Quantization and Waves

Concepts: Compton Scattering
Complementarity

Photon Momentum

- In 1919, Einstein showed that photons with energy E=hυ have momentum:

$$p = E/c = h\nu/c = h/\lambda$$

- Note that this definition agrees with our definition of the wavelength of a matter wave:

$$\lambda_{matter} = h / p_{matter}$$

Photons are ***massless***, so we cannot write their momentum as p=mv, in analogy with matter

The Compton Effect

- Experiments show that, when an X–ray photon strikes an electron, the X–ray photon scatters in one direction and the electron recoils in another direction
- The scattering of light off electrons is called the ***Compton Effect***
- Since we have assigned a momentum to a photon, we can treat this collision using conservation of momentum!

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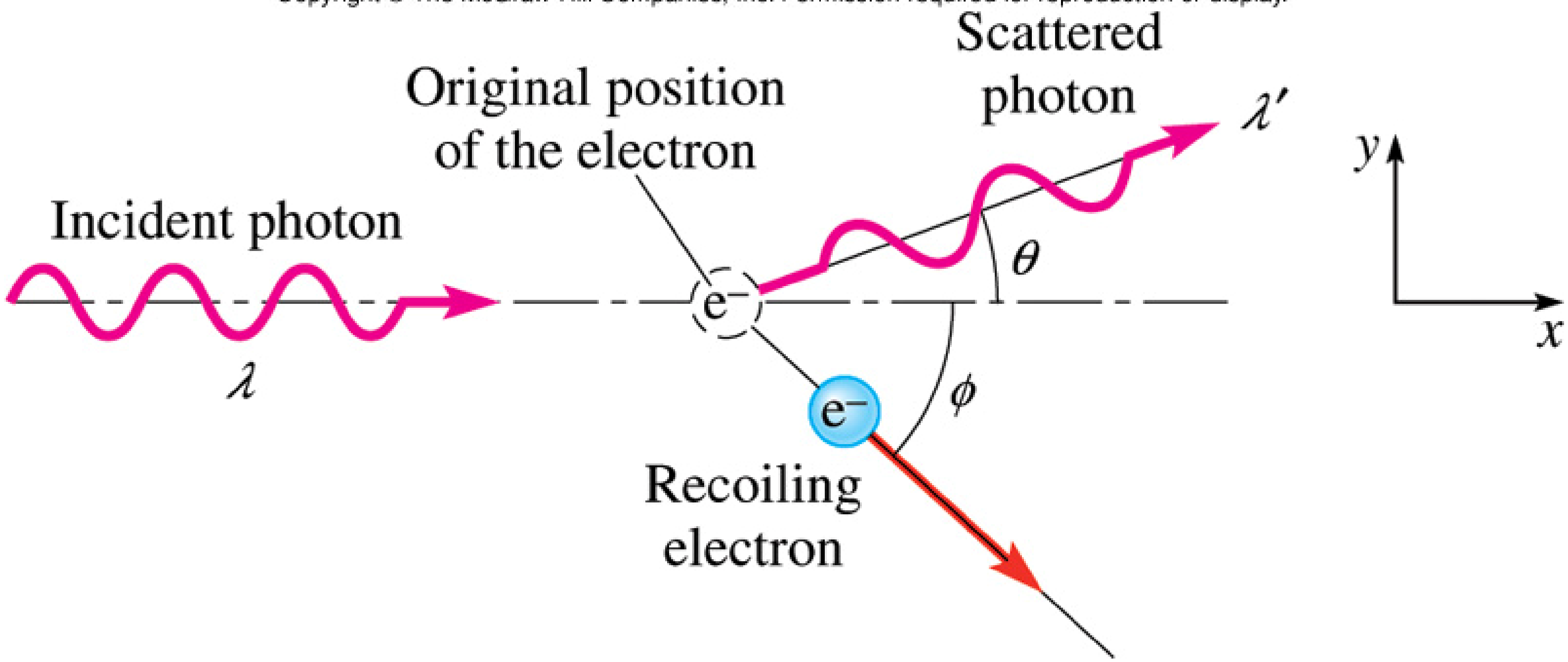
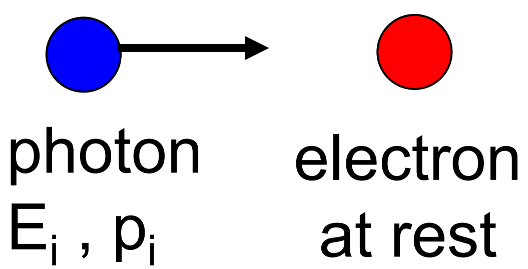


- Einstein’s description of the incident photon tells us that the incident photon has energy and momentum:

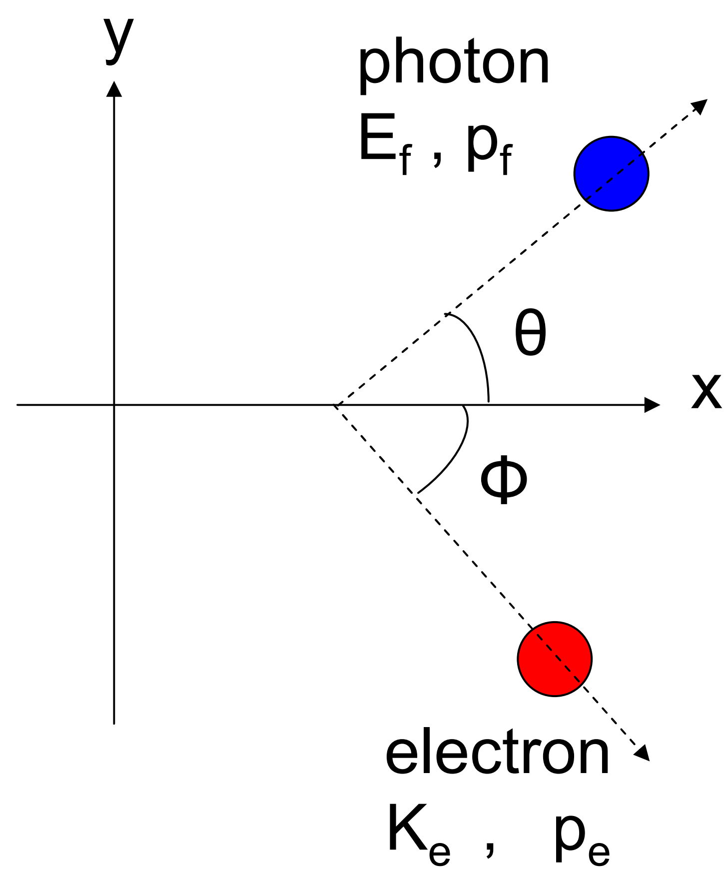
$$E_i = h\nu_i \qquad p_i = h/\lambda_i$$

- Conserve energy and momentum before and after the collision

Before Collision



After Collision



$$\vec{p}_{i,\ system} = \vec{p}_{f\ ,\ system}$$

$$E_{i,\ system} = E_{f\ ,\ system}$$

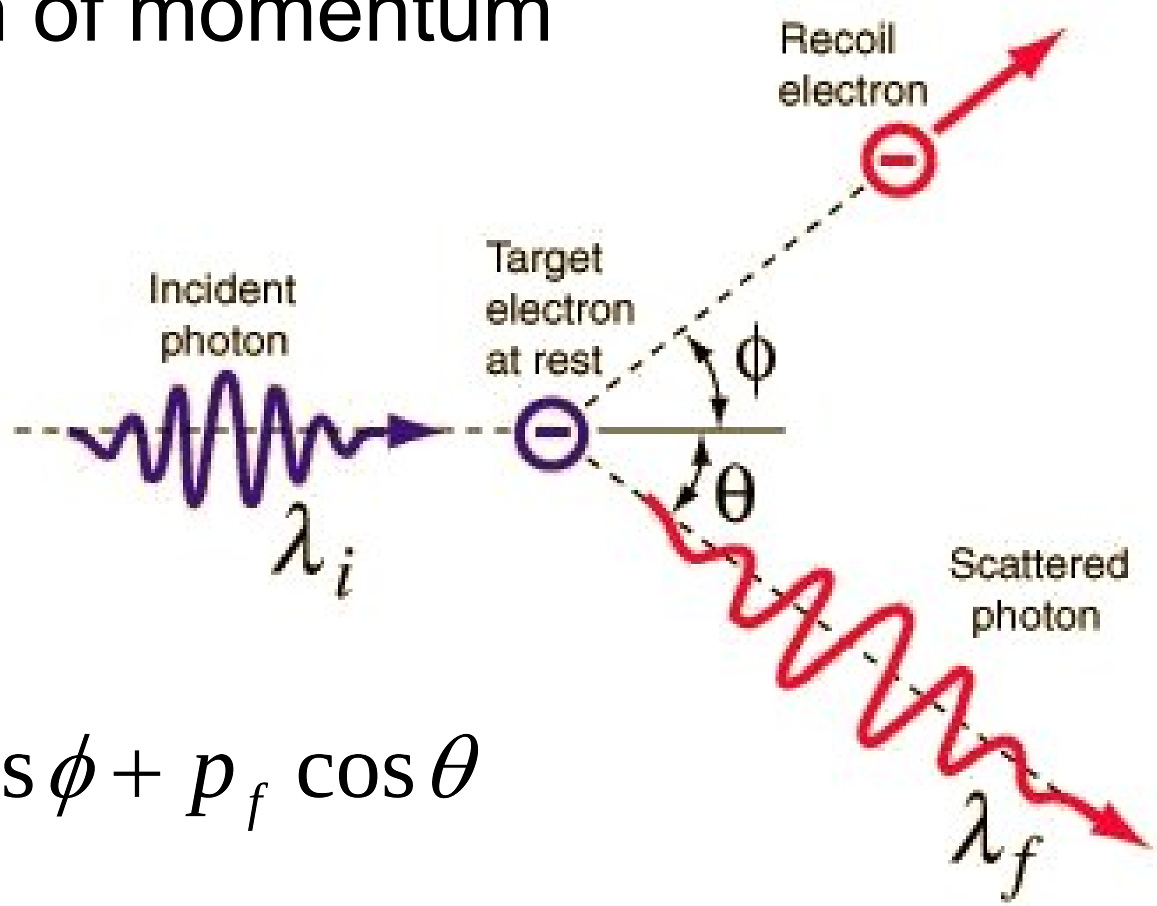
- Conservation of energy:

$$E_{i,\,system} = E_{f\,,\,system}$$

$$E_i = K_{electron} + E_f$$

$$\frac{hc}{\lambda_i} = K_e + \frac{hc}{\lambda_f}$$

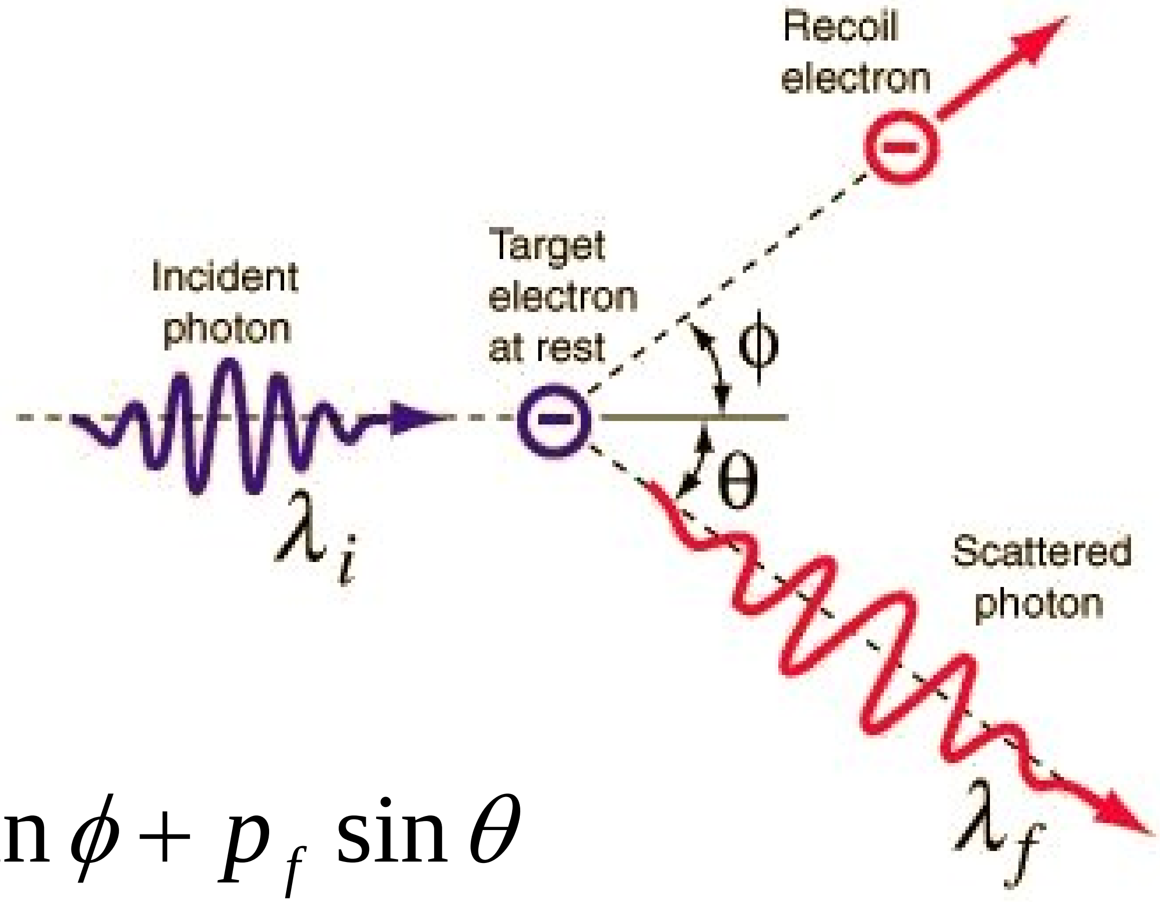
- Conservation of momentum



$$x: \; p_i = p_f$$

$$p_i = p_e \cos\phi + p_f \cos\theta$$

$$\frac{h}{\lambda_i} = p_e \cos\phi + \frac{h}{\lambda_f} \cos\theta$$



$$y: \; p_i = p_f$$

$$0 = -p_e \sin\phi + p_f \sin\theta$$

$$0 = -\,p_e \sin\phi + \frac{h}{\lambda_f}\sin\theta$$

- Some algebra gives us the change in the wavelength of the photon, which we call the ***Compton shift:***

$$\Delta\lambda = \lambda_f - \lambda_i = (h/m_e c)(1 - \cos\theta)$$

- We can write the Compton shift in terms of the ***Compton wavelength, λ_C:***

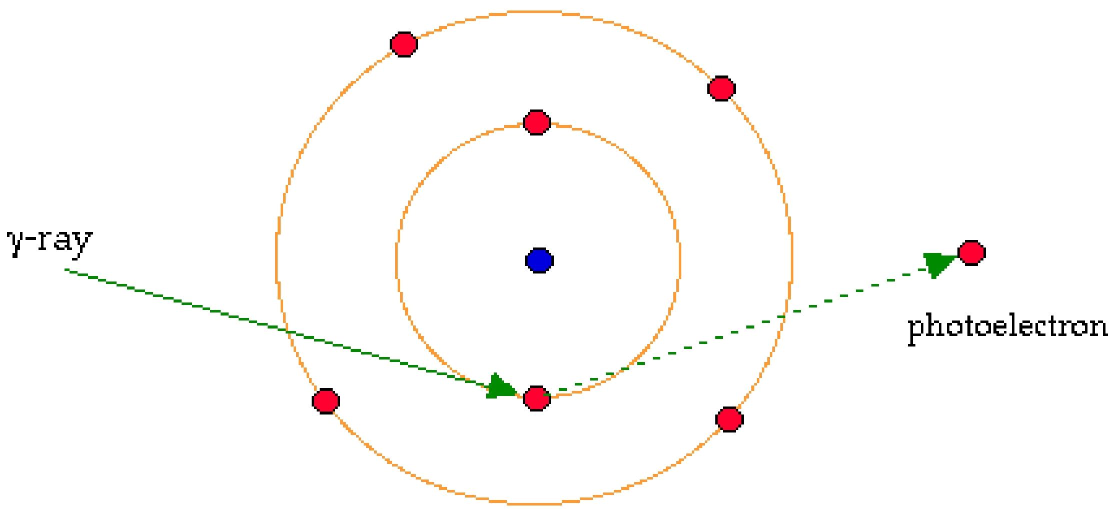
$$\lambda_c = (h/m_e c) = 0.00243\ nm$$

$$\Delta\lambda = \lambda_c(1 - \cos\theta)$$

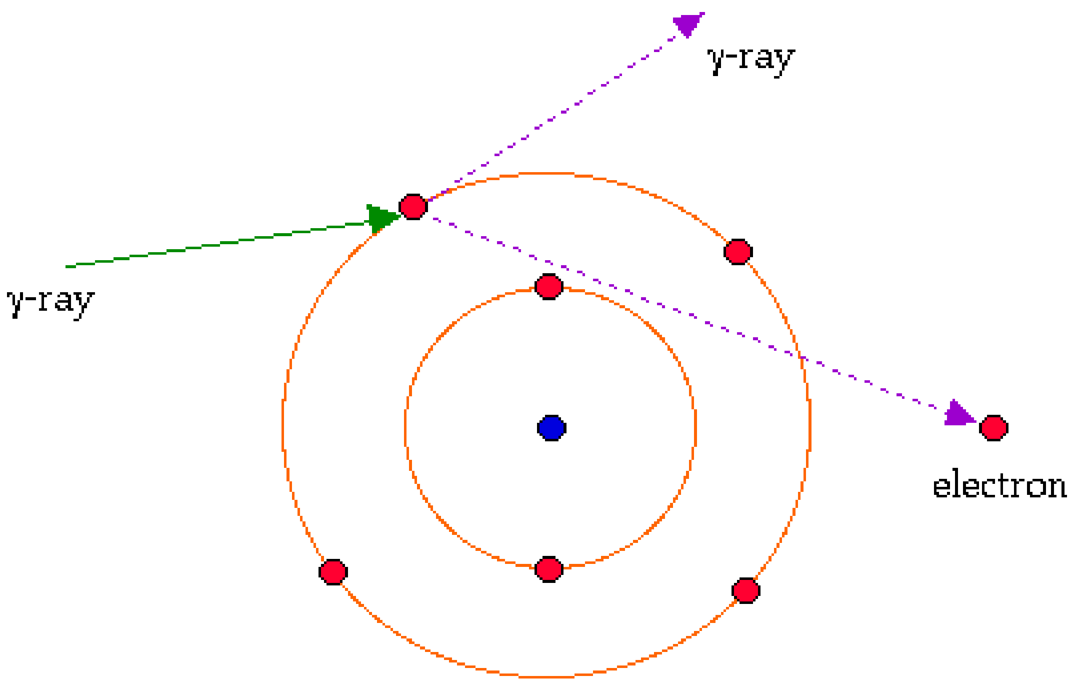
PE and CE in Biology

- In many medical and biological applications, high-energy photons such as X-rays and gamma-rays are used for imaging and diagnosis
- Imagine you fire a beam of high-energy (i.e. high frequency) photons into a block of matter→what happens to them?

- Some photons make it through the sample unaffected
- The rest are either absorbed or scattered
- The photoelectric effect and the Compton effect are the dominant means of absorption and scattering in living tissues
- The PE and the CE are responsible for “subtracting” photons from the beam you project into a subject



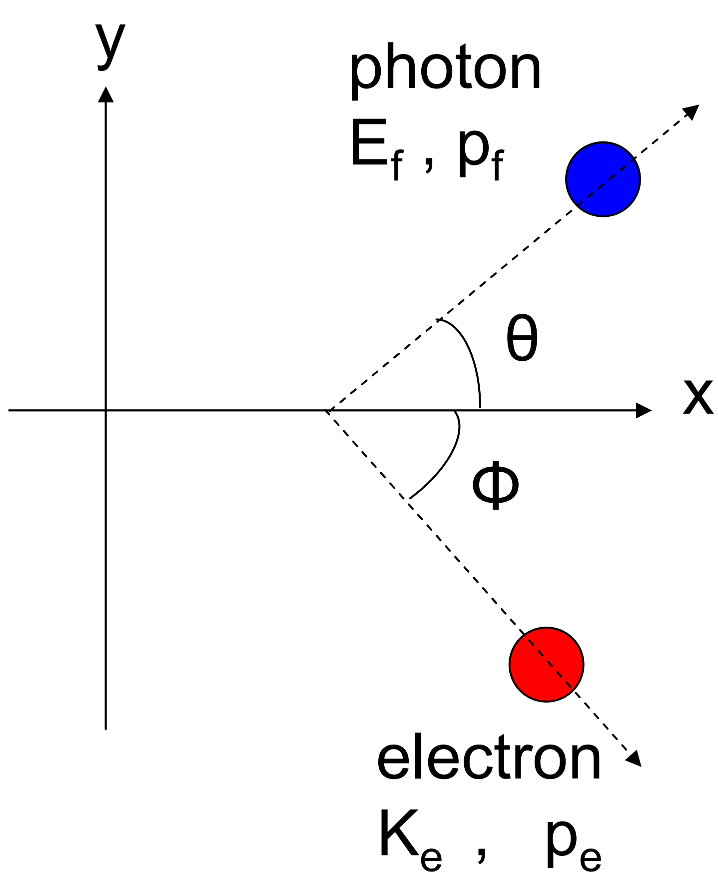
photoelectric effect



Compton effect

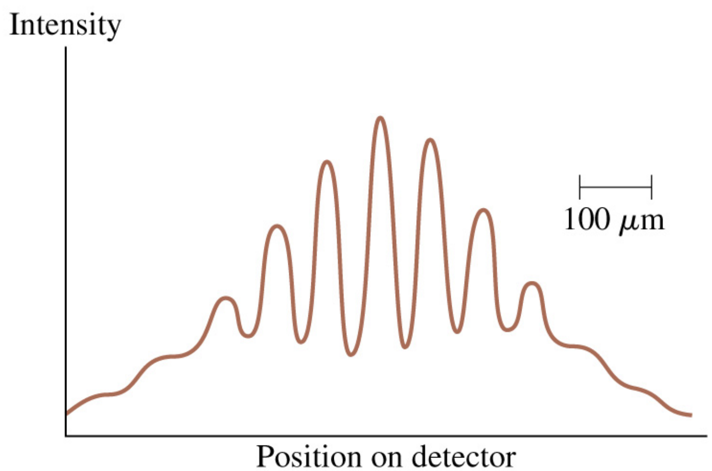
Example

- An X–ray photon of wavelength 10.0 pm (1 pm = 10^{-12} m) is scattered through 110.0^{o} by an electron. What is the kinetic energy of the recoiling electron?



Wave-Particle Duality

- Note that ***ALL*** matter particles have wave properties, including electrons, protons, and neutrons
- Here is the interference pattern produced by neutrons:



(b) Double-slit interference of neutrons

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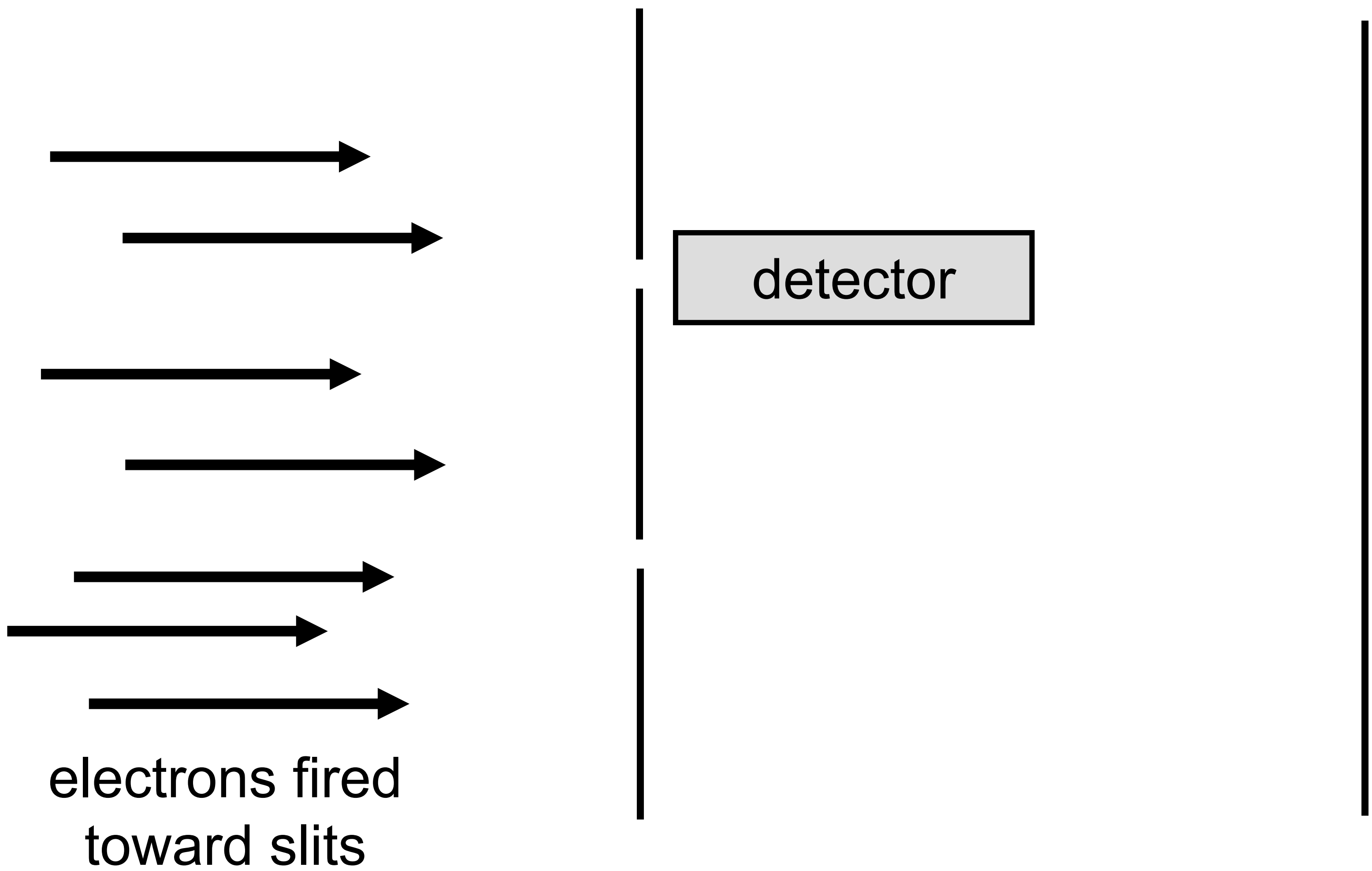
- ***Wave-particle duality*** ➔ all objects have ***BOTH*** wave and particle properties
- Can we cause matter to behave simultaneously as ***both*** a wave and a particle?
- Imagine we repeat the one-at-a-time electron double-slit experiment
  - We know the electrons will act as waves and interfere
  - This implies that ***each*** electron goes through ***both*** slits
  - Put another way, the interference pattern tells us nothing about which slit each electron went through

- What if we try to detect which slit an electron goes through?



- If we detect each electron after it passes through the slits, so we can be sure which slit it went through, ***the interference pattern goes away!***
- If we detect the particle nature of electrons, their wave nature fails to manifest!
- If we detect the wave nature of electrons, their particle nature fails to manifest!

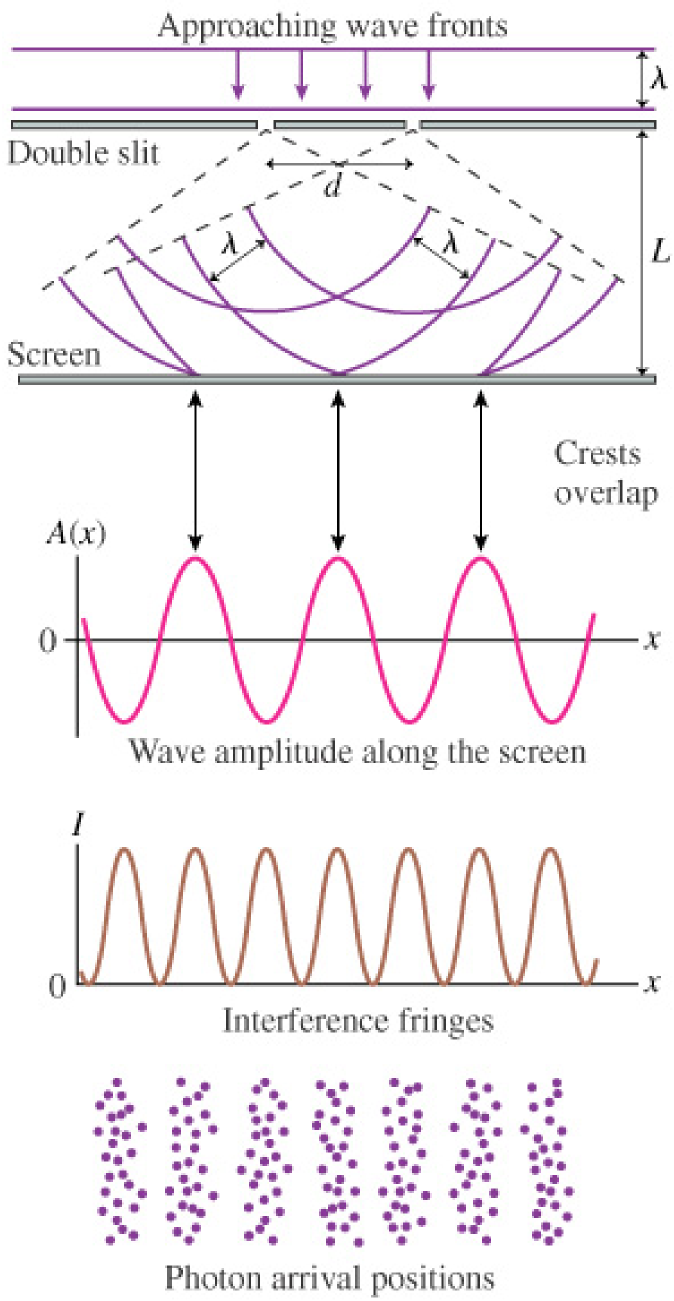
Complementarity

- The ***Complementarity Principle*** says that we cannot simultaneously observe the wave and particle nature of quantum mechanical objects
- "Waveness" and "particleness" are complementary properties of an object which cannot simultaneously be observed

- So what ***is*** an electron?
  - an electron is a wave-particle!
  - it is BOTH a wave AND a particle, but it never shows both characters at once
  - it is NOT a “wavicle”—something that has both aspects simultaneously
- This is extremely counter-intuitive!

Waves of What?

- What are matter waves waves ***<u>of</u>***?
- There is no classical answer to this question
- Matter waves are waves of ***probability***
- Think back to the double-slit experiment



Amplitude, A, of an electromagnetic wave

Intensity of light in the wave picture, $I \propto A^2$

Photon impacts in the photon picture

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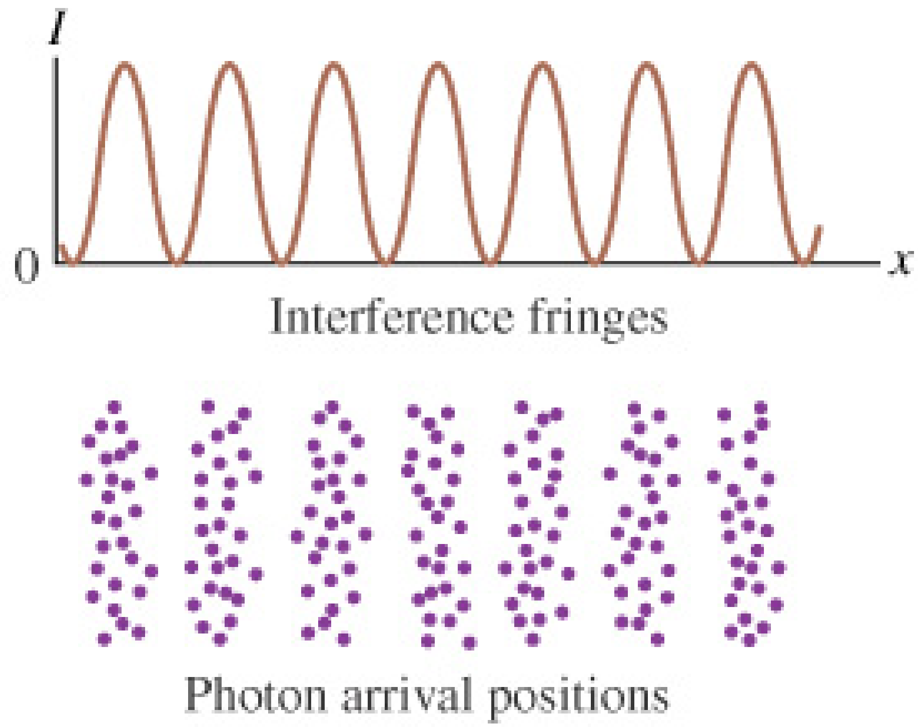
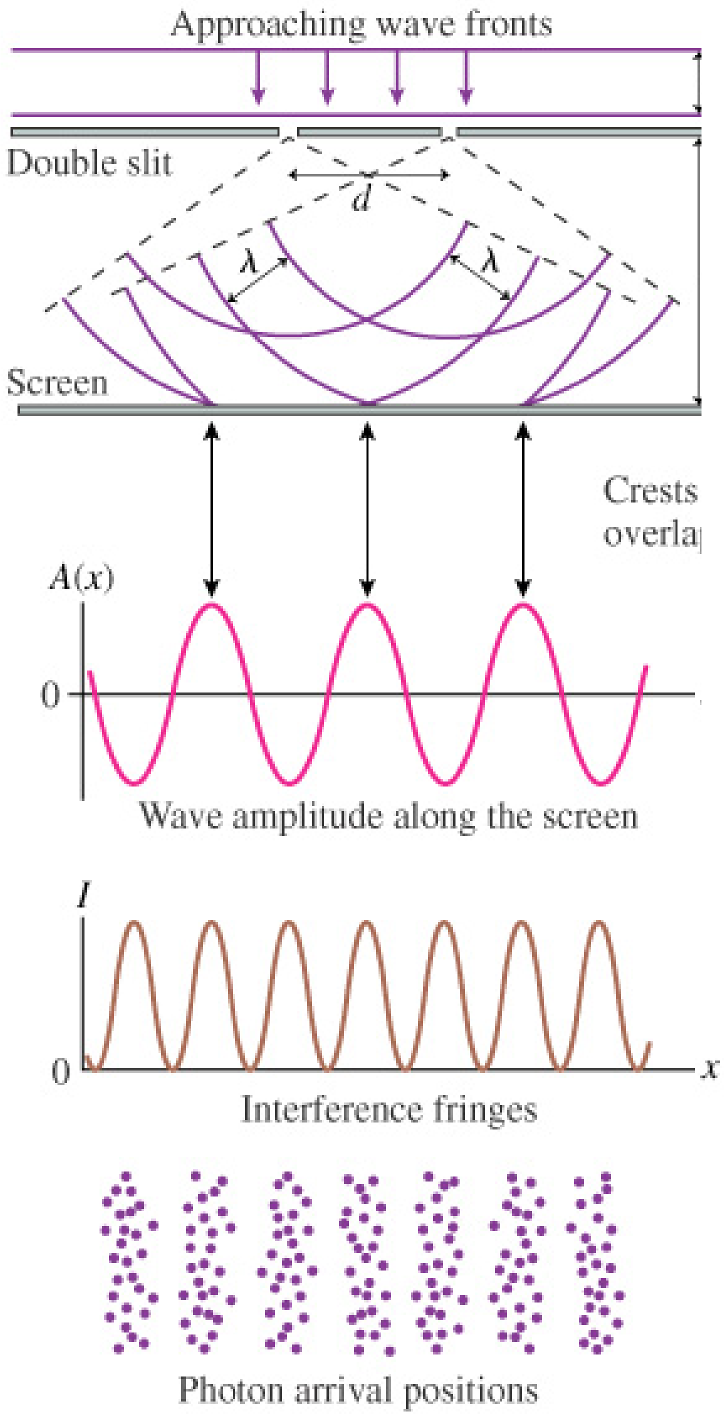
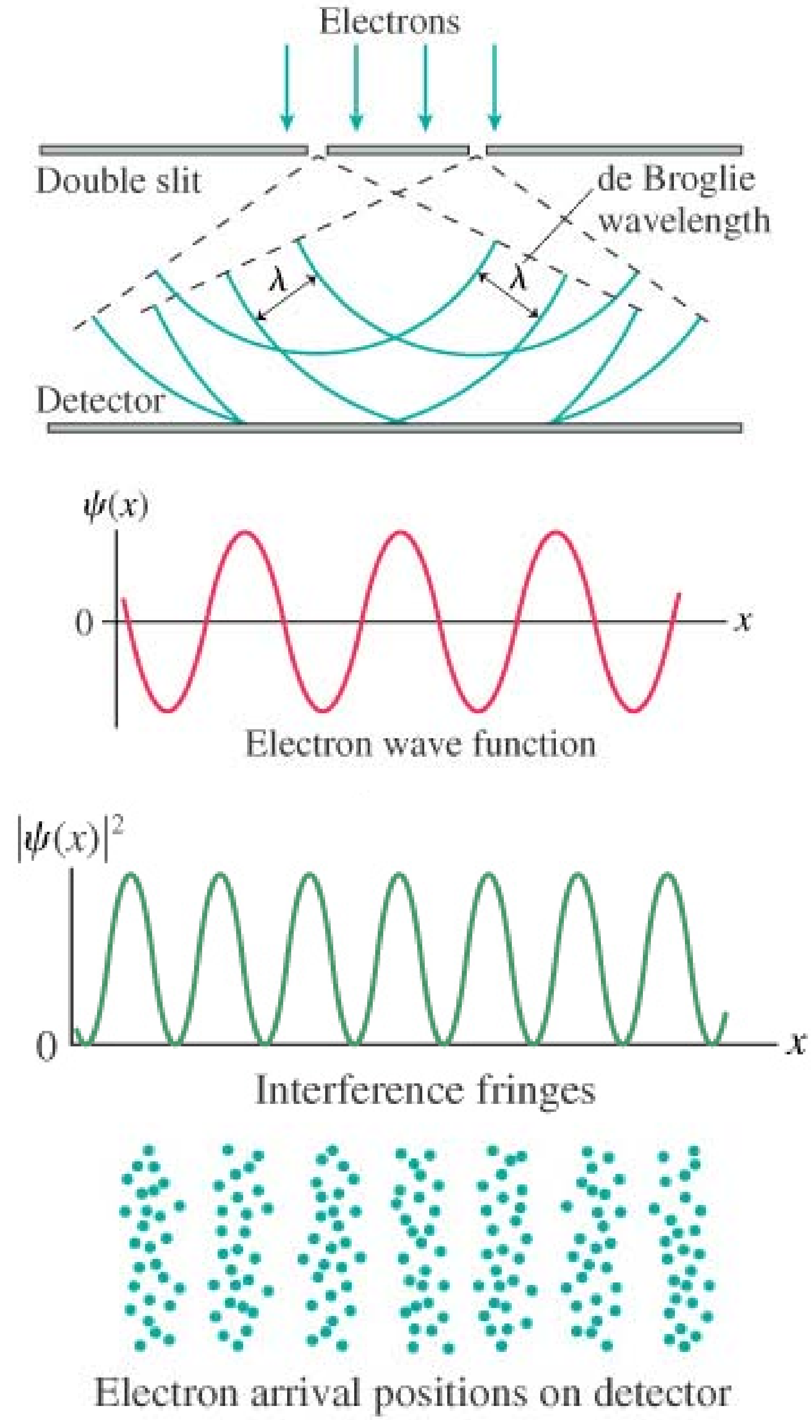


- Interference maxima are places where we are more ***likely*** to detect a photon impact
- We can think of the intensity distribution as a distribution of ***probability***
  - where classical physics predicts an intensity maximum, quantum mechanics predicts a high probability of detecting a photon

Wave Function

- For light, we can say that the ***amplitude of the electromagnetic wave*** pattern on the screen determines where we are likely to find a photon
- For matter, we have to define ***some other function*** whose amplitude tells us the probability of finding, say, an electron
- Define a ***wave function***, $\Psi(x)$, to quantify the probability of finding an electron

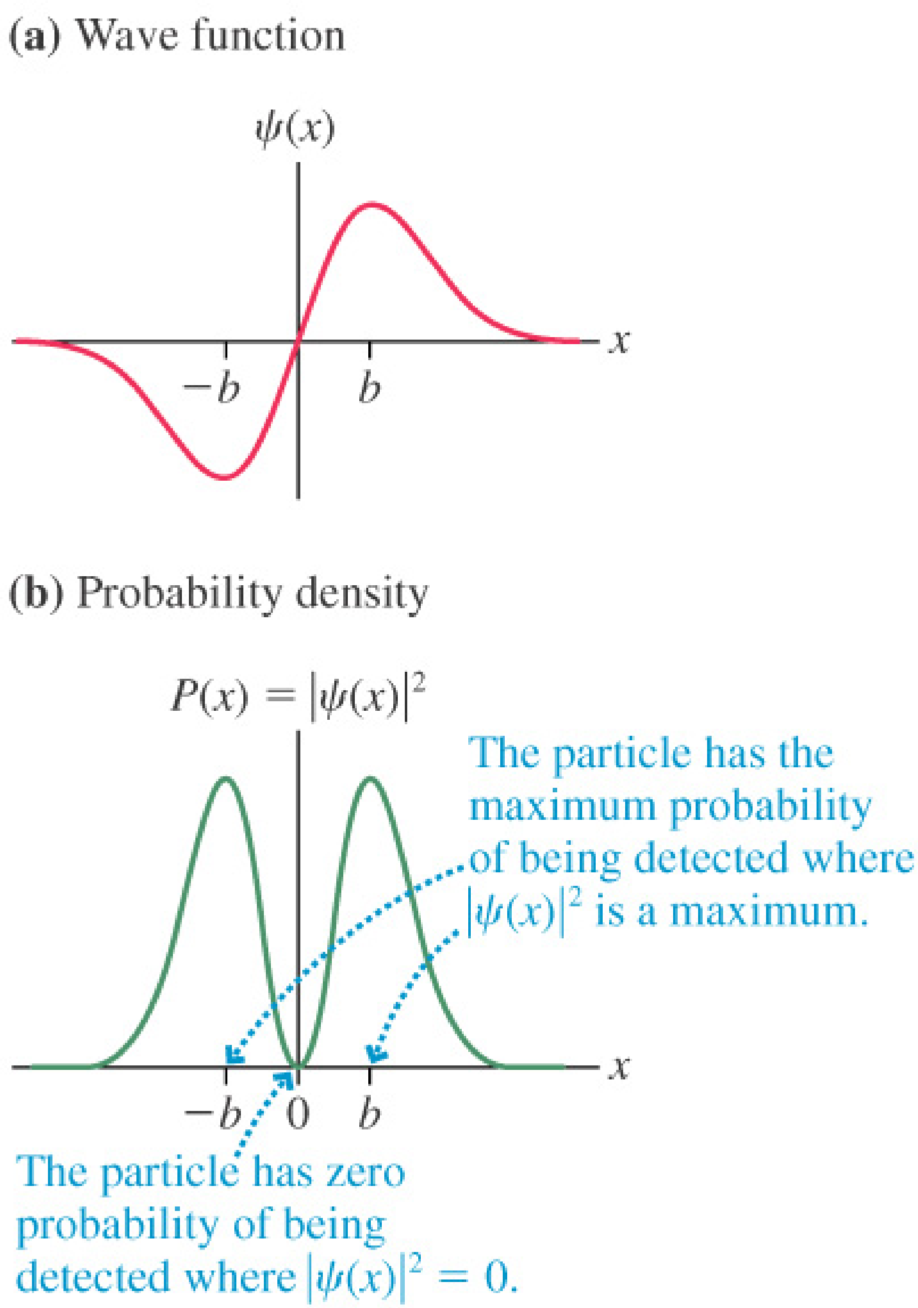

Approaching wave fronts
Double slit
d
λ
λ
Screen
A(x)
0
Wave amplitude along the screen
I
0
x
Interference fringes
Photon arrival positions


Electrons
Double slit
de Broglie
wavelength
λ
λ
Detector
ψ(x)
0
x
Electron wave function
|ψ(x)|²
0
x
Interference fringes
Electron arrival positions on detector

Wave Function → Probability

- The wave function, Ψ(x), plays a role similar to the amplitude of the wave pattern, A(x)
- Like the amplitude, Ψ(x) can have +ve and –ve values
- Probabilities are always +ve, so we define Ψ(x) such that the probability density, P(x) of finding a particle at position x, is:

$$I(x) \propto |A(x)|^2 \longrightarrow \boxed{P(x) = |\psi(x)|^2}$$



An arbitrary wave function, $\Psi(x)$, for some particle.

The probability density, $P(x) = |\Psi(x)|^2$, for this particle.

We are ***more likely*** to find the particle at positions, x, where P(x) is large, than at positions were P(x) is small.

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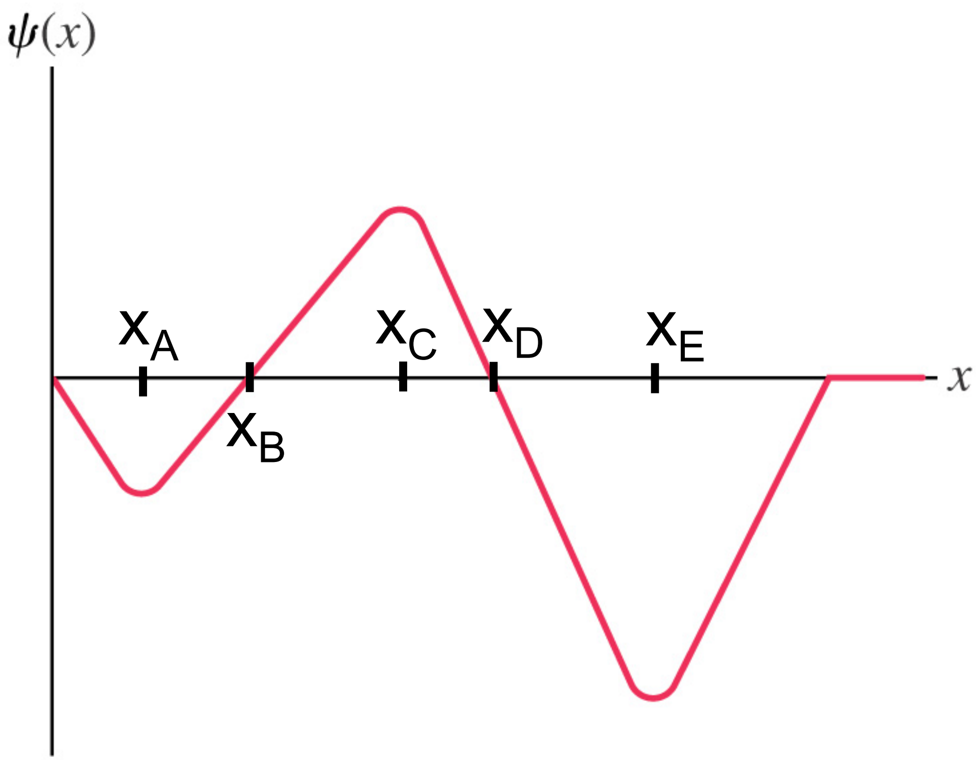
Quick Quiz 85

- The wave function of a proton is shown below. Where is it most likely to be found?



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A. x_A

B. x_B

C. x_C

D. x_D

E. x_E