Construction of a Non-Fermi Liquid Ground State

(Condensed Matter Journal Club Presentation)

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Motivation and Preliminaries

Non-Fermi liquid (d-wave Metal) of Jiang et. al.

Motrunich and Fisher’s d-wave Bose Metal

Numerical signatures in quasi-1D systems

Comments & Discussion
Motivation

Understanding Strange Metal Phase (non-Fermi Liquids)

Cuprate Superconductors

Temperature [K]

Doping

Taken from: http://www.physik.uni-augsburg.de/~grasersi/
Motivation

Understanding Strange Metal Phase (non-Fermi Liquids)

Heavy Fermions

Taken from: http://www.toulouse.lnclm.cnrs.fr
Non-interacting electrons

\[ \mathcal{H}_0 = \sum_{i,j} t_{ij} c_{i,\sigma}^\dagger c_{j,\sigma} + \text{h.c.} = \sum_k \xi_k c_{k,\sigma}^\dagger c_{k,\sigma} \]

\[ G^0(k, t) = -i \left[ \theta(\xi_k) \theta(t) - \theta(-\xi_k) \theta(-t) \right] e^{-it\xi_k} \]

\[ G^0(k, \omega) = \frac{1}{\omega - \xi_k + i0^+ \text{sgn}(\omega)} \]
Preliminaries - Landau-Fermi Liquid Theory

Interacting electrons \((D \geq 2)\)

\[
\mathcal{H} = \sum_{i,j} \left[ t_{ij} c_{i,\sigma}^{\dagger} c_{j,\sigma} + \text{h.c.} \right] + \mathcal{H}_{\text{int}}
\]

\[
G(k, \omega) = G_{\text{coh}}(k, \omega) + G_{\text{incoh}}(k, \omega)
\]

\[
G_{\text{coh}}(k, \omega) = \frac{Z_k}{\omega - \xi_k + \frac{i}{\tau_k}}
\]

Lev Landau
Preliminaries - Tomonaga-Luttinger Theory

Hubbard model in 1D

\[ \mathcal{H} = t \sum_{i, \sigma} \left[ c_{i, \sigma}^\dagger c_{i+1, \sigma} + \text{h.c.} \right] + U \sum_i c_{i, \uparrow}^\dagger c_{i, \uparrow} c_{i, \downarrow}^\dagger c_{i, \downarrow} \]

Non-interacting part:

\[ \xi_k = \frac{2t}{\pi} \int_{-k_F}^{k_F} \frac{dk'}{2} \sqrt{1 - \frac{k'^2}{k_F^2}} \]

Duncan Haldane

Sin-Itiro Tomonaga

J. M. Luttinger
Scattering channels:

\begin{align*}
\mathcal{H}_1 &= \nu g_1 \left[ \psi_R^{\dagger,\sigma} \psi_L^{\sigma,\sigma} \psi_L^{\dagger,-\sigma} \psi_R^{\sigma,-\sigma} \right] \\
\mathcal{H}^c_2 &= \nu g^c_2 (J_{R,\uparrow} + J_{R,\downarrow}) (J_{L,\uparrow} + J_{L,\downarrow}) \\
\mathcal{H}^s_2 &= \nu g^s_2 (J_{R,\uparrow} - J_{R,\downarrow}) (J_{L,\uparrow} - J_{L,\downarrow}) \\
\mathcal{H}_3 &= \nu \frac{g_3}{2} \left[ \psi_R^{\dagger,\sigma} \psi_R^{\dagger,-\sigma} \psi_L^{\sigma,\sigma} \psi_L^{\sigma,-\sigma} + \text{h.c.} \right] \\
\mathcal{H}^c_4 &= \nu \frac{g^c_4}{2} \left[ (J_{R,\uparrow} + J_{R,\downarrow})^2 + (J_{L,\uparrow} + J_{L,\downarrow})^2 \right] \\
\mathcal{H}^s_4 &= \nu \frac{g^s_4}{2} \left[ (J_{R,\uparrow} - J_{R,\downarrow})^2 + (J_{L,\uparrow} - J_{L,\downarrow})^2 \right] \\
J &= \psi^{\dagger} \psi
\end{align*}
Preliminaries - **Tomonaga-Luttinger Theory**

- **Backscattering** term can gap the **spin** sector
- **Umklapp** term can gap the **charge** sector
- Tomonaga-Luttinger model:

\[
\mathcal{H}_{TL} = \mathcal{H}_0 + \mathcal{H}_2^C + \mathcal{H}_2^S + \mathcal{H}_4^C + \mathcal{H}_4^S
\]
Separation of spin and charge in 1D.

\[ \theta_c = \frac{1}{4} \left( K_c + \frac{1}{K_c} - 2 \right) \quad , \quad K_c = \sqrt{\frac{\pi - g_2^c + g_4^c}{\pi + g_2^c + g_4^c}} \]

\[ \nu_\mu = \nu \sqrt{\left(1 + \frac{g_4^\mu}{\pi}\right)^2 - \left(\frac{g_2^\mu}{\pi}\right)^2} \]
Luttinger liquids and Fermi liquids, although different in many aspects, are similar in one feature:
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Presence of strong interactions is not vital for LLs and FLs.
d-wave Metal of Jiang et. al.

**Slave-Boson trick** (break up electron operator into a bosonic **chargon** and a fermionic **spinon**):

\[
c_\sigma(r) = b(r) f_\sigma(r), \quad b^\dagger(r)b(r) = \sum_\sigma f^\dagger_\sigma(r)f_\sigma(r) = \sum_\sigma c^\dagger_\sigma(r)c_\sigma(r)
\]

Assuming that spinons are in a **Fermi sea**:

- chargons bose condense ( \( \langle b(r) \rangle \neq 0 \) ) \( \sim \) a FL
- chargons **do not** condense but still **conduct** \( \sim \) a non-FL

One needs a **“Bose metal”** to construct a nFL. ¹

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d-wave Bose Metal (DBL)

Motrunich and Fisher’s DBL: \(^2\)

- **TR invariant** analogue of \(\nu = \frac{1}{2}\) Laughlin wave function:

\[
\Psi(z_1, z_2, \ldots z_N) = \prod_{i<j} (z_i - z_j)^2 , \quad z = x + iy
\]

- **d-wave** in the sense that relative single particle wave function, upon close approach to a specific particle looks:

\[
\Phi_{z_2, z_3, \ldots z_N}(z) \sim (z - z_i)^2
\]

- Boson operator is composed of two **fermionic partons**:

\[
b^\dagger(r) = d_1^\dagger(r) d_2^\dagger(r)
\]

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d-wave Bose Metal (DBL)

Motrunich and Fisher’s DBL: \(^2\)

- Fermionic partons with identical Fermi surfaces lead to a s-wave.
- MFT gives:

\[
G_b^{MF}(\mathbf{r}, t = 0) \sim \frac{\cos[(\mathbf{k}_{F1} - \mathbf{k}_{F2}) \cdot \mathbf{r}]}{\sqrt{c_1 c_2} |\mathbf{r}|^3} + \frac{\cos[(\mathbf{k}_{F1} + \mathbf{k}_{F2}) \cdot \mathbf{r}]}{\sqrt{c_1 c_2} |\mathbf{r}|^3} - \frac{3\pi}{2}
\]

Microscopic Model Hamiltonian

\[ \mathcal{H} = \mathcal{H}_{tJ} + \mathcal{H}_K \]

\[ \mathcal{H}_{tJ} = -t \sum_{\langle i,j \rangle} \left( c_{i,\sigma}^\dagger c_{j,\sigma} + \text{h.c.} \right) + J \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j \]

\[ \mathcal{H}_K = 2K \sum \left( S_{13}^\dagger S_{24} + \text{h.c.} \right) \]

where,

\[ \mathbf{S}_i = c_{i,\alpha}^\dagger \frac{\sigma_{\alpha,\beta}}{2} c_{i,\beta} \quad , \quad S_{ij}^\dagger = \frac{1}{\sqrt{2}} \left( c_{i,\uparrow}^\dagger c_{j,\downarrow}^\dagger - c_{i,\downarrow}^\dagger c_{j,\uparrow}^\dagger \right) \]
Numerical Results

Constructing a trial wave function \( \psi(r) = d^1(r) d^2(r) f_\sigma(r) \)

Filling factor \( \rho = \frac{1}{3} \) with \( N = 48 \times 2 \), \( k = 0 \) and \( k = \pi \) are the bonding and antibonding bands.
Numerical Results

Luttinger Liquid Phase

DMRG Results in FL \((K/t = 0.5)\) and the d-wave Metal phase \((K/t = 1.8)\) for \(J/t = 2\) in both case
Numerical Results

Jump in central charge (obtained from von Neumann entanglement entropy)
Numerical Results

Rényi entropy:

\[ S_\alpha(\rho_A) = \frac{1}{1 - \alpha} \ln \left[ \text{Tr}(\rho_A^\alpha) \right] \quad (\alpha \rightarrow 1 \text{ von Neumann entropy}) \]

Rényi entropy in a CFT:

\[ S_{\text{CFT}}^\alpha(X, L_x) = \frac{c}{6} \left( 1 + \frac{1}{\alpha} \right) \ln \left[ \frac{L_x}{\pi} \sin \frac{\pi X}{L_x} \right] + c'_\alpha \]

\[ N = 48 \times 2 \quad \text{and} \quad N = 36 \times 2 \]

\( \alpha = 2 \) Rényi Entanglement Entropy Comparisons
Numerical Results

Phase Diagram for $\rho = \frac{1}{3}$
Conclusions & Comments

– Numerics strongly indicate the presence of the d-wave metal in the t-J-K model on the two leg ladder.

– Does this phase exist in 2D?
Thank You

The End.