Relaxation towards equilibrium of isolated strongly correlated 1-D Bose gas

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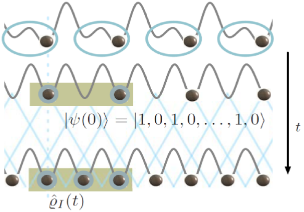
"Still, what is remarkable about one-dimensional systems: They are strongly interacting, yet at best weakly dissipative"
Bill Sutherland in *Beautiful Models*, World Scientific, 2004

Talk outline

- Overview-Statement of the problem
- Motivation
- Quenching in the Bose Hubbard model
- Initial state preparation
- Read-out of even-odd resolved local density
- Read-out of local currents
- Conclusion

Overview

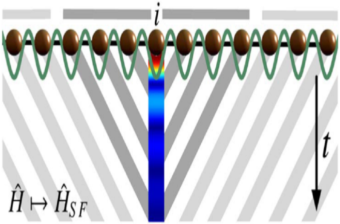


- 1-D lattice with patterned initial state (CDW) deep in mott regime.
- Sudden quench to large tunneling regime- non-equilibrium dynamics
- Nature of steady state (is there thermalisation?)
- Nature of dynamics towards steady state?

Motivation

- Equilibrium thermodynamics has a well established framework. Non-equilibrium (quantum) statistical mechanics not very well understood. No overarching framework.
- Resort to specific questions: Do quantum many body systems in non-eqbm evolving under a local hamiltonian equilbrate or not?
- What states do they equilibrate to and what is the dynamics towards equilibrium?
- Why 1-D? Classically known that 2-body collisions alone not enough to thermalize. How about quantum mechanics + strong correlations?
- Existense of controllable clean experimental probes: Cold atoms. Added advantage of being quantum simulators.

Quenching in the Bose-Hubbard model



$$H = H_{\mathrm{SF}} + \hat{U} = -J \sum_{<ij>} \hat{b}_i^{\dagger} \hat{b}_j + U/2 \sum_{i=1}^{N} \hat{n}_i(\hat{n}_i - 1) - \mu_i \hat{n}_i$$

Local Relaxation Hypothesis: PRL **100**,030602 (2008)

Start with ground state in $U >> J$ MI regime. Quench H to H_{SF}. Under the non-equilibrium dynamics that follows, a single site relaxes to a steady state with maximal entropy possible (respecting conserved quantities).

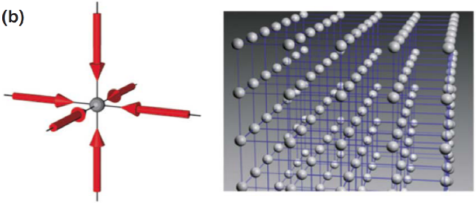
Physical Picture: Rest of lattice is like a reservoir for the single site, Incommensurate excitation influence of lattice sites in the effective light cone

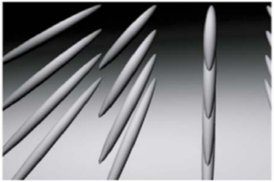
Caveats

- Exact proof of local relaxation only for $U = 0$.
- Maximal entropy state from dynamics different from Jaynes maximum entropy principle: given incomplete prior information leading to maximal entropy state or kinematical approaches: randomizing over all possible pure intial states puts sub-systems in maximal entropy states.
- Addressing single sites a challenge even for the cold atom magicians. This is accomplished in arXiv:1101.2659. $\hat{b}_i(t) = e^{iH_{\mathrm{SF}}t/\hbar}\hat{b}_i(0)e^{-iH_{\mathrm{SF}}t/\hbar}$ predicts local variables relax as: $\langle \hat{b}_i^\dagger \hat{b}_i \rangle \sim 1/2 - (-1)^i/2 J_0(4Jt)$ and $\Im\{\langle \hat{b}_{i+1}^\dagger \hat{b}_i \rangle\} \propto J_1(4Jt)$, both decay asymptotically as $t^{-1/2}$

Experiment-State Preparation

Initial loading of Rb condensate into a three dimensional optical lattice and ramping up the intensity ($J- > 0$) to go to a MI state in the 3-D lattice with one particle per site.

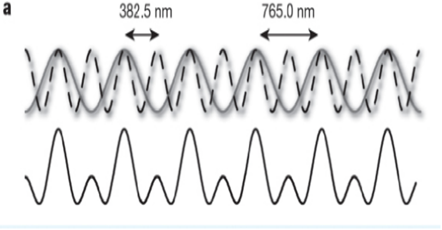


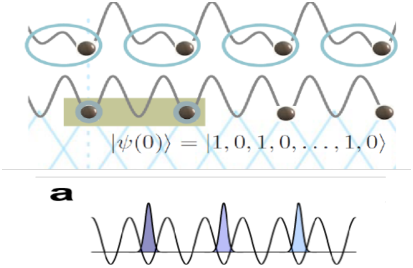


Call long axis x. I.Bloch,J.Dalibard & W.Zwerger, RMP **80**, 3(2008)

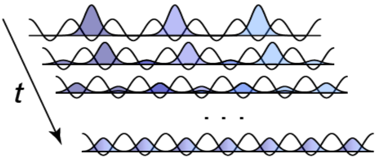
Patterned initial state

Add short lattice along x with half the period of the original lattice:



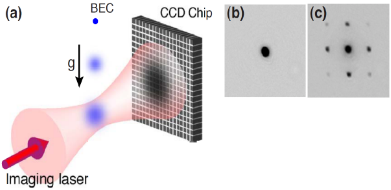


(*i*) Preparation



(*ii*) Evolution

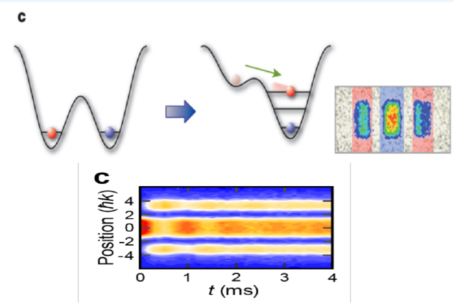
Short diversion: Time of flight imaging



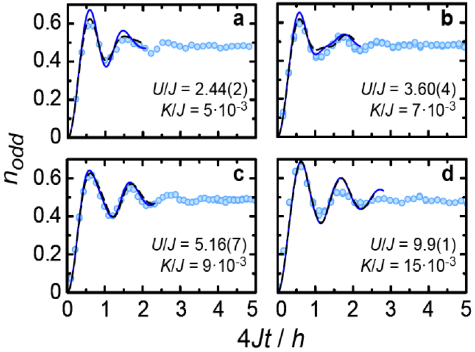
$$\langle \hat{\rho}_{3D}(x) \rangle_{\mathrm{tof}} = \langle \hat{\Psi}^{\dagger}_{\mathrm{tof}} \hat{\Psi}_{\mathrm{tof}} \rangle_{\mathrm{tof}} \approx \langle \hat{\Psi}^{\dagger}(k) \hat{\Psi}(k) \rangle \text{ with } k = Mx/\hbar t_{\mathrm{tof}}$$

Readout: Local density

Ramp back both short lattices after relaxation time t. Resolve odd and even sites by first transferring atoms in odd sites to higher band of even sites followed by time of flight images (S.Folling et.al, Nature **448**,1029(2007);J.Sebby-Strabley et.al,PRL **98**,200405(2007).)



Local Density: Results

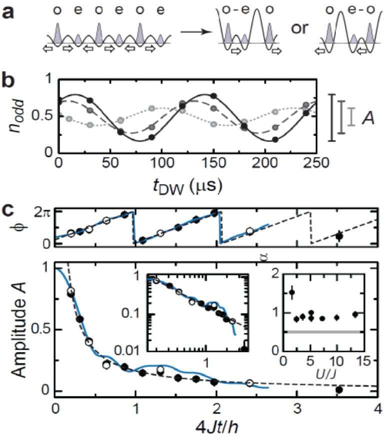


Ensemble averaged odd population density: blue lines (t-DMRG calculation),dotted black (t-DMRG + Next nearest nbr hopping) and blue dots from experiment.

The asymptotic decay rate is faster than $t^{-1/2}$, for $U/J \approx 5$ falls like $t^{-0.8}$. No obvious explanation for this feature, one of the main results of the paper

Readout: Local currents

Ramp up long lattice after relaxation suppressing every second coupling, array of double wells with $(U/J)_{\mathrm{dw}} = 0.2 < 1$.Record time evolution in isolated double wells as before.



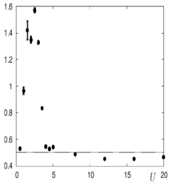
b) $t = 100, 200, 400\mu s$ order of decreasing A c)filled odd-even and empty even-odd; d). Blue line t-DMRG simulations;dashed lines exponential fits; horizontal line in the exponents inset denotes $\alpha = 0.5$

$$A_j(t) = \left(\left(\langle \hat{n}_j(t) \rangle - \langle \hat{n}_{j+1}(t) \rangle \right)^2 + 4\Im\left(\langle \hat{a}_j^\dagger \hat{a}_{j+1}(t) \rangle \right)^2 \right)^{1/2}$$

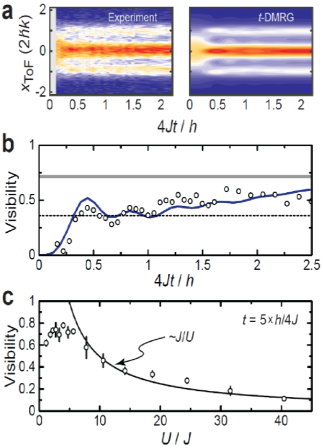
$$\phi_j(t) = \arctan\left(-2 \frac{\Im\left(\langle \hat{a}_j^\dagger \hat{a}_{j+1}(t) \rangle \right)}{\left(\langle \hat{n}_j(t) \rangle - \langle \hat{n}_{j+1}(t) \rangle \right)} \right)$$

Local currents:Remarks

- Phase ϕ contains information about local mass flow, linear evolution indicates ballistic expansion of excitations.
- Amplitude depends on local population and mass current. Decays to zero on the same scale as local density. For nonzero U on short time scales ($0 < 4Jt/\hbar < 3$) exponential damping $\propto t^{-\alpha}$ with $\alpha \neq 1/2$ for any value of U (even for small U).
- This fast relaxation not explained due to experimental details like transverse couplings or inhomgeneous tunnel couplings in different chains. Rate equations (derived from markovian master equation) cannot capture the fast damping of local densities. Dynamical mft cannot capture large U dynamics.



Quasi-momentum distribution



- Expand cloud freely after relaxation and take time of flight images. a). shows build up of next nearest neighbour correlations giving peaks at $k = 2\pi/\lambda_{xs}$ coming from memory of initial state.
- Visibility is proportional to the real part of the mass current $\langle \hat{a}_j^\dagger \hat{a}_{j+1} \rangle$ and its behaviour for large U in (c) is qualitatively similar to the expectation from a global thermal state.

Conclusions

- Relaxation of a non-equilibrium state in a 1-D boson system studied by following the dynamical evolution of local densities,currents and short range correlations extracted from quasi-momentum distribution.
- Measurements compare well with t-DMRG simulations and can run longer than the numerical method - quantum simulator
- Local observables decay to steady state value as expected from the local relaxation hypothesis
- Dynamics towards the state is cannot be understood from simple small or large U limits.
- Direct measurement of global observables inhibited due to the ensemble averaging but one can overcome by looking at single chain to determine exact global nature of the final state

Bibliography

Experimental Papers

- S Trotzky et.al., arXiv 1101.2659
- S Folling et.al., Nature **449**,1029 (2007)

Theory papers:

- Local relaxation hypothesis: M.Cramer et.al., PRL **100**,030602 (2008)
- Theory for the main experiment: A.Flesch et.al., PRA **78**,033608 (2008)