Majorana fermions inch closer to reality

A Journal Club Talk:
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List of Things I will Address:

- What are Majorana Fermions?
- Where should we look for them?
- What is special about this papers proposal?

Take Home Message

- Majorana Fermions have long evaded detection, but if the paper is correct there may be an easy way to produce them in an experimental system which already exists.

Preliminaries

Majorana Fermions

- First predicted to exist in a paper by Majorana in 1937, based on principles of mathematical beauty
- Majorana Fermions, unlike regular (Dirac) Fermions, are their own antiparticles
- No Majorana Fermions have ever been observed in nature, although neutrinos may be Majorana in nature (Rev. Mod. Phys. **80**, 481, (2008))

Majorana Fermions

- In condensed matter, we hope to find Majorana Fermions as an *emergent* degree of freedom in a strongly interacting electron system

- It is easy to make Majorana Fermions from regular (Dirac) Fermions:

$$\gamma_1 = \frac{c^\dagger + c}{2} \qquad \gamma_2 = \frac{c^\dagger - c}{2i}$$

- This transformation is not very useful in many systems.

Why Majorana Fermions?

- Under certain conditions (in very special systems), they may have *non-abelian* statistics

- Particles with non-abelian statistics have special topological properties and may be used to do robust quantum computation (Rev. Mod. Phys. **80**, 1083 (2008)

(12) **United States Patent**
Freedman et al.

(10) **Patent No.: US 7,525,202 B2**
(45) **Date of Patent: Apr. 28, 2009**

(54) **QUANTUM COMPUTATIONAL SYSTEMS**

(75) Inventors: **Michael Freedman**, Redmond, WA (US); **Chetan Nayak**, Santa Monica, CA (US); **Kirill Shtengel**, Seattle, WA (US)

(73) Assignee: **Microsoft Corporation**, Redmond, WA (US)

Freedman, M.H. et al., "Topological Quantum Computation", *arXiv:quant-ph*/0101025 v2, Sep. 24, 2002, 1-12.

Freedman, M. H., "P/NP, and the Quantum Field Computer", *Proc. Natl. Acad. Sci. USA*, Jan. 1998, 95, 98-101.

Freedman, M.H. et al., "The Two_Eigenvalue Problem and Density of Jones Representation of Braid Groups", *Commun. Math. Phys.*, 2002, 228, 177-199.

Freedman, M. et al., "A Class of P, T-Invariant Topological Phases of

Where can we find them?

- Fractional Quantum Hall Effect (Nucl. Phys. B **360**, 362 (1991))
- p_x+ip_y superconductors (Phys. Rev. B **61**, 10267 (2000))
- Rotating Cold Atoms (Phys. Rev. Lett. **87**, 120405 (2001))
- Interacting Spins (Ann. Phys. **321**, 2 (2003))
- Graphene coupled to s-wave SC (arxiv 0709.2626 (2007))
- Topological Insulator in proximity to s-wave SC (Phys. Rev. Lett. **100**, 096407 (2008))
- Semiconductor coupled to s-wave SC and FM insulator (Phys Rev. Lett. 1**04,** 040502, Phys. Rev. B **81**, 125318 (2010))

Where can we find them?

$$\gamma_1 = \frac{c^\dagger + c}{2} \qquad \gamma_2 = \frac{c^\dagger - c}{2i}$$

- May look familiar to those familiar with the theory of superconductivity. SC described by Boguliobons:

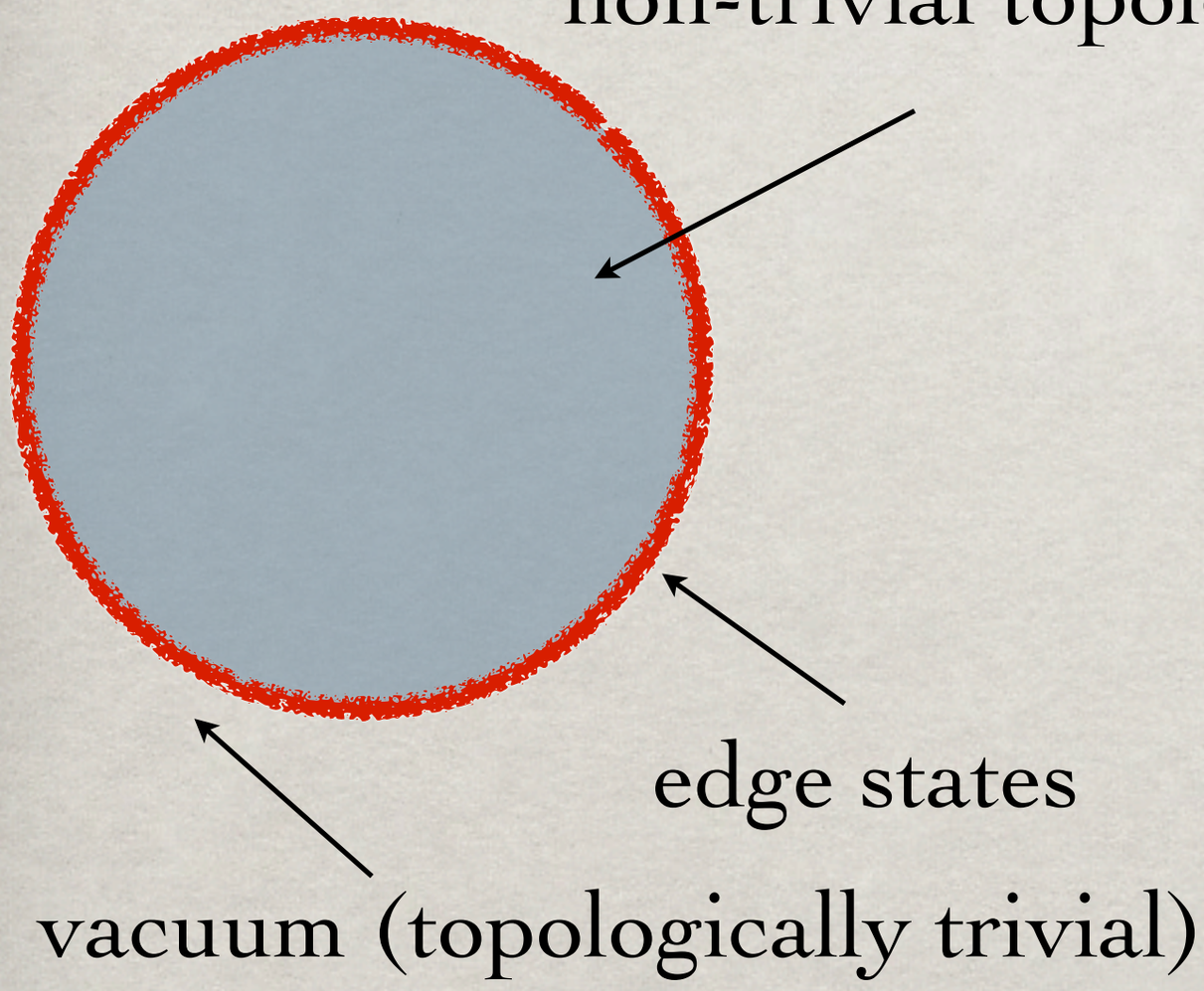
$$\gamma_k = u_k c_k - v_k c^\dagger_{-k}$$
$$\gamma^\dagger_k = u^*_k c^\dagger_k - v^*_k c_{-k}$$

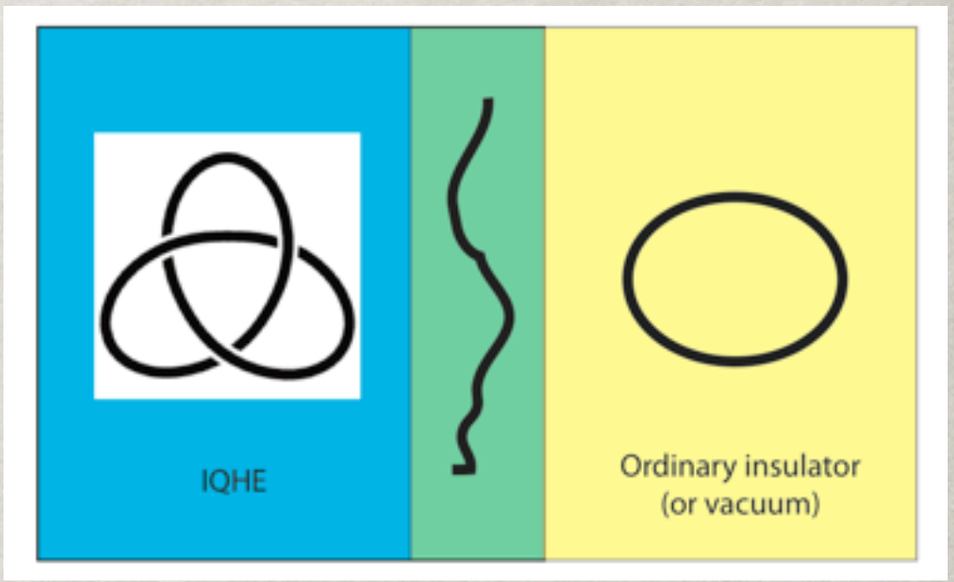
- What makes a chiral p-wave state special? It has topological properties.

Where can we find them?

Have states which cost no energy that reside at the boundary of the system

non-trivial topological invariant



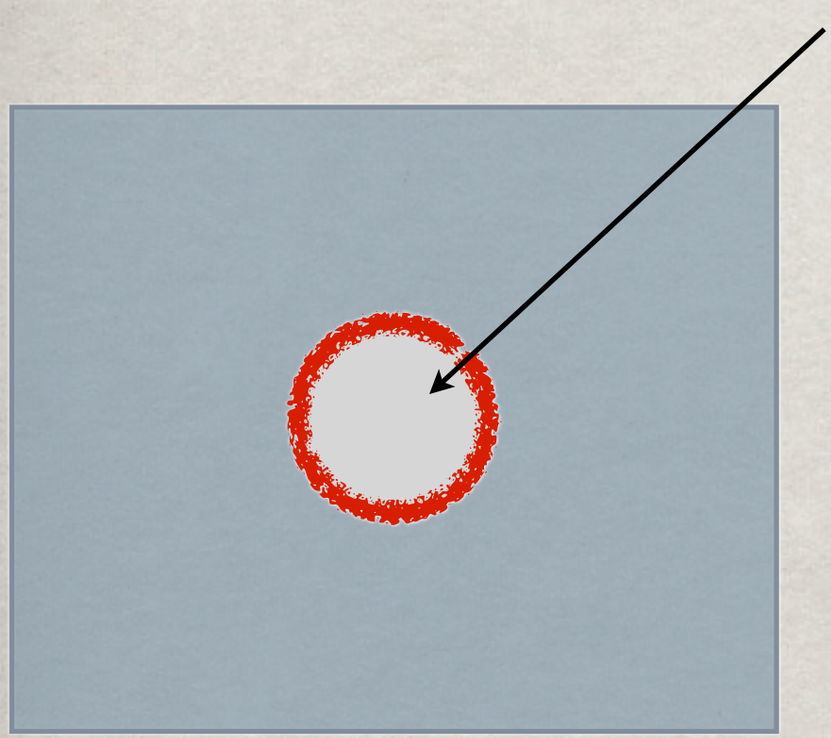


$$u(x) \propto e^{i\pi/4} \exp\left(\frac{1}{\Delta}\int^{x} \mu(x)dx\right)$$

Where can we find them?

Have states which cost no energy that reside inside vortex cores!

Hole threaded with flux, or vortex core



$$\Phi = \frac{h}{2e}$$

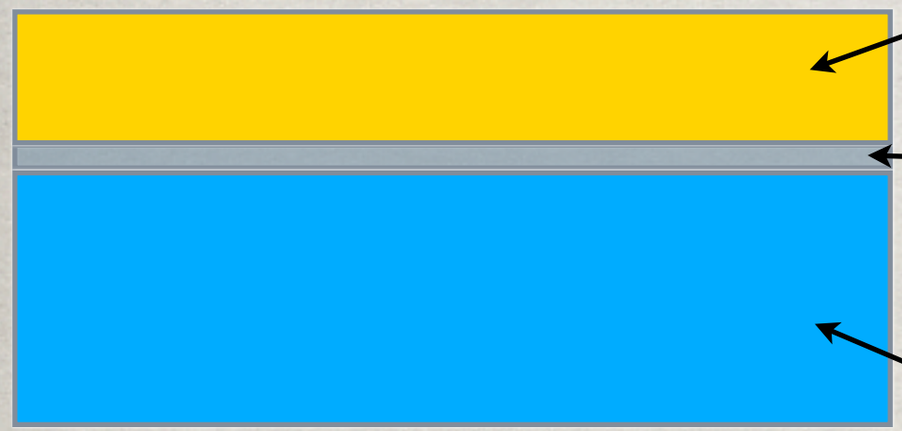
Can, by expressing in radial coordinates rewrite as the same equation as before!

In a SC for every state with E there is a state with -E. 0 Energy states are MF.

$E \propto \Delta^2/E_f$ for a regular vortex (Phys. Lett. **9**, 307 (1964)

Where can we find them?

Fu & Kane 2008



Regular s-wave SC

Proximity Effect induced SC

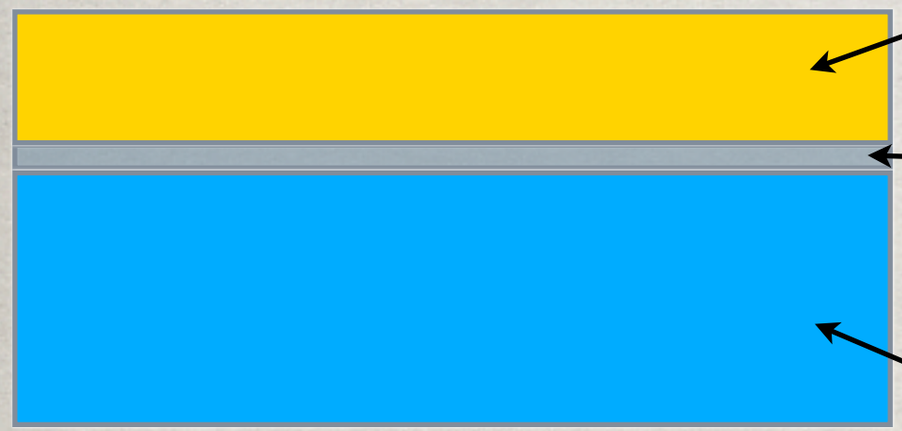
Topological Insulator

Special dispersion of edge states of TI cause the induced SC to look like chiral p-wave state but preserving time reversal

Kane and Fu predict votrices should also carry MF

Where can we find them?

Fu & Kane 2008



Regular s-wave SC

Proximity Effect induced SC

Topological Insulator

This doesn't work because 3D topological insulators that we have made aren't good insulators (and hence aren't topological insulators??)

The Paper

Idea Thief

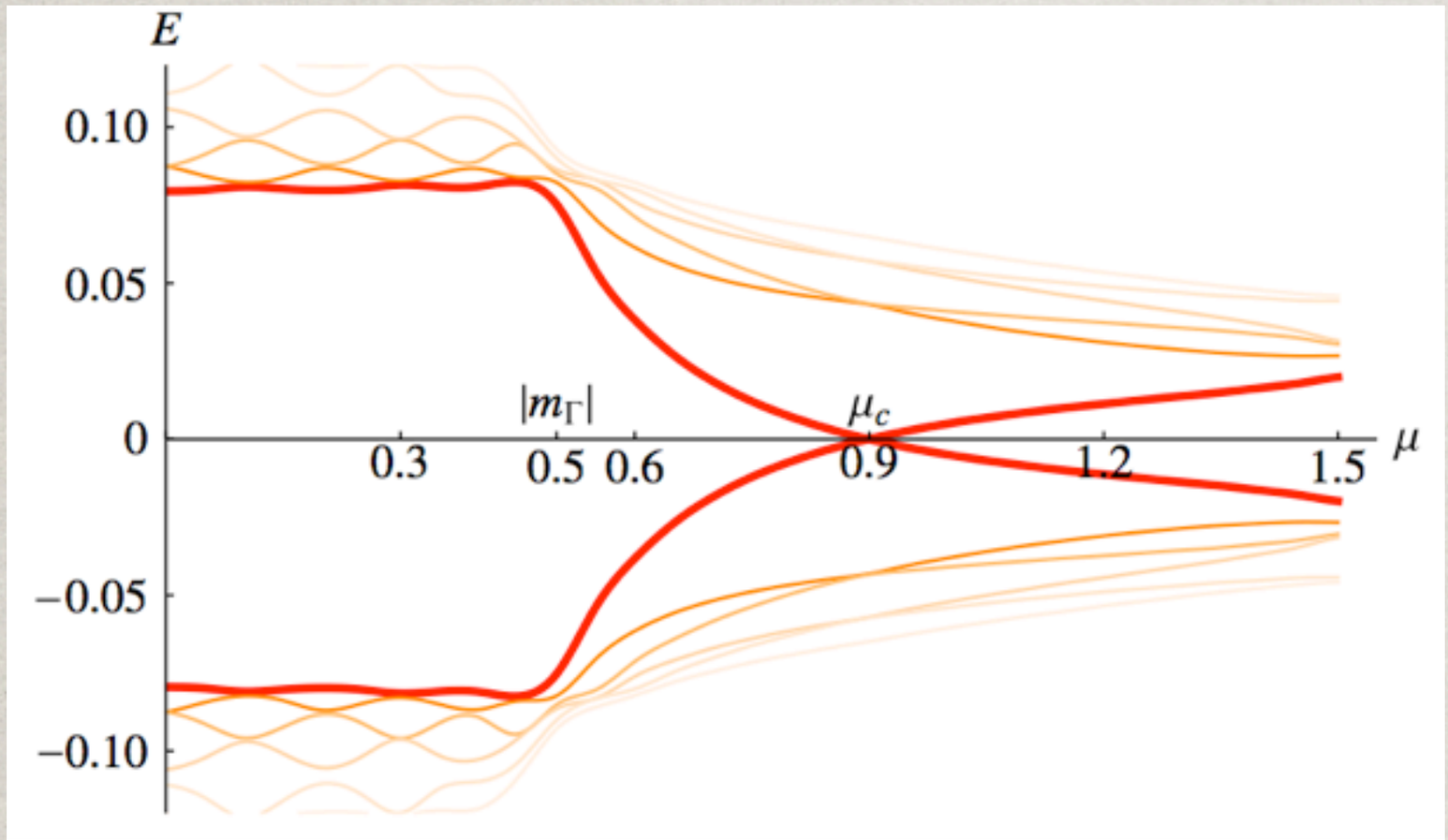
Certain of the topological insulators have been found to become superconducting as a function of doping (or also pressure)

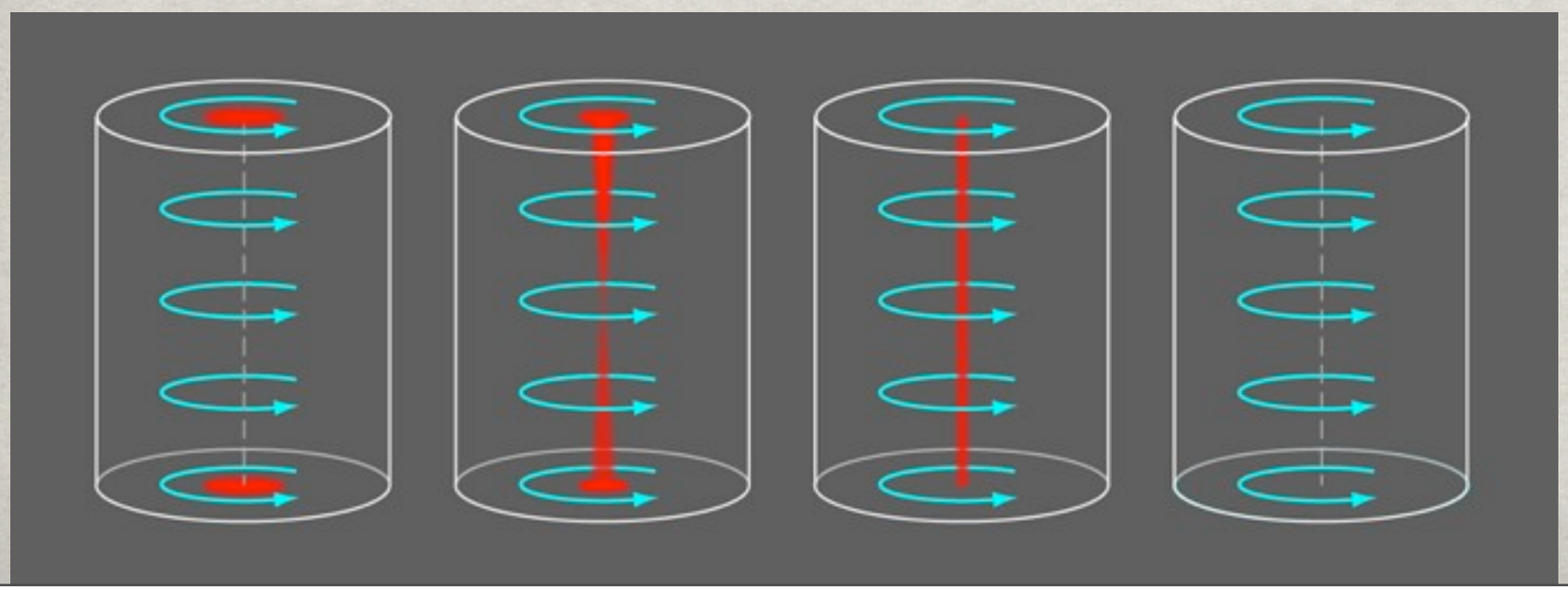
In particular the paper focuses on copper doped Bi_2Te_3

Can a vortex in this SC support Majorana Fermions?

This paper answers in the affirmative!

Results

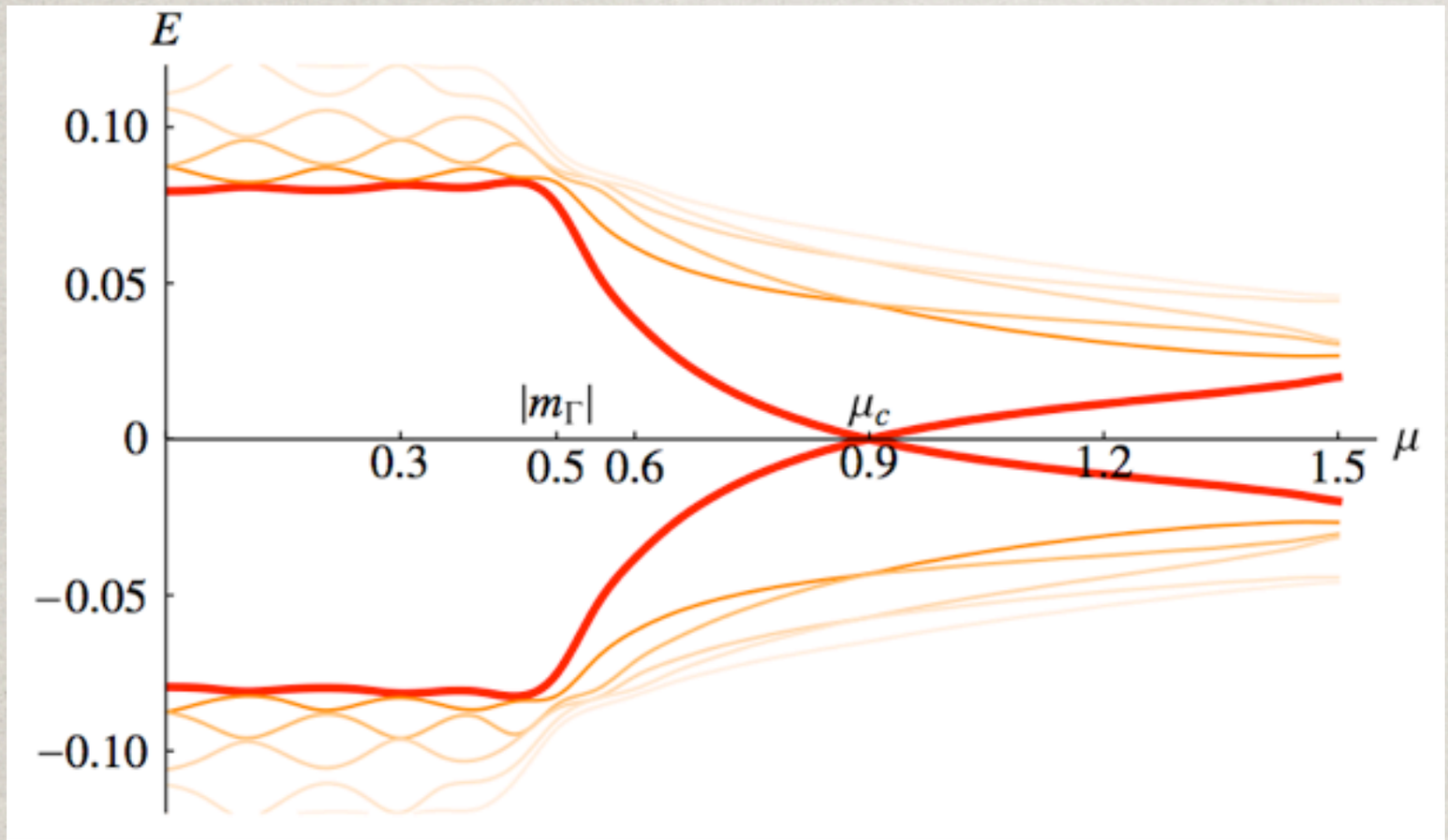




As chemical potential is tuned the Majorana Fermions eventually tunnel through the low lying states in the Vortex core

$$E \propto \Delta^2 / E_f$$

Results



The continuum limit gives a critical value of approx 1 for their model parameters

In Cu doped Bi_2Se_3 mu = 0.25 eV. The authors compute that the critical value of mu for a c-axis vortex is 0.24 eV.

By tilting the vortex this can be raised to 0.30 eV!

Take Home Message

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Thank you For listening!