MAJORANA FERMIONS INCH CLOSER TO REALITY

Friday, November 4, 2011
List of Things I will Address:

- What are Majorana Fermions?
- Where should we look for them?
- What is special about this paper's proposal?
Majorana Fermions have long evaded detection, but if the paper is correct there may be an easy way to produce them in an experimental system which already exists.
Preliminaries
Majorana Fermions

First predicted to exist in a paper by Majorana in 1937, based on principles of mathematical beauty.

Majorana Fermions, unlike regular (Dirac) Fermions, are their own antiparticles.

No Majorana Fermions have ever been observed in nature, although neutrinos may be Majorana in nature (Rev. Mod. Phys. 80, 481, (2008))
**Majorana Fermions**

- In condensed matter, we hope to find Majorana Fermions as an *emergent* degree of freedom in a strongly interacting electron system.

- It is easy to make Majorana Fermions from regular (Dirac) Fermions:

  \[
  \gamma_1 = \frac{c^\dagger + c}{2} \quad \gamma_2 = \frac{c^\dagger - c}{2i}
  \]

- This transformation is not very useful in many systems.
Why Majorana Fermions?

Under certain conditions (in very special systems), they may have *non-abelian* statistics.

Particles with non-abelian statistics have special topological properties and may be used to do robust quantum computation (Rev. Mod. Phys. **80**, 1083 (2008))
WHERE CAN WE FIND THEM?

- Graphene coupled to s-wave SC (arxiv 0709.2626 (2007))
- Topological Insulator in proximity to s-wave SC (Phys. Rev. Lett. 100, 096407 (2008))
Where can we find them?

\[ \gamma_1 = \frac{c^\dagger + c}{2} \quad \gamma_2 = \frac{c^\dagger - c}{2i} \]

May look familiar to those familiar with the theory of superconductivity. SC described by Boguliobons:

\[ \gamma_k = u_k c_k - v_k c_{-k} \]
\[ \gamma_k^\dagger = u_k^* c_{-k}^\dagger - v_k^* c_{-k} \]

What makes a chiral p-wave state special? It has topological properties.
Where can we find them?

Have states which cost no energy that reside at the boundary of the system

non-trivial topological invariant

equation:

\[ u(x) \propto e^{i\pi/4} \exp \left( \frac{1}{\Delta} \int x \mu(x) dx \right) \]
WHERE CAN WE FIND THEM?

Have states which cost no energy that reside inside vortex cores!

Hole threaded with flux, or vortex core

\[ \Phi = \frac{\hbar}{2e} \]

Can, by expressing in radial coordinates rewrite as the same equation as before!

In a SC for every state with \( E \) there is a state with \( -E \). 0 Energy states are MF.

\[ E \propto \Delta^2/E_f \] for a regular vortex (Phys. Lett. 9, 307 (1964))
Where can we find them?

Fu & Kane 2008

- Regular s-wave SC
- Proximity Effect induced SC
- Topological Insulator

Special dispersion of edge states of TI cause the induced SC to look like chiral p-wave state but preserving time reversal

Kane and Fu predict votrices should also carry MF
Where can we find them?

Fu & Kane 2008

- Regular s-wave SC
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- Topological Insulator

This doesn’t work because 3D topological insulators that we have made aren’t good insulators (and hence aren’t topological insulators??)
The Paper
Idea Thief

Certain of the topological insulators have been found to become superconducting as a function of doping (or also pressure)

In particular the paper focuses on copper doped Bi$_2$Te$_3$

Can a vortex in this SC support Majorana Fermions?

This paper answers in the affirmative!
As chemical potential is tuned the Majorana Fermions eventually tunnel through the low lying states in the Vortex core

\[ E \propto \Delta^2 / E_f \]
The continuum limit gives a critical value of approx 1 for their model parameters.

In Cu doped Bi$_2$Se$_3$ $\mu = 0.25$ eV. The authors compute that the critical value of $\mu$ for a c-axis vortex is 0.24 eV.

By tilting the vortex this can be raised to 0.30 eV!
Majorana Fermions have long evaded detection, but if the paper is correct there may be an easy way to produce them in an experimental system which already exists.
Thank you For listening!