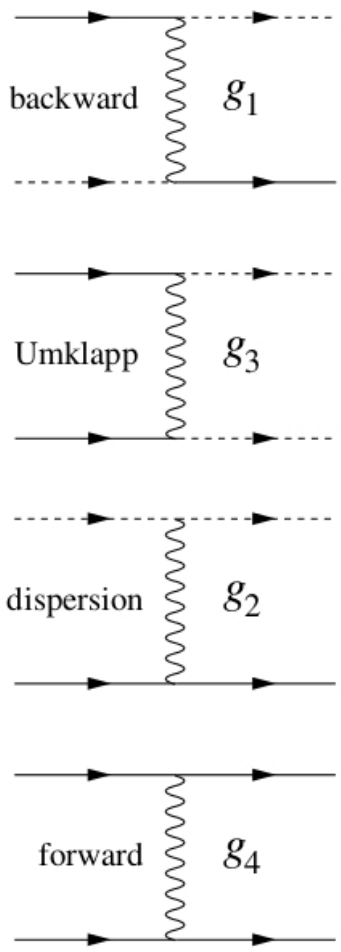
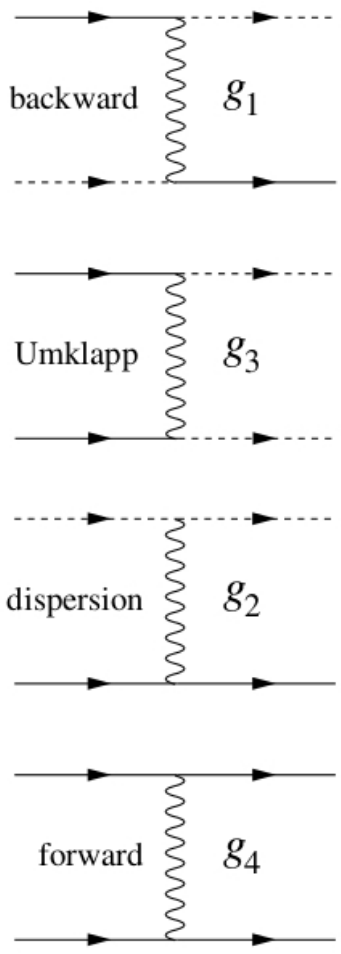
$$J = \psi^\dagger \psi$$



Preliminaries - **Tomonaga-Luttinger Theory**

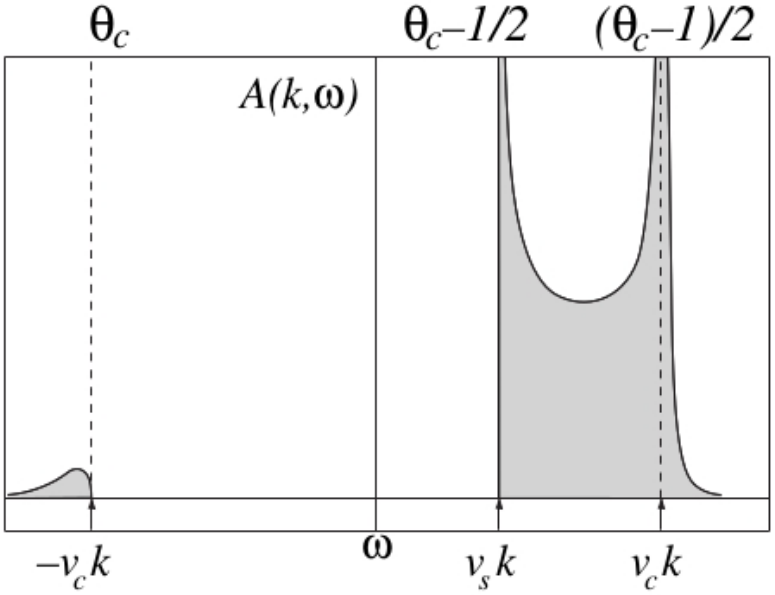
- **Backscattering** term can gap the **spin** sector
- **Umklapp** term can gap the **charge** sector
- Tomonaga-Luttinger model:

$$\mathcal{H}_{\mathsf{TL}} = \mathcal{H}_0 + \mathcal{H}_2^c + \mathcal{H}_2^s + \mathcal{H}_4^c + \mathcal{H}_4^s$$



Preliminaries - **Tomonaga-Luttinger Theory**

Separation of **spin** and **charge** in 1D.



$$\theta_c = \frac{1}{4}\left(K_c + \frac{1}{K_c} - 2\right) \quad , \quad K_c = \sqrt{\frac{\pi - g_2^c + g_4^c}{\pi + g_2^c + g_4^c}}$$

$$\nu_\mu = \nu\sqrt{\left(1 + \frac{g_4^\mu}{\pi}\right)^2 - \left(\frac{g_2^\mu}{\pi}\right)^2}$$

Preliminaries - Morals

Luttinger liquids and **Fermi liquids**, although different in many aspects, are similar in one feature:

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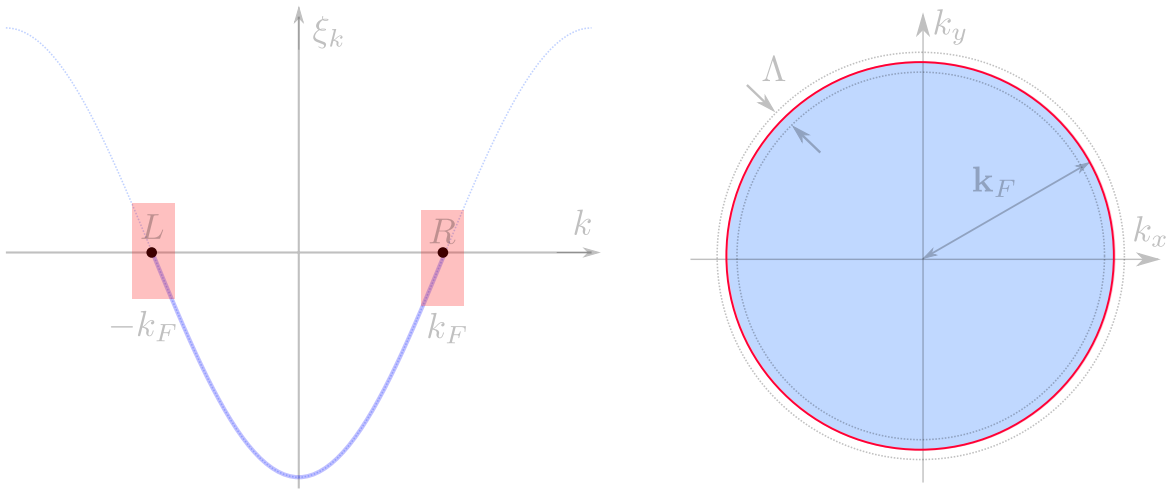
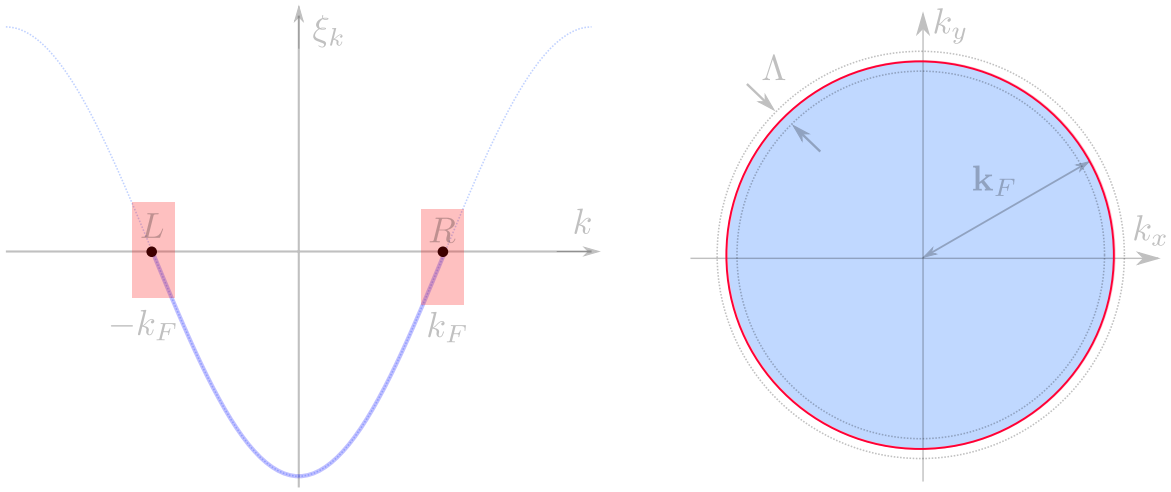


Preliminaries - Morals

Luttinger liquids and **Fermi liquids**, although different in many aspects, are similar in one feature: **Luttinger Theorem** (volume enclosed by the Fermi surface is determined solely by the electron density)



presence of strong interactions is not **vital** for LLs and FLs.

d-wave Metal of Jiang *et. al.*

Slave-Boson trick (break up electron operator into a bosonic **chargon** and a fermionic **spinon**):

$$c_\sigma(\mathbf{r}) \;=\; b(\mathbf{r})\, f_\sigma(\mathbf{r}) \quad , \quad b^\dagger(\mathbf{r})b(\mathbf{r}) = \sum_\sigma f_\sigma^\dagger(\mathbf{r}) f_\sigma(\mathbf{r}) = \sum_\sigma c_\sigma^\dagger(\mathbf{r}) c_\sigma(\mathbf{r})$$

Assuming that spinons are in a Fermi sea:

- chargons bose condense ($\langle b(\mathbf{r})\rangle \neq 0$) $\rightsquigarrow$ a FL
- chargons **do not** condense but still **conduct** $\rightsquigarrow$ a non-FL

One needs a "**Bose metal**" to construct a nFL. [1]

[1] **H.C. Jiang, M. Block, R. Mishmash, J. Garrison, D.N. Sheng, O. Motrunich and M. P.A. Fisher, "Non-Fermi d-wave phases of strongly interacting electrons"**, *arXiv 1207.6608* **(2012)**

d-wave Bose Metal (DBL)

Motrunich and Fisher's DBL: [2]

- **TR invariant** analogue of $\nu = \frac{1}{2}$ Laughlin wave function:

$$\Psi(z_1, z_2, ...z_N) = \prod_{i<j} (z_i - z_j)^2 \quad , \quad z = x + iy$$

- **d-wave** in the sense that relative single particle wave function, upon close approach to a specific particle looks:

$$\Phi_{z_2, z_3, ...z_N}(z) \sim (z - z_i)^2$$

- Boson operator is composed of two **fermionic partons**:

$$b^\dagger(\mathbf{r}) = d_1^\dagger(\mathbf{r})\, d_2^\dagger(\mathbf{r})$$

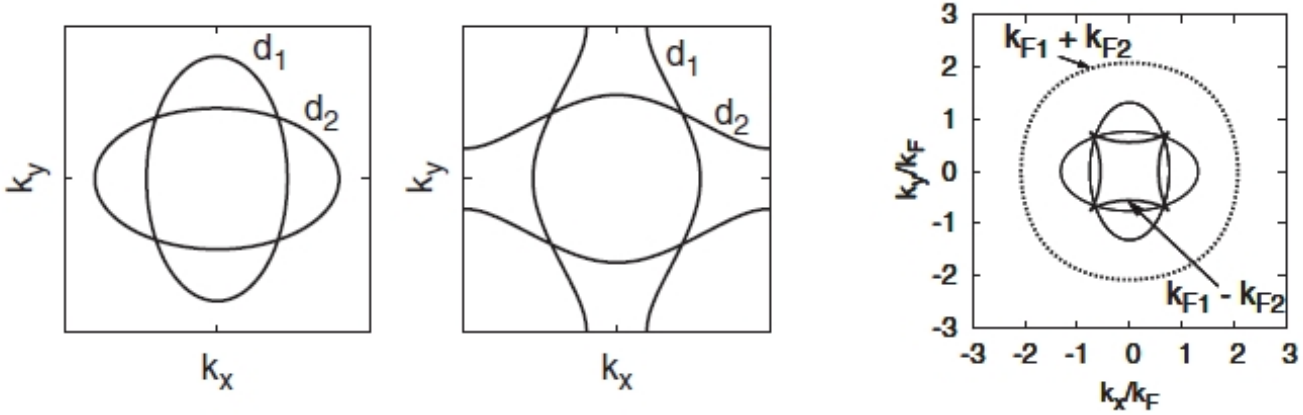
[2] O. I. Motrunich, and M. P. A. Fisher, "d-wave correlated critical Bose liquids in two dimensions", *Phys. Rev. B 75, 235116*, **(2007)**

d-wave Bose Metal (DBL)

Motrunich and Fisher's DBL: [2]

- Fermionic partons with identical Fermi surfaces leadsto a s-wave.
- MFT gives:

$$\mathcal{G}_b^{\mathsf{MF}}(\mathbf{r}, t=0) \sim \frac{\cos[(\mathbf{k}_{F_1} - \mathbf{k}_{F_2}) \cdot \mathbf{r}]}{\sqrt{c_1\, c_2}|\mathbf{r}|^3} + \frac{\cos[(\mathbf{k}_{F_1} + \mathbf{k}_{F_2}) \cdot \mathbf{r}] - \frac{3\pi}{2}}{\sqrt{c_1\, c_2}|\mathbf{r}|^3}$$



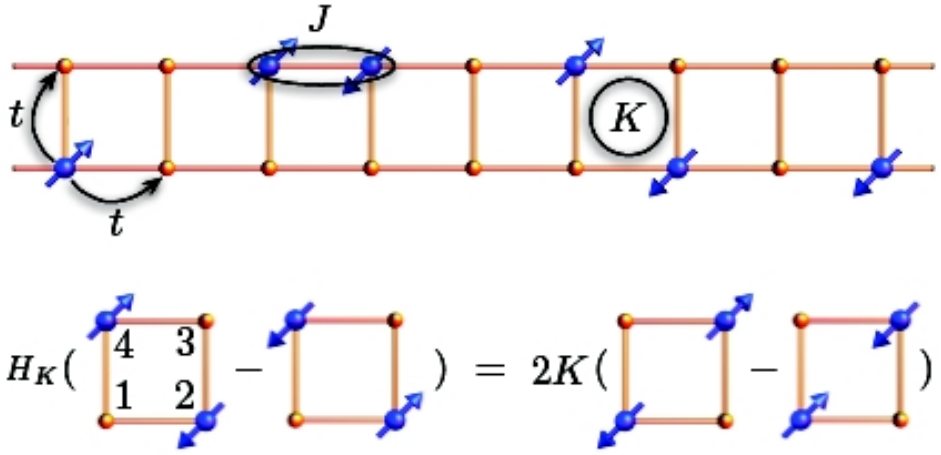
[2] O. I. Motrunich, and M. P. A. Fisher, "d-wave correlated critical Bose liquids in two dimensions", *Phys. Rev. B 75, 235116*, **(2007)**

Microscopic Model Hamiltonian

$$
\begin{aligned}
\mathcal{H} &= \mathcal{H}_{tJ} + \mathcal{H}_K \\
\mathcal{H}_{tJ} &= -t \sum_{\substack{<i,j> \\ \sigma}} \left(c^\dagger_{i,\sigma} c_{j,\sigma} + \text{h.c.} \right) + J \sum_{<i,j>} \mathbf{S}_i \cdot \mathbf{S}_j \\
\mathcal{H}_K &= 2K \sum_{\square} \left(\mathcal{S}^\dagger_{13} \mathcal{S}_{24} + \text{h.c.} \right)
\end{aligned}
$$

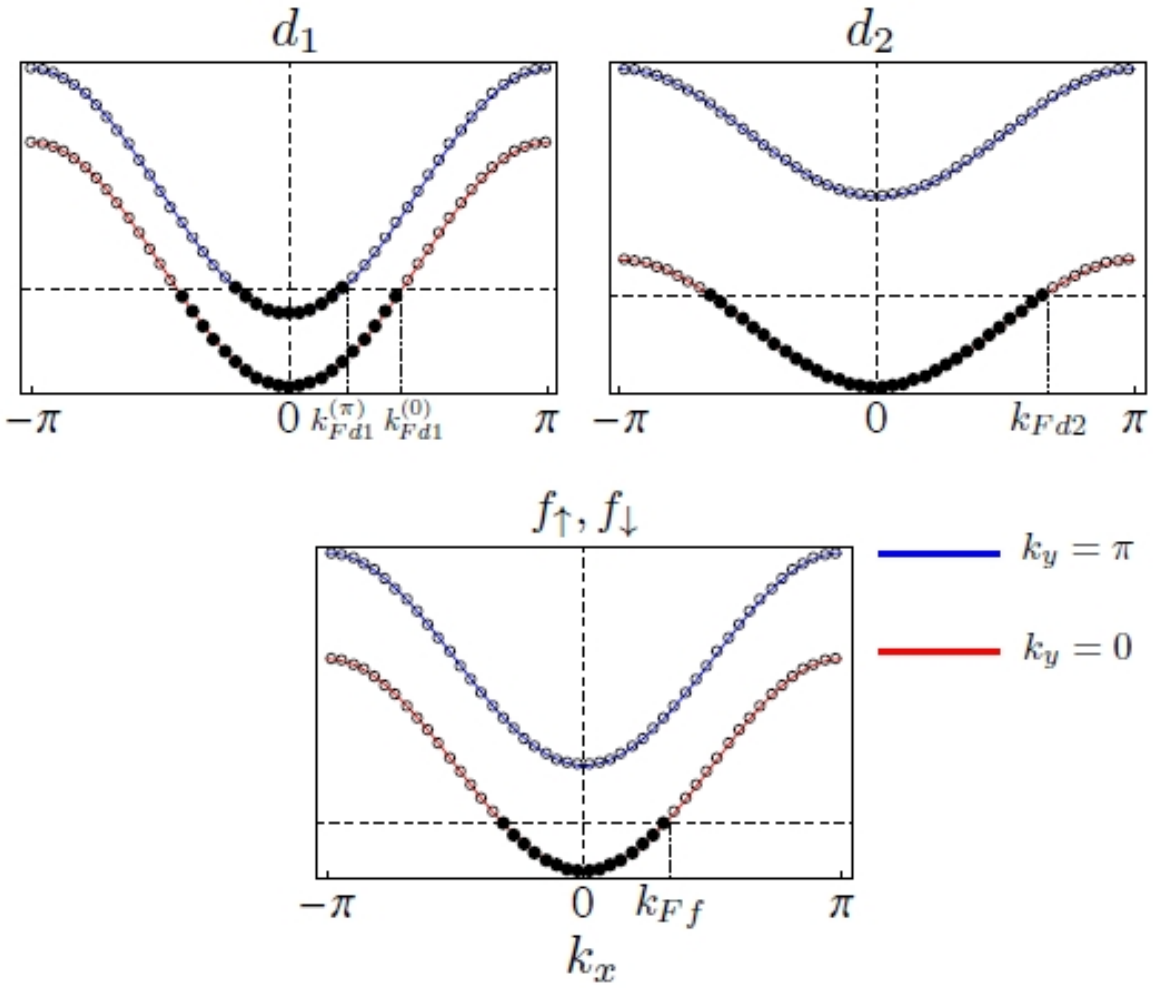
where,

$$
\mathbf{S}_i = c^\dagger_{i,\alpha} \frac{\sigma_{\alpha,\beta}}{2} c_{i,\beta} \quad , \quad \mathcal{S}^\dagger_{ij} = \frac{1}{\sqrt{2}} \left(c^\dagger_{i,\uparrow} c^\dagger_{j,\downarrow} - c^\dagger_{i,\downarrow} c^\dagger_{j,\uparrow} \right)
$$



Numerical Results

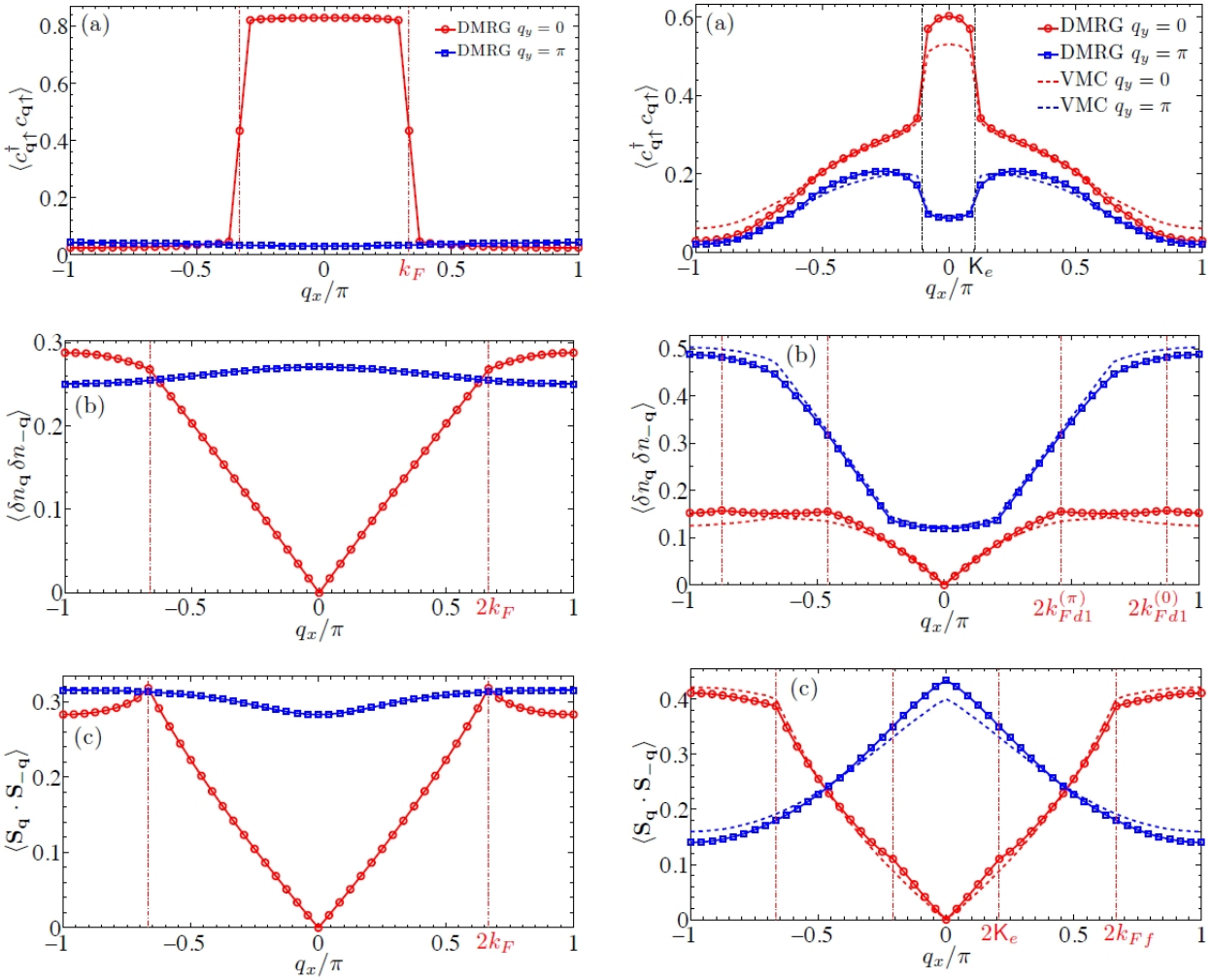
Constructing a trial wave function ($c_\sigma(\mathbf{r}) = d^1(\mathbf{r})d^2(\mathbf{r})f_\sigma(\mathbf{r})$ **)**



Filling factor $\rho = \frac{1}{3}$ **with** $N = 48 \times 2$**,** $k = 0$ **and** $k = \pi$ **are the bonding and antibonding bands**

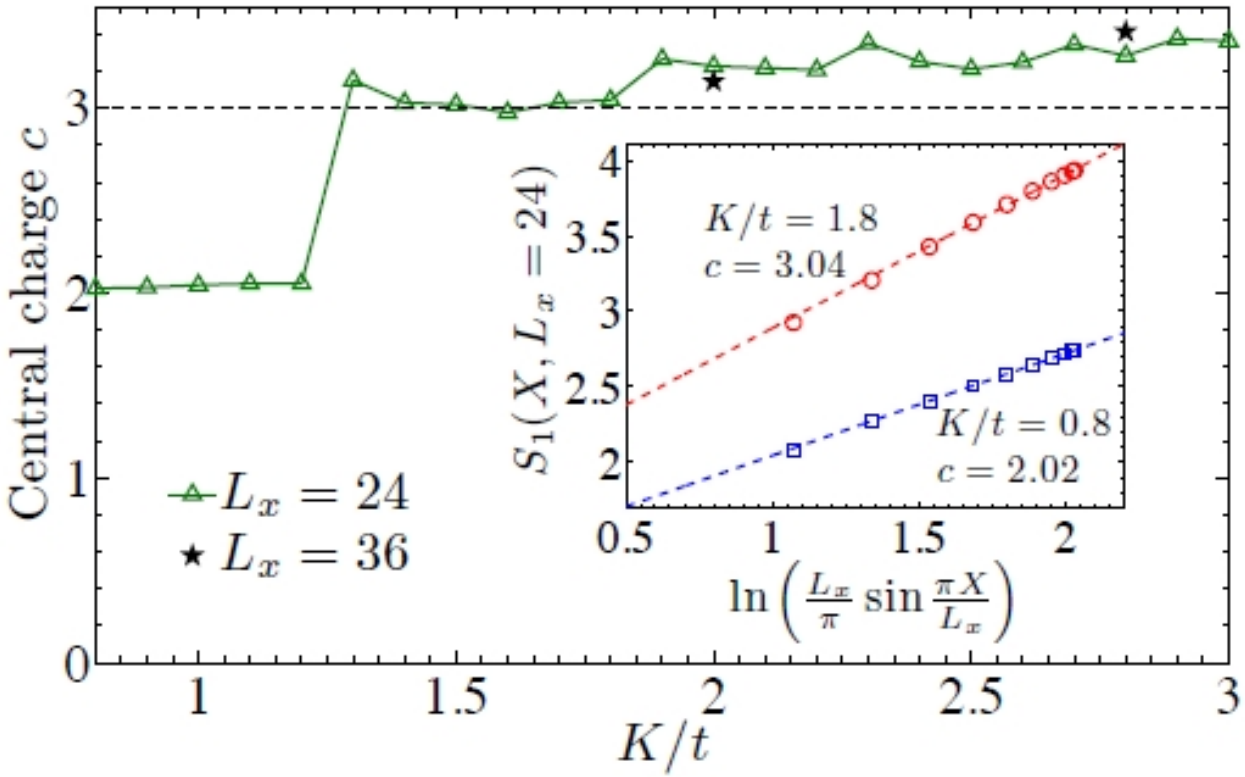
Numerical Results

Luttinger Liquid Phase **d-wave Metal Phase**



DMRG Results in FL ($K/t = 0.5$) and the d-wave Metal phase ($K/t = 1.8$) for $J/t = 2$ in both case

Numerical Results



Jump in central charge (obtained from von Neumann entanglement entropy)

Numerical Results

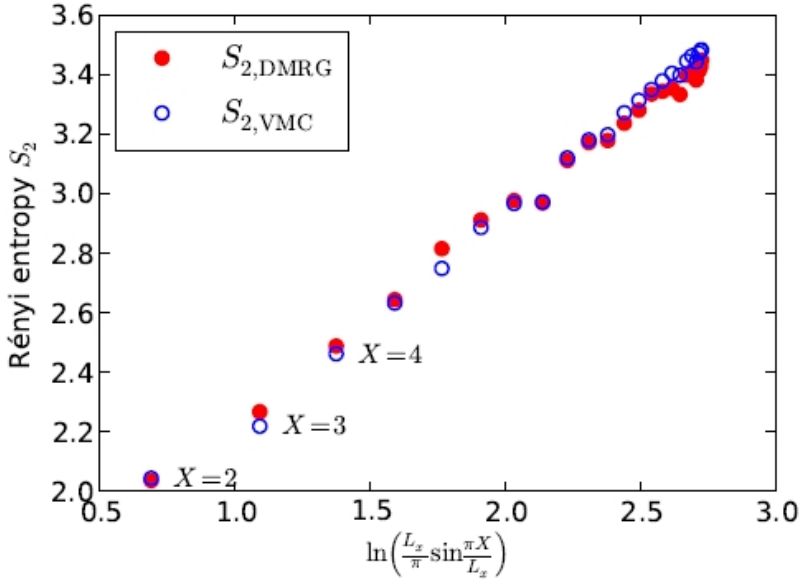
Rényi entropy:

$$S_\alpha(\rho_A) = \frac{1}{1-\alpha}\ln\left[\mathrm{Tr}(\rho_A^\alpha)\right] \qquad (\alpha \to 1 \text{von Neumann entropy})$$

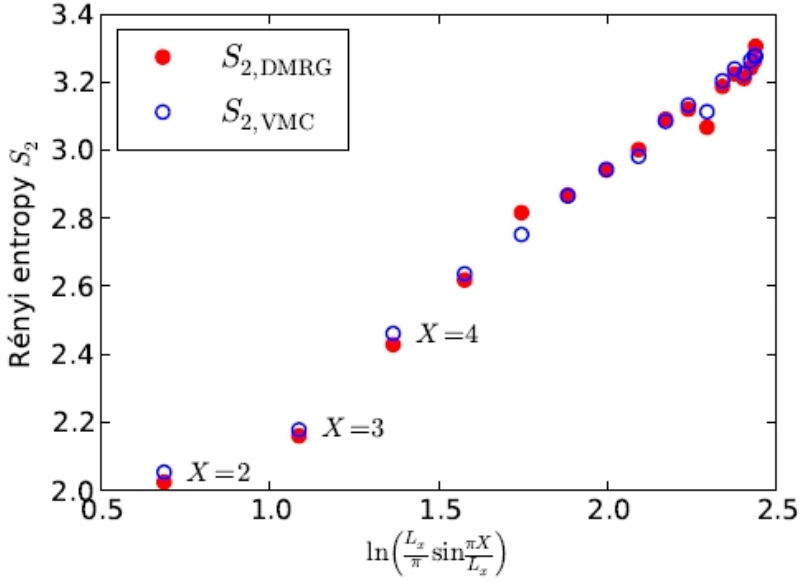
Rényi entropy in a CFT:

$$S_\alpha^{\mathrm{CFT}}(X, L_x) = \frac{c}{6}\left(1+\frac{1}{\alpha}\right)\ln\left[\frac{L_x}{\pi}\sin\frac{\pi X}{L_x}\right] + c'_\alpha$$

$N = 48 \times 2$

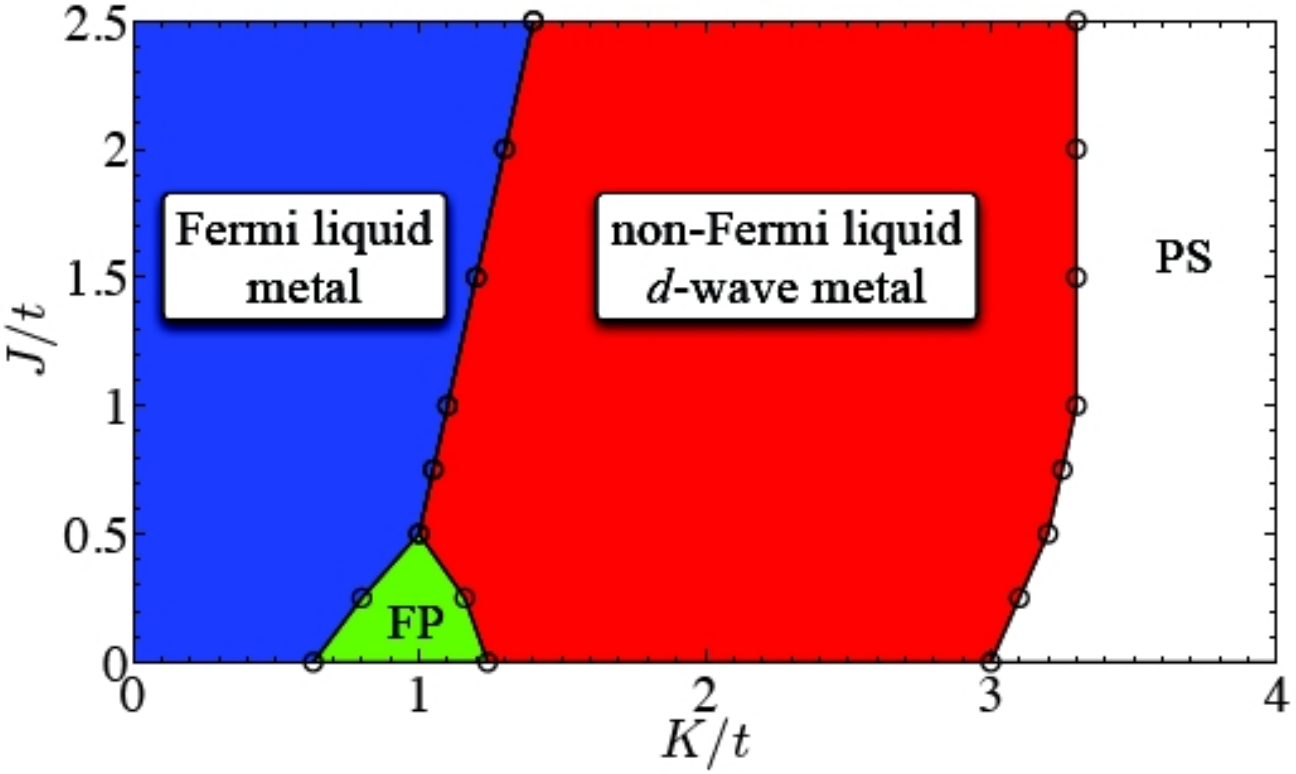


$N = 36 \times 2$



$\alpha = 2$ **Rényi Entanglement Entropy Comparisons**

Numerical Results



Phase Diagram for $\rho = \frac{1}{3}$

Conclusions & Comments

- Numerics strongly indicate the presence of the d-wave metal in the t-J-K model on the two leg ladder.
- Does this phase exist in 2D?

The End.