Electric Potential

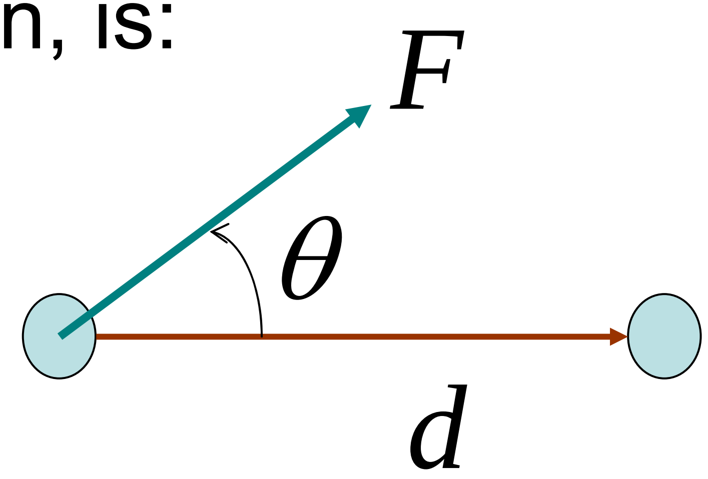
Concepts: Work & Conservative Forces
Potential Energy, Kinetic Energy
Conservation of Energy
Electric Potential Energy
Electric Potential
Electric Field and Potential

1st ed: 29.1-7, 30.1-3

2nd ed: 29.1-7, 30.1-4

Quick Quiz 49

The mechanical work done by a constant force, $\vec{F}$, which moves an object along a distance, d, as shown, is:



A. 0
B. $\vec{F} \cdot \vec{d}$
C. $\frac{1}{2} m(\Delta v)^2$
D. mgd
E. It depends on whether the force is conservative

Quick Quiz 50

The mechanical work done by a constant force, $\vec{F}$, which moves an object through a displacement, $\Delta\vec{r}$, is:

A. 0

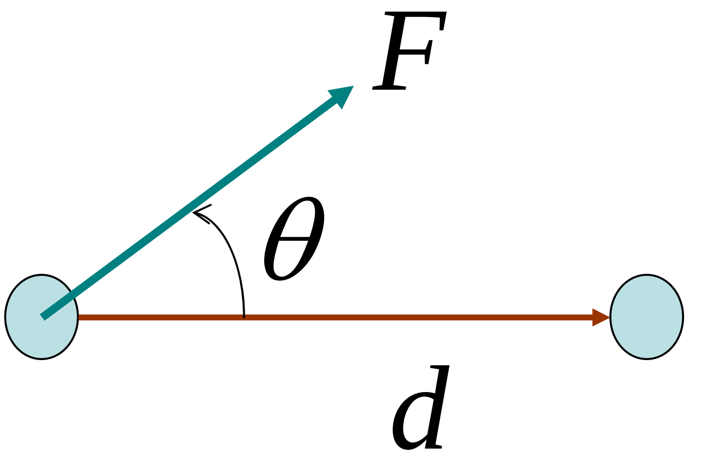
B. $\vec{F} \cdot \Delta\vec{r}$

C. $F \Delta r \cos\theta$

D. It depends on whether the force is conservative

- ***Work*** done by force $\vec{F}$ in moving a particle through distance, d:

$$W = \vec{F} \cdot \vec{d} = \left|\vec{F}\right|\left|\vec{d}\right| \cos\theta$$



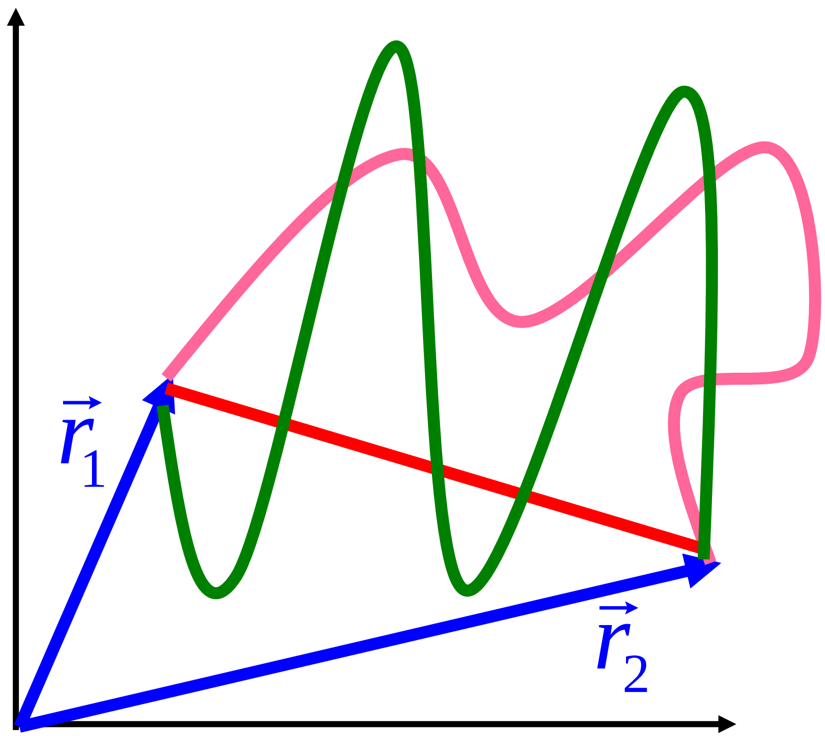
- Work done by a ***conservative force*** is independent of the path traveled by the object the force is acting on:

$$W = \vec{F} \cdot \Delta\vec{r} = \vec{F} \cdot \underbrace{(\vec{r}_1 - \vec{r}_2)} = \left|\vec{F}\right|\left|\Delta\vec{r}\right| \cos\theta$$

only depends on initial and final positions

Work done by a non-conservative force depends on the path taken:

$$W = \int \vec{F} \cdot d\vec{r} \neq \vec{F} \cdot \Delta\vec{r}$$



Work done by a conservative force is the same along all three paths.

Work done by a non-conservative force is different along all three paths.

Quick Quiz 51

Which force does not conserve mechanical energy:

A. Gravity
B. Electrostatic
C. Friction

Quick Quiz 52

If kinetic friction acts on an object, the total energy of that object plus its environment is

A. conserved
B. not conserved

- Work-kinetic energy theorem:

$$W = \Delta K = K_f - K_i = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2$$

The work done by a force on a particle as it is displaced from its initial position to its final position is equal to the change of the particle's kinetic energy.

- If the work done on a object is done by a conservative force, we can write:

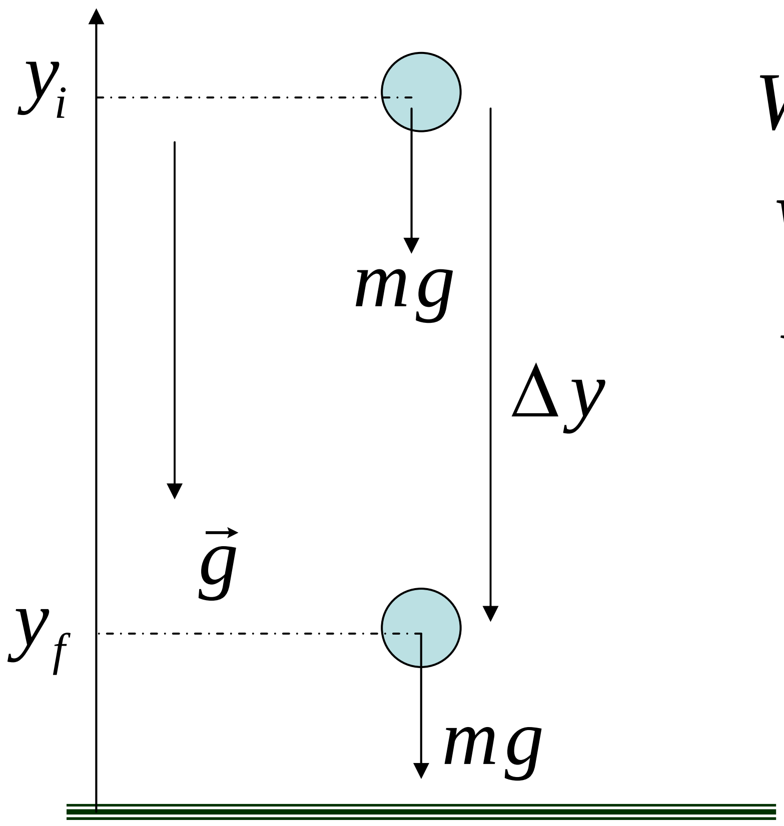
$$\Delta U = U_f - U_i = -W_{c(i \to f)}$$

work done by a conservative force in moving particle from i to f

Work and Potential Energy

- We can only define a potential energy corresponding to a conservative force
  - e.g. gravity
- We cannot define a potential energy corresponding to a non-conservative force
  - e.g. friction

Example: Gravity Near Earth's Surface



$$W_{grav(i\to f)} = \vec{F}_g \cdot \Delta\vec{y}$$

$$W_{grav(i\to f)} = (-mg)(y_f - y_i)$$

$$W_{grav(i\to f)} = mg(y_i - y_f)$$

$$W_{grav(i\to f)} = -\Delta U$$

$$\therefore \Delta U = mg\Delta y$$

This is how we **define** gravitational potential energy near the surface of the Earth.

Quick Quiz 53

The total mechanical energy of a system is conserved only if:

A. the net force acting on the system is zero
B. all of the forces acting on the system are conservative
C. angular momentum is conserved
D. the time rate of change of the linear momentum of the system is zero

Conservation of Energy

- Mechanical energy: $E_{mech} = K + U$
- W-KE theorem: $W = \Delta K$
- If only conservative forces act on the system: $\Delta U = -W$
- So if only conservative forces act on the system:

$$\Delta E_{mech} = \Delta K + \Delta U = W - W = 0$$

$\therefore$ mechanical energy is conserved when all of the forces acting on a system are conservative

Electric Potential Energy

- Electric forces are conservative (no surprise, since they take the same form as that other conservative force, gravity)
- Therefore, we can define an electric potential energy
- Gravitational potential energy:

$$\Delta U_g = mg\Delta y$$

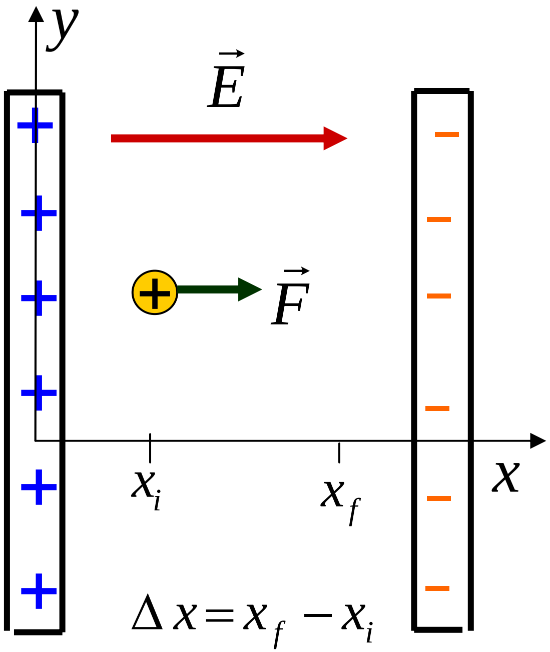
"charge" field displacement

- Electric potential energy:

$$\Delta U_e = qE\Delta r$$

charge — field — displacement

Electric Potential Energy in a Capacitor



Work done in moving a charge through the field of a capacitor:

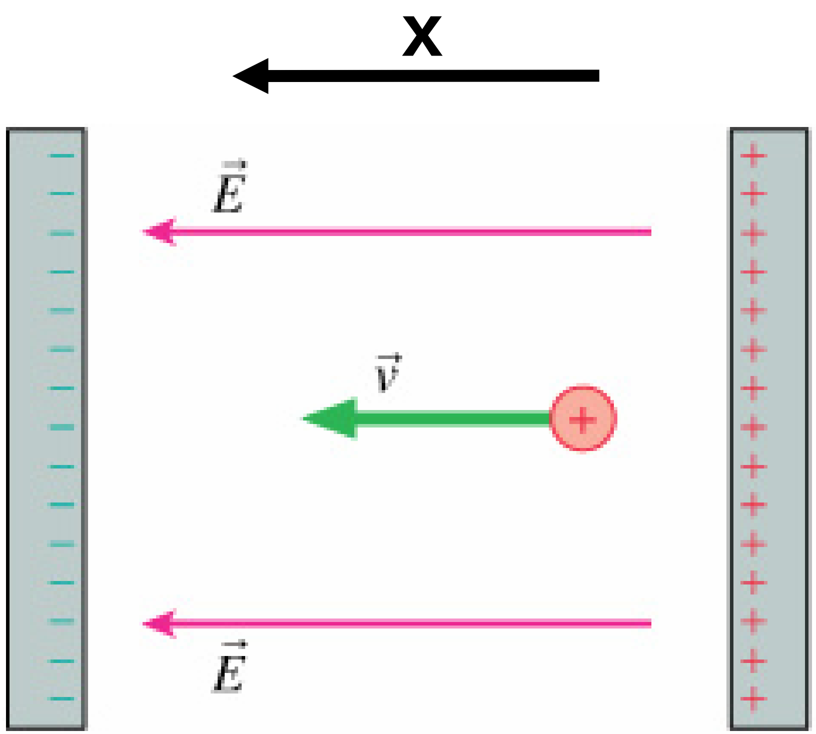
$$W_{elec} = F\Delta x$$

$$W_{elec} = qE\Delta x$$

Change in electrical potential energy:

$$\Delta U_{elec} = -W_{elec}$$

$$\Delta U_{elec} = -qE\Delta x$$



The potential energy of a positive charge decreases in the direction of $\vec{E}$. The charge gains kinetic energy as it moves toward the negative plate.

$$q > 0$$

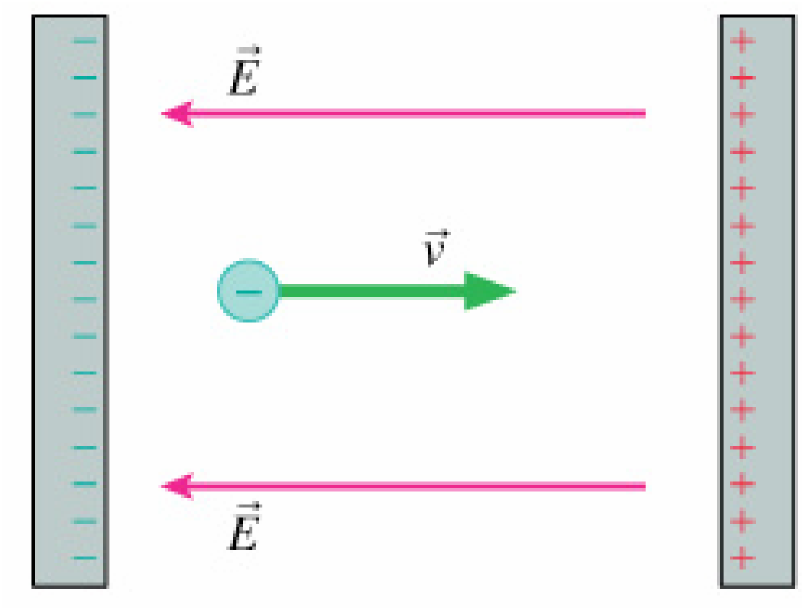
$$E > 0$$

$$\Delta x > 0$$

$$\therefore \Delta U_{elec} = -qE\Delta x < 0$$

Quick Quiz 54

Will this charge gain or lose electric potential energy?



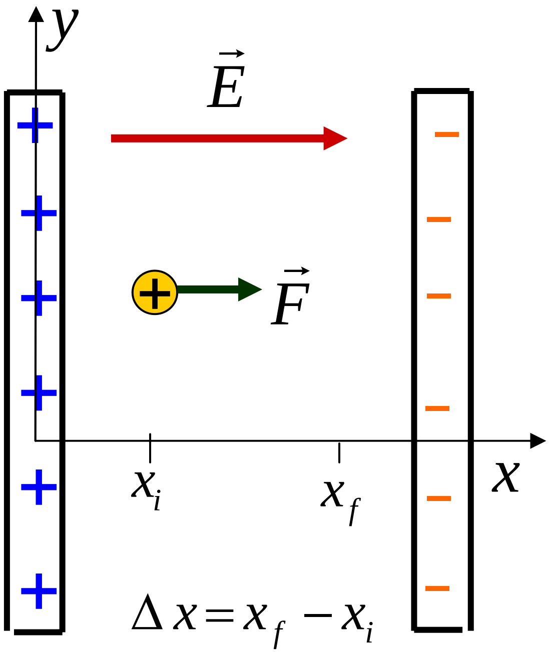
A. Gain

B. Lose

C. Remain unchanged

D. I’m guessing blindly

Electric Potential Energy in a Capacitor



Work done in moving a charge through the field of a capacitor:

$$W_{elec} = F\Delta x$$

$$W_{elec} = qE\Delta x$$

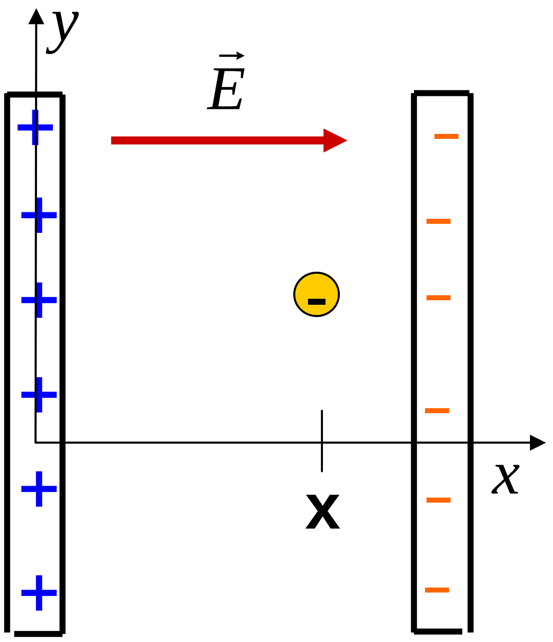
Change in electrical potential energy:

$$\Delta U_{elec} = -W_{elec}$$

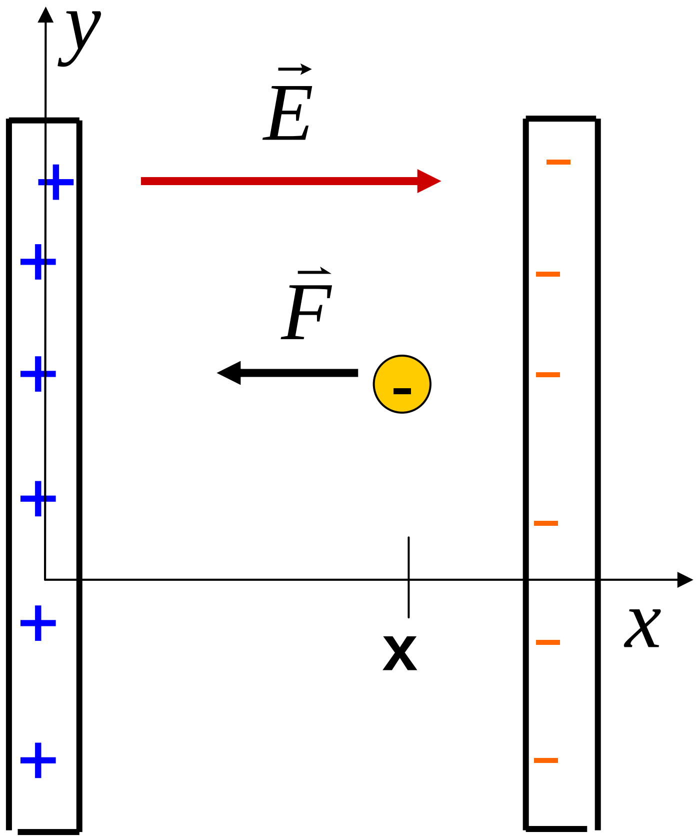
$$\Delta U_{elec} = -qE\Delta x$$

Example: Conservation of Energy

- An electron is released in the uniform electric field between the plates of a capacitor. If it is released from rest at a distance x from the positive plate, with what speed will it strike the positive plate?



- Can we apply conservation of energy?
- Only if the forces are conservative. Electric force is conservative ∴ yes!



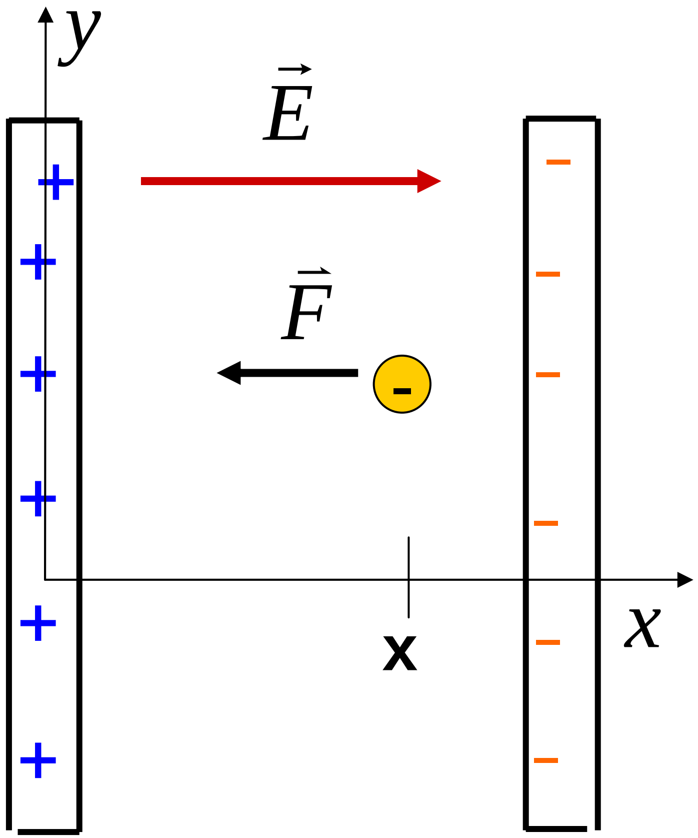
$$E_i = E_f$$

$$K_i + U_i = K_f + U_f$$

$$K_i + (U_i - U_f) = K_f$$

$$K_i - \Delta U = K_f$$

- Can we apply conservation of energy?
- Only if the forces are conservative. Electric force is conservative $\therefore$ yes!



$$K_i - \Delta U = K_f$$

$$0 + eEx = \tfrac{1}{2} m_e v_f^2$$

$$\boxed{v = \sqrt{2eEx/m_e}}$$

Work Done Along an Arbitrary Path

- In general, for a small displacement $d\vec{r}$, the work done by the field on a positive test charge is:

$$d\,W = \vec{F}.d\,\vec{r} = q_0\,\vec{E}.d\vec{r}$$

- The potential energy of the charge changes by:

$$dU = -\,q_0\,\vec{E}.d\,\vec{r}$$

- The change in Potential Energy of the test charge in the electric field moving from some initial position to some final position is:

$$\Delta U = U_f - U_i = -\,q_0 \int_i^f \vec{E} \cdot d\,\vec{r}$$

Electric Potential Energy of Point Charges

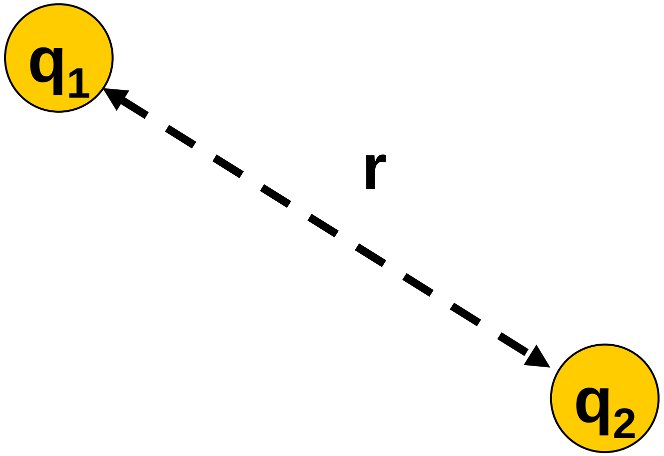
What is the electric potential energy of two point charges?

$$\Delta U = -q_1 \int_i^f \vec{E} \cdot d\vec{r}$$

$$= -q_1 \int_i^f \frac{k_e q_2}{r^2} dr$$

$$= q_1 k_e q_2 \left[\frac{1}{r_f} - \frac{1}{r_i} \right]$$

$$= \frac{k_e q_1 q_2}{r_f} - \frac{k_e q_1 q_2}{r_i}$$



- If the ***change*** in U is:

$$\Delta U = \frac{k_e q_1 q_2}{r_f} - \frac{k_e q_1 q_2}{r_i}$$

- If we set $U_e(r_i = \infty) = 0$ then:

$$U_e = k_e \frac{q_1 q_2}{r}$$ [J]

- This is the energy of the ***system*** of both charges, not just one of them.

Quick Quiz 55

A proton and an electron start at some distance away from one another and are brought closer together. How does the potential energy of the system change?

A. increases

B. decreases

C. stays the same

D. we cannot tell based on the information given

Quick Quiz 56

What kind of work is done by the field in the previous question?

A. positive

B. negative

C. none

If we bring two protons together:

$$U_e = k\frac{(e)(e)}{r} = k\frac{e^2}{r}$$

$$\Delta U_e = U_{e,f} - U_{e,i} > 0$$

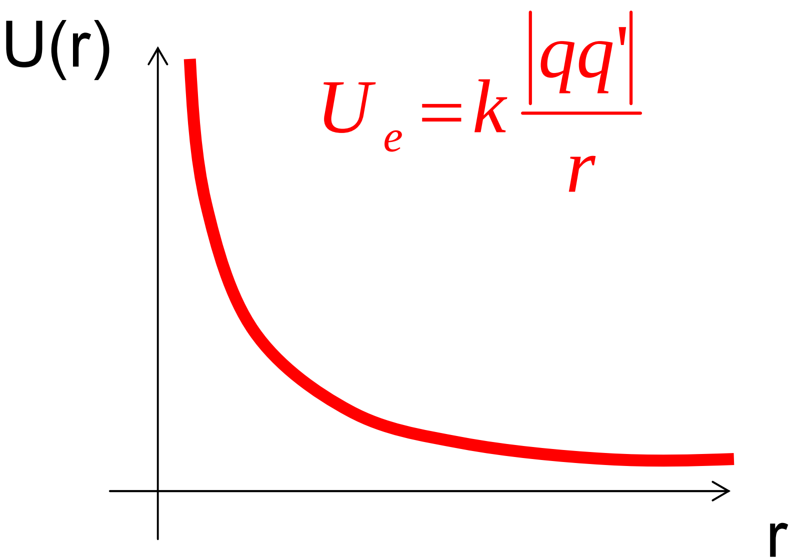
$$\therefore \Delta U_e > 0 \Rightarrow W_{elec} < 0$$

Some external agent has to do work to push like charges together.

Graphing U_e

- When a charge, q’, is brought close to another charge, q, the potential energy of the system of these two charges will:

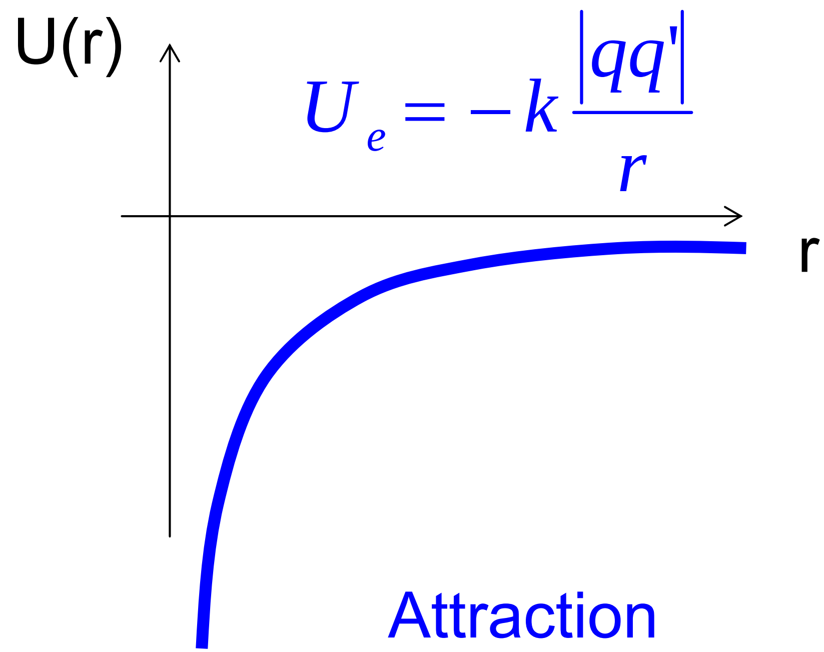
qq’>0



$$U_e = k\frac{|qq'|}{r}$$

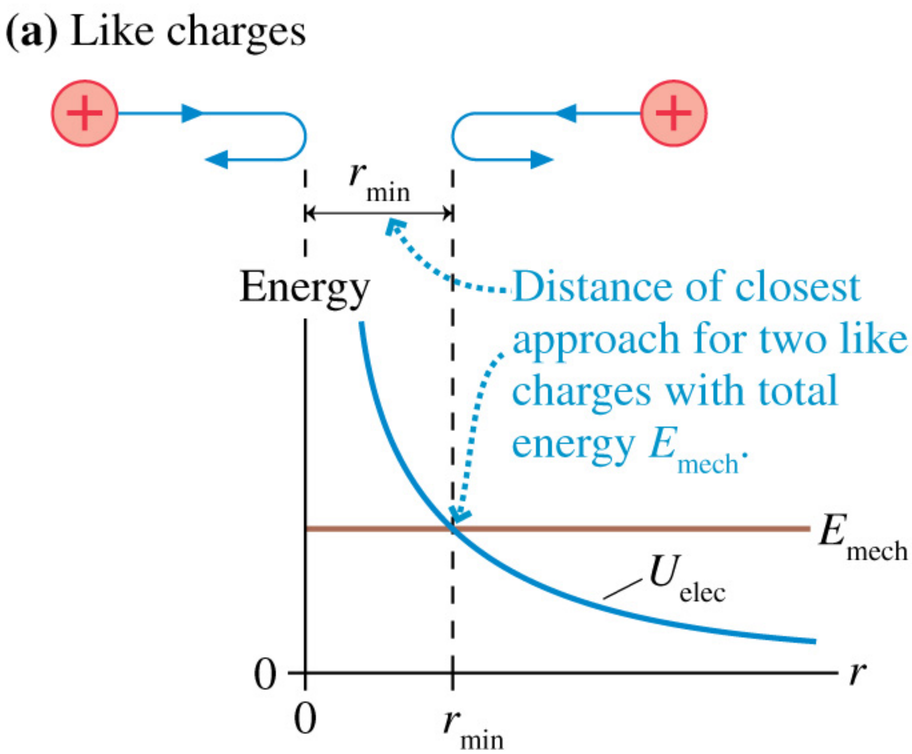
Repulsion

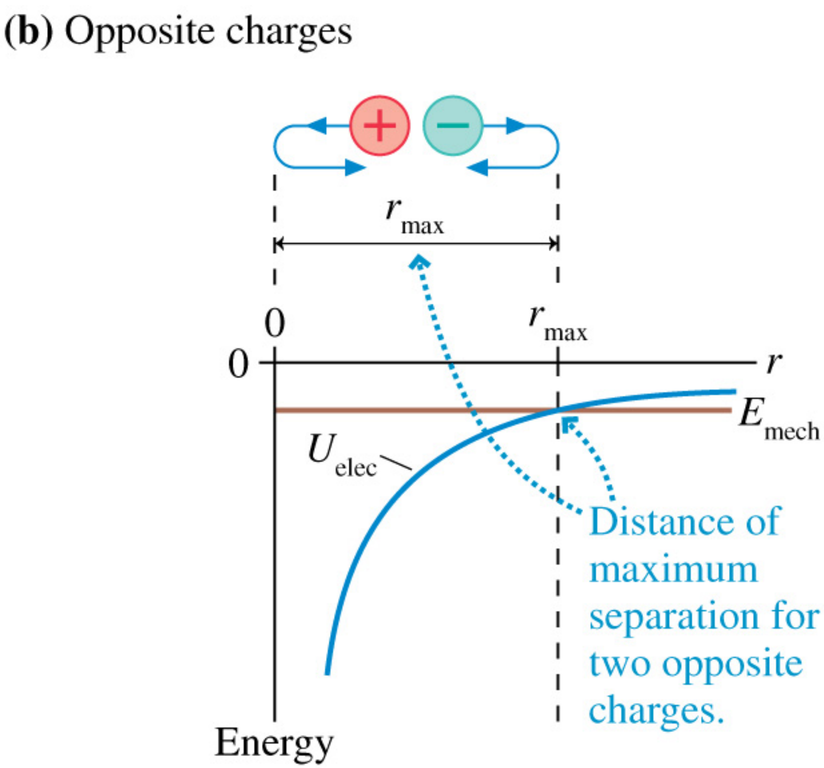
qq’<0



$$U_e = -k\frac{|qq'|}{r}$$

Attraction





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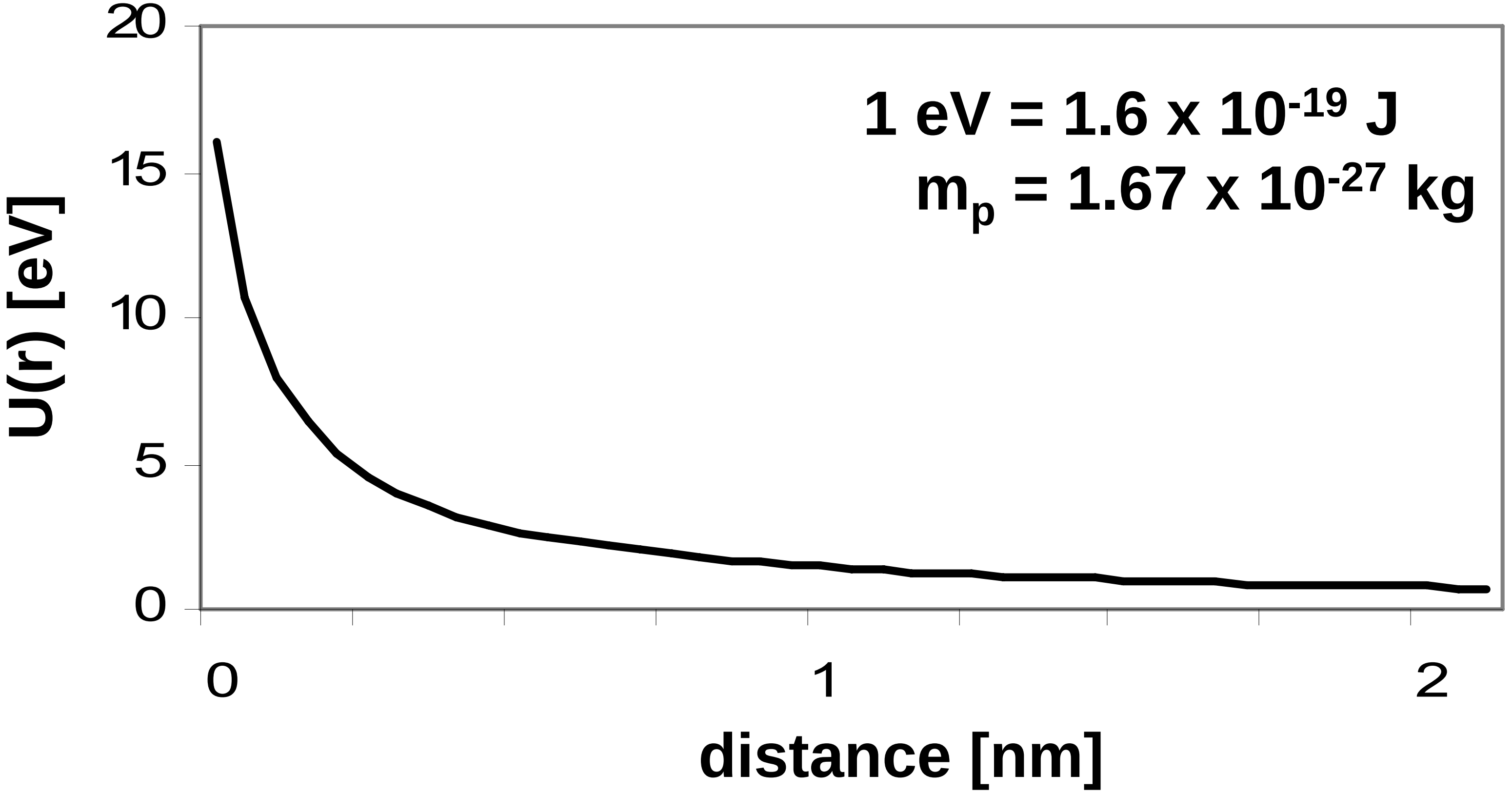
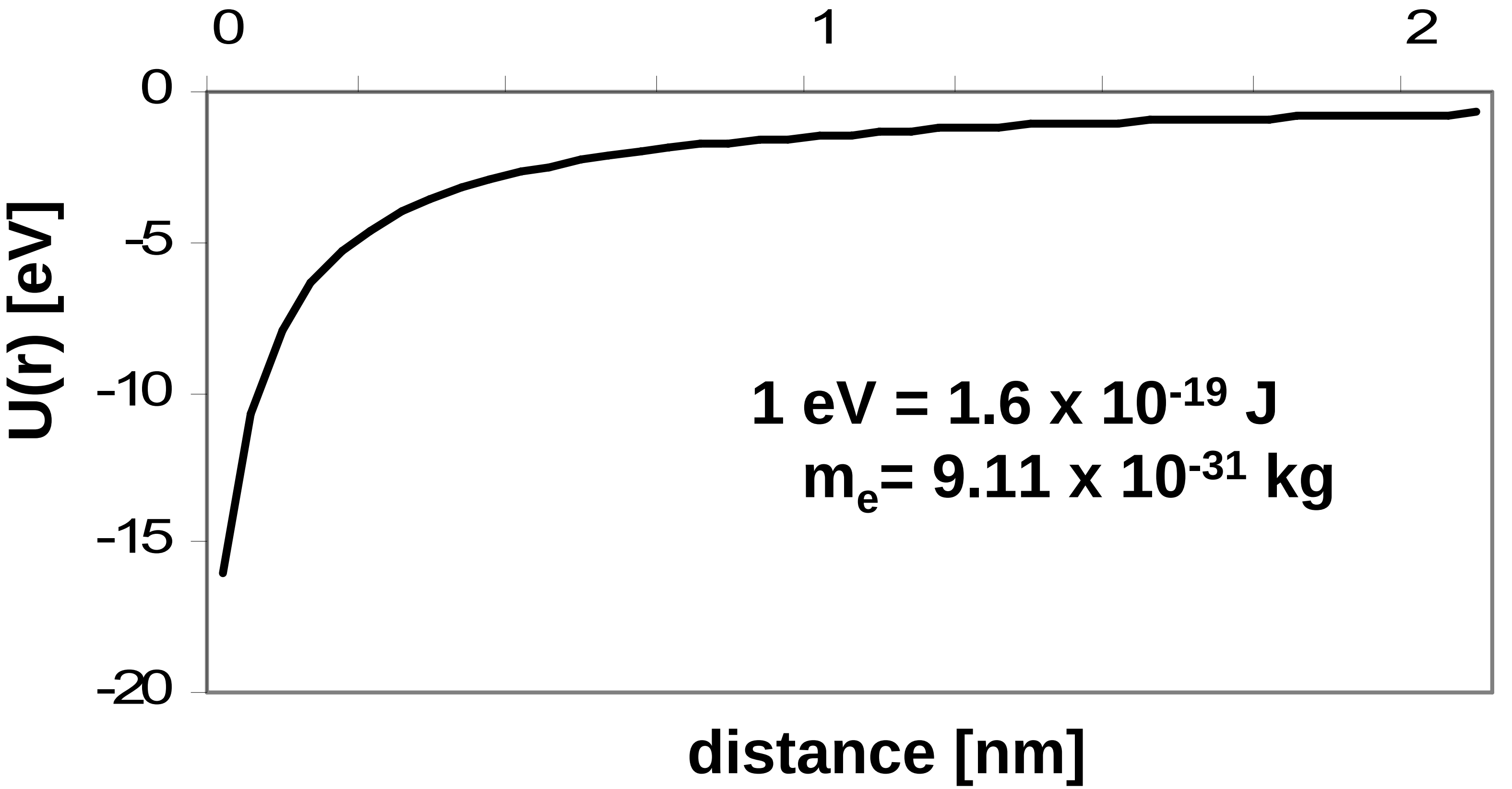
Quick Quiz 57

This graph describes potential energy as a function of separation between two charged protons. If one of the protons is at rest, and the other is moving radially toward it at a speed of 10^5 m/s, what is the distance at minimum separation?



A) r = 1 nm   B) r = 0.5 nm   C) r = 0.25 nm   D) r < 0.1 nm   E) r > 1nm

This graph describes potential energy as a function of separation between a proton and an electron. The proton is at rest. If the initial separation is 0.1 nm, what is the maximum speed of the electron if it is to remain bound to the proton?



1 eV = 1.6×10^{-19} J

$m_e = 9.11 \times 10^{-31}$ kg

A) 10^4 m/s B) 10^5 m/s C) 10^6 m/s D) 10^7 m/s E) 10^8 m/s

U_e of a System of Charges

- What is the potential energy of an arrangement of point charges?
  - Place the first one far away from the others. No work necessary to do this: $U_e = 0$
  - Bring in the second charge: $U_e = k\frac{q_1 q_2}{r_{12}}$
  - Bring in the third charge:

$$U_e = k\frac{q_1 q_2}{r_{12}} + k\frac{q_1 q_3}{r_{13}} + k\frac{q_2 q_3}{r_{23}}$$

  - Carry on for the rest of the charges

U_e of a System of Charges

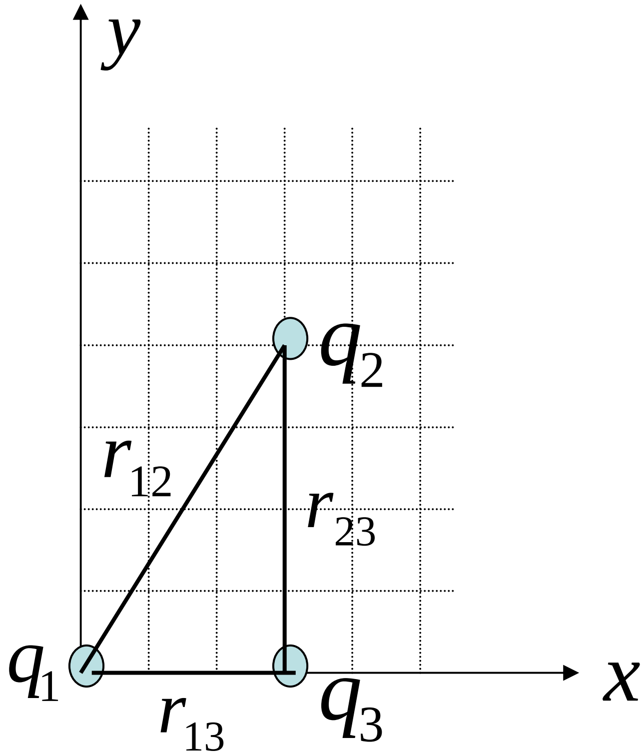
- Potential energy of a system of charges:

$$U_e = \sum_{i<j} \frac{k\, q_i\, q_j}{r_{ij}}$$

where r_{ij} is the distance between charges q_i and q_j

Example

- What is the potential energy of three point charges arranged as follows?
- Charges are $q_1 = +4.0\,\mu C$, $q_2 = +2.0\,\mu C$, $q_3 = -3.0\,\mu C$; and are located at (0,0) m, (3,4) m, and (3,0) m respectively.



$$U_e = k\frac{q_1 q_2}{r_{12}} + k\frac{q_1 q_3}{r_{13}} + k\frac{q_2 q_3}{r_{23}}$$

Example

- What is the potential energy of three point charges arranged as follows?
- Charges are $q_1 = +4.0\,\mu C$, $q_2 = +2.0\,\mu C$, $q_3 = -3.0\,\mu C$; and are located at $(0.0, 0.0)\ m$,$(3.0, 4.0)\ m$, $(3.0, 0.0)\ m$ respectively.

$$U_e = k\frac{q_1 q_2}{r_{12}} + k\frac{q_1 q_3}{r_{13}} + k\frac{q_2 q_3}{r_{23}}$$

$$U_e = 9.0\times10^9\ N.m^2/C^2 [\frac{(+4.0)(+2.0)}{5.0} + \frac{(+4.0)(-3.0)}{3.0} + \frac{(+2.0)(-3.0)}{4.0}]\times10^{-12}\frac{C^2}{m}$$

$$= -0.035\ J$$

Electric Potential

- A useful quantity when actually dealing with electricity (e.g. circuits) is the ***electric potential***

$$V = \frac{U_e}{q}$$

- Electric potential is the electric potential energy per unit charge
- Units:

$$1\ \mathrm{Volt} = 1V = 1\frac{J}{C}$$

Quick Quiz 59

The electric potential of a point charge, Q, is:

A. $V = \frac{k_e Q}{r}$

B. $V = k_e \frac{Qq}{r}$

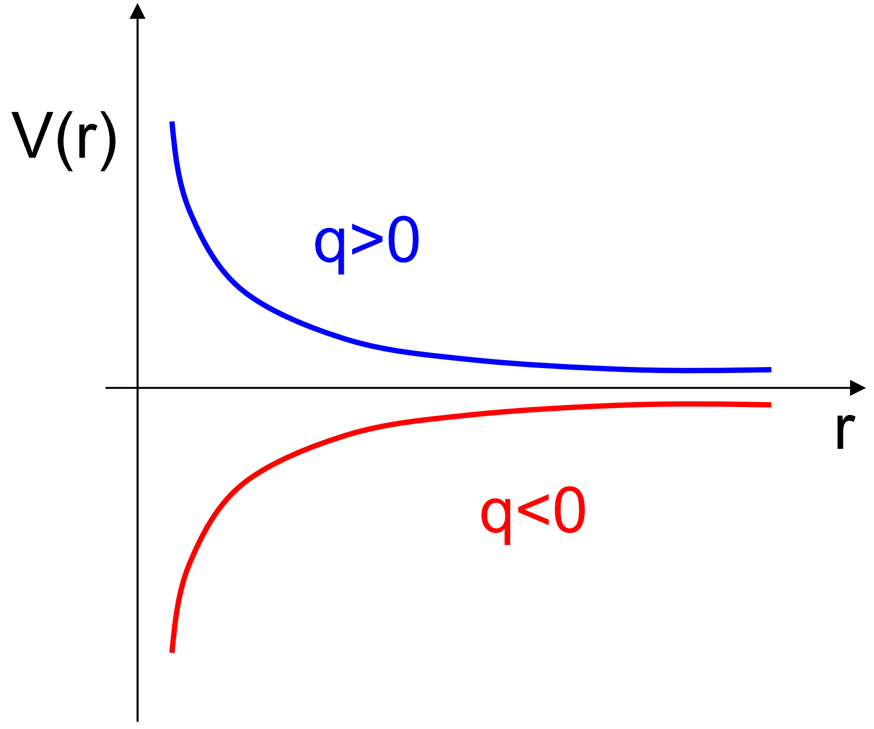
C. $V = \frac{k_e Q}{r^2}$

D. None of the above

V of a Point Charge

- The electric potential generated by a point charge, q, depends on the distance, r, to the charge:

$$V(r) = \frac{k_e q}{r}$$



Electric Potential Difference

- When a charge moves between two points in an electric field, the potential difference is:

$$\Delta V = V_f - V_i = \frac{U_{e,f}}{q} - \frac{U_{e,i}}{q} = \frac{\Delta U_e}{q}$$

- The charge's potential energy will change:

$$\Delta U_e = q\,\Delta V$$

- Conservation of energy then dictates:

$$\Delta K + \Delta U_e = \Delta K + q\,\Delta V = 0 \qquad \boxed{\therefore \Delta K = -q\,\Delta V}$$

∴A charged particle speeds up or slows down when it moves through a potential difference.

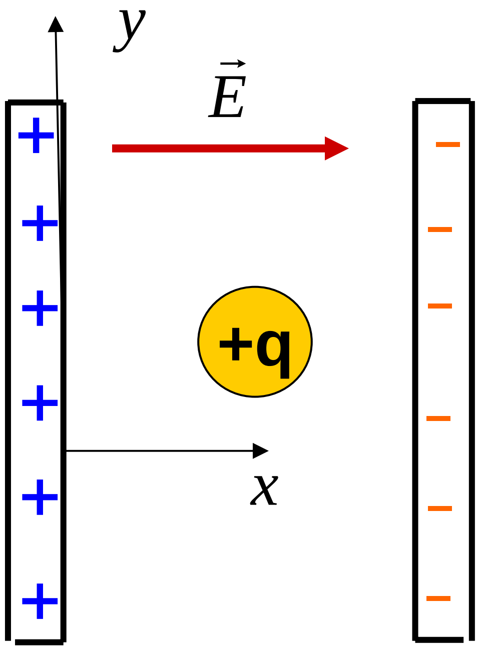
Electric Potential for Uniform E Field

- Consider a uniform field, for example electric field inside a capacitor:
- Electric potential energy at x:

$$U_e(\vec{x}) = -q\,E\,x$$

- Electric potential at x:

$$V(\vec{x}) = \frac{U_e(\vec{x})}{q} = -\,E\,x$$



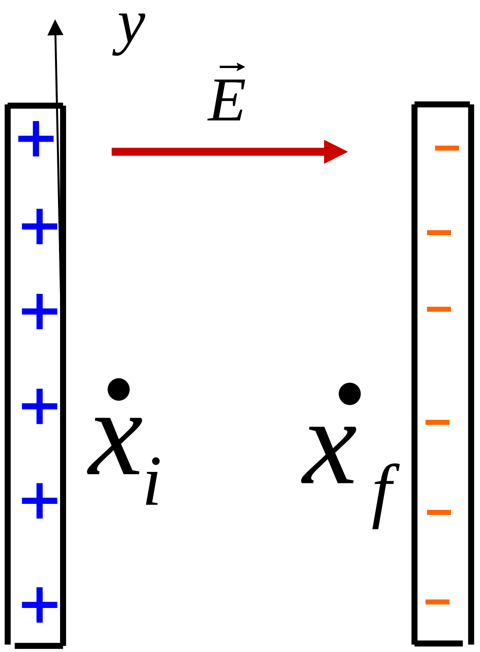
- Now consider the electric potential difference between two points:

$$\Delta V = V(x_f) - V(x_i)$$

Choose $V(x_i) = 0$.

$$\Delta V = V(x_f)$$

$$\Delta V = -E\,x$$



Quick Quiz 60

What is the potential difference across a capacitor consisting of two square plates, each 4 m x 4 m, separated by 10 cm, creating a field of 10 N/C.

A. -0.1 V
B. -0.16 V
C. -1 V
D. -1.6 V

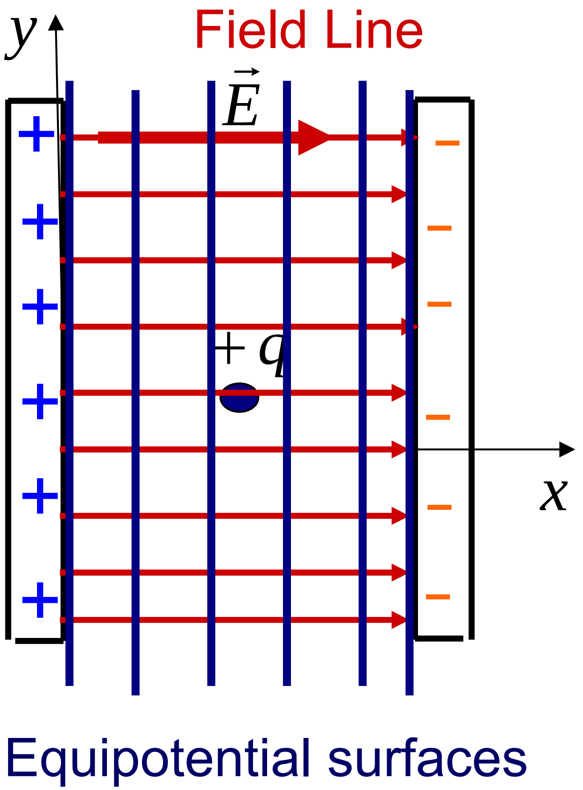
Equipotential Surfaces

- Properties of equipotential surfaces:
  - ΔV between any two points on surface is zero
  - no work done by electric field if charge moves between points on an equipotential surface
  - ***electric field is always perpendicular to the equipotential surface***

Equipotentials are drawn to look like contour maps.
The density of contour lines describe rate of change of the electric potential.

Equipotential Surfaces for Uniform E

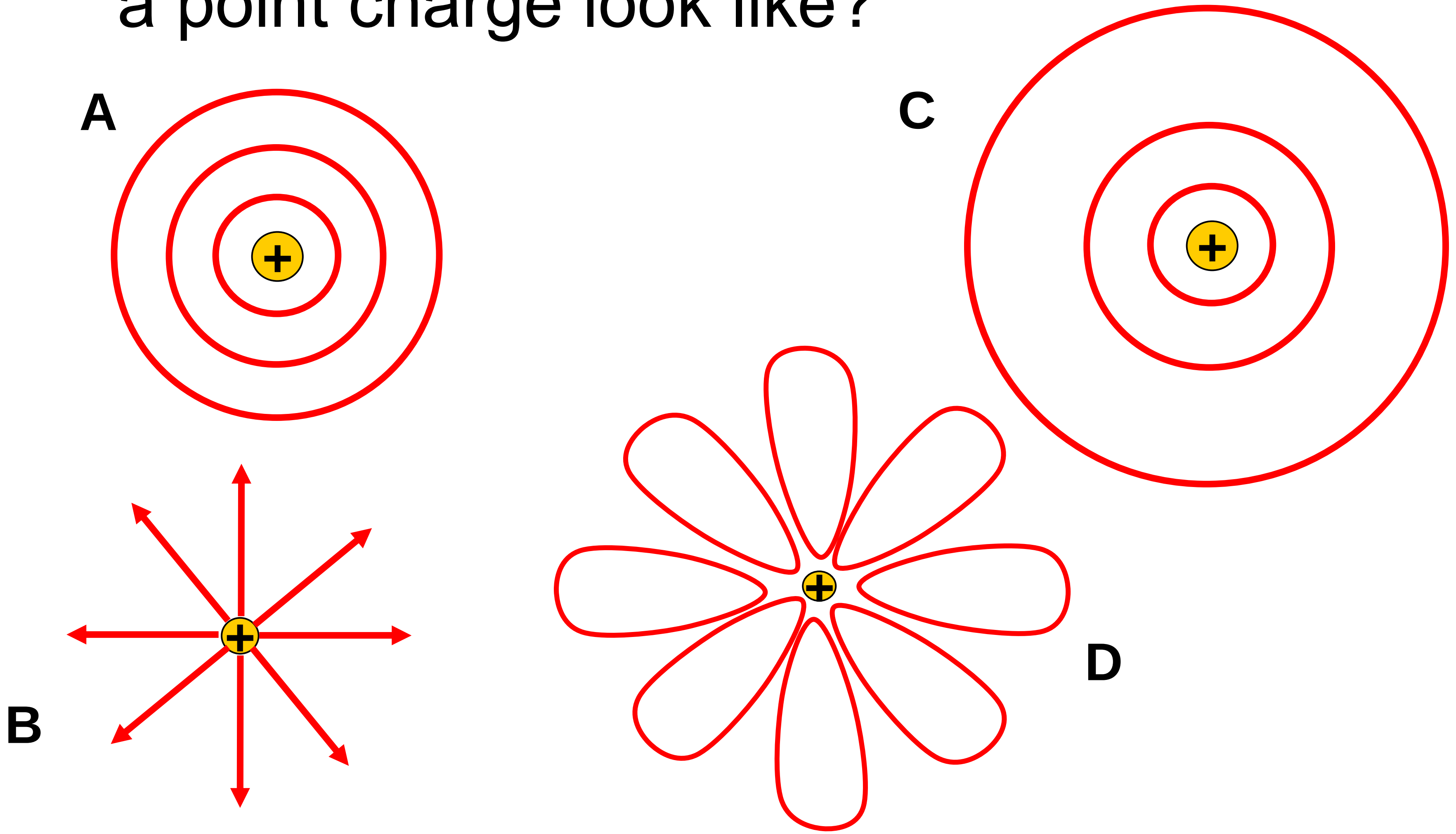
- Let's look at the electric field and electric potential inside a capacitor:



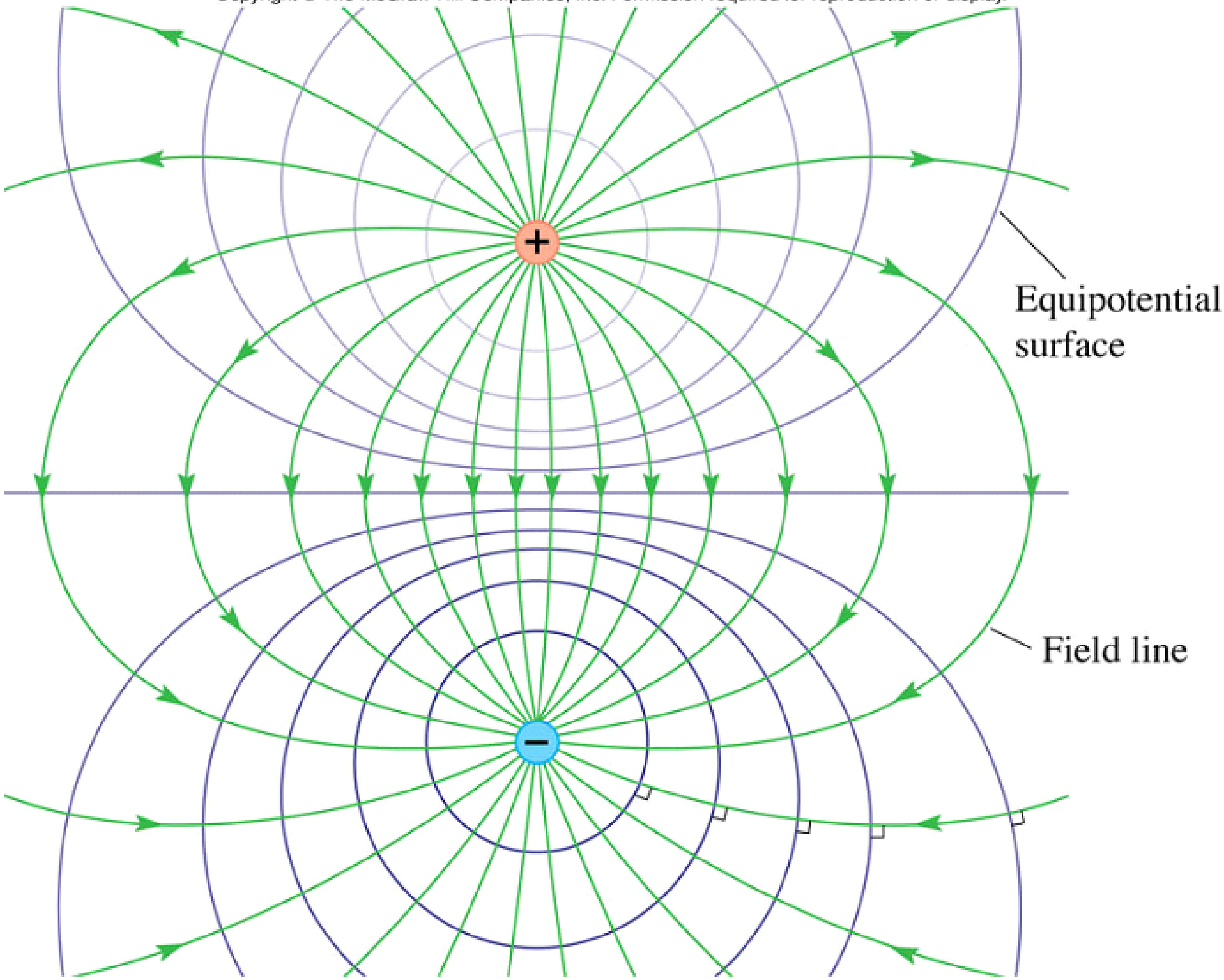
- The lines perpendicular to the plates are electric field lines.
- The planes parallel to the plates are equipotential surfaces.
- The plates themselves are equipotential surfaces.
- The equipotential surfaces are perpendicular to the electric field lines (as expected).

Quick Quiz 61

- What do the electric equipotentials around a point charge look like?



Equipotential surface and field lines of a dipole



Electric Potential of Multiple Charges

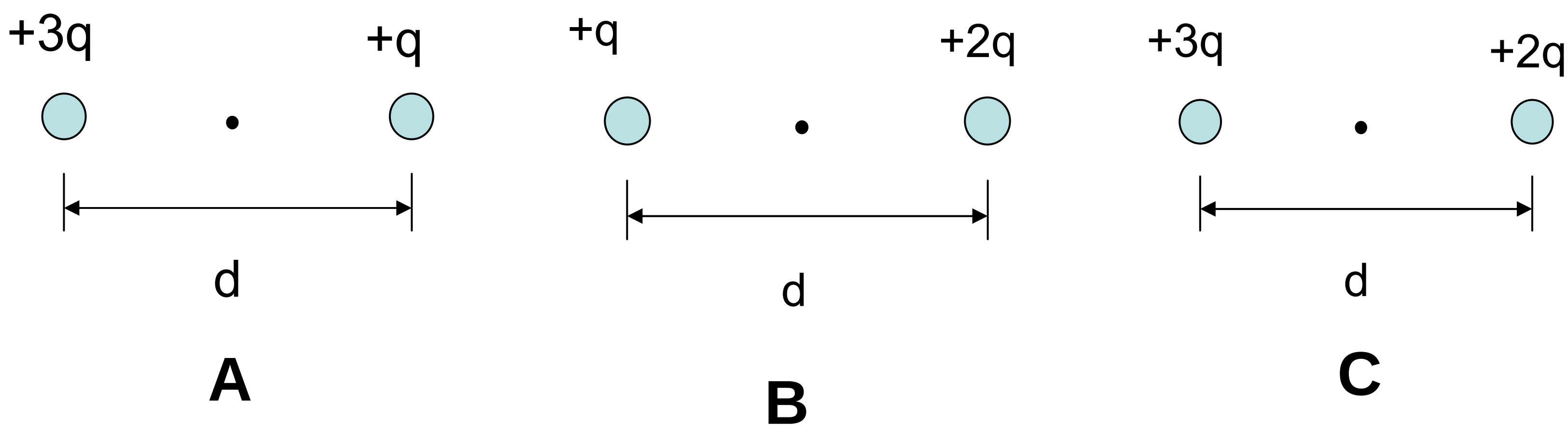
- The potential at a point in space due to several point charges is:

$$V = \sum_i k_e \frac{q_i}{r_i}$$

- Where r_i is the distance from charge q_i to the point where you are sampling the potential

Quick Quiz 62

Which configuration of charges produces a potential of V = 6kq/d at the dot?



Electric Potential of Charged Sphere

- A uniformly charged sphere of radius R and total charge Q:

$$V = K\frac{Q}{r} = k_e \frac{Q}{r} \qquad r \geq R$$

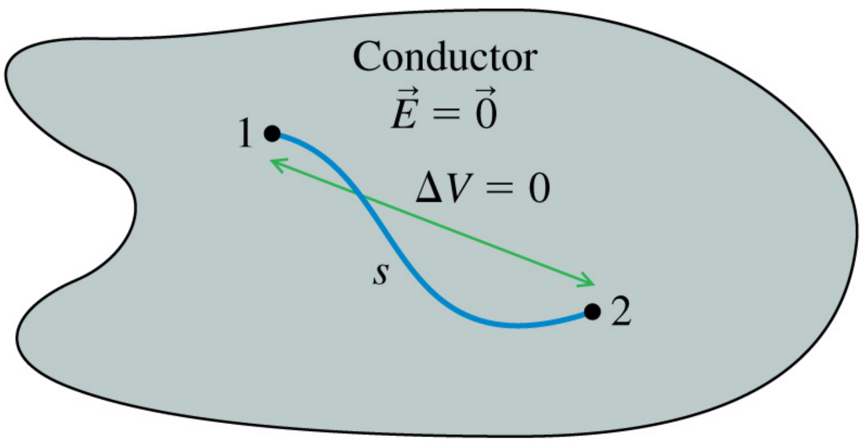
- Potential on the surface of the sphere:

$$V_0 = V(r = R) = k_e \frac{Q}{R}$$

- Potential outside a sphere charged to potential V_0:

$$V = (R/r)\, V_0$$

- Remember that E=0 everywhere inside a conductor
- If E = 0, then the potential difference between any two interior points:



$$\Delta V = -\int_{s_i}^{s_f} E(s)ds = -\int_{s_i}^{s_f} 0ds = 0$$

- Hence, all points inside a conductor are at the same electric potential

Moving Charge in an E Field

- When a charge moves from one position to another in electric field, the total energy stays constant (because the electric force is conservative)
- All that happens is kinetic and electric potential energy are inter-converted

$$E_i = E_f$$

$$\Delta E = \Delta K + \Delta U = 0$$

$$\Delta K = -\Delta U$$

Example

- Point P is at a potential of 500.0 kV and point S is at a potential of 200.0 kV. The space between these points is evacuated. When a charge of +2e moves from P to S, by how much does its kinetic energy change?

$$\Delta K = -\Delta U$$

$$\Delta K = -\Delta U = -q\Delta V$$

$$\Delta K = -(+2e)(200.0 - 500.0)\,kV = \boxed{9.6\times 10^{-14}\ J}$$

The particle loses electric potential energy, but gains kinetic energy.

Electric Potential Differences in Biology

Electric signals in neurons:

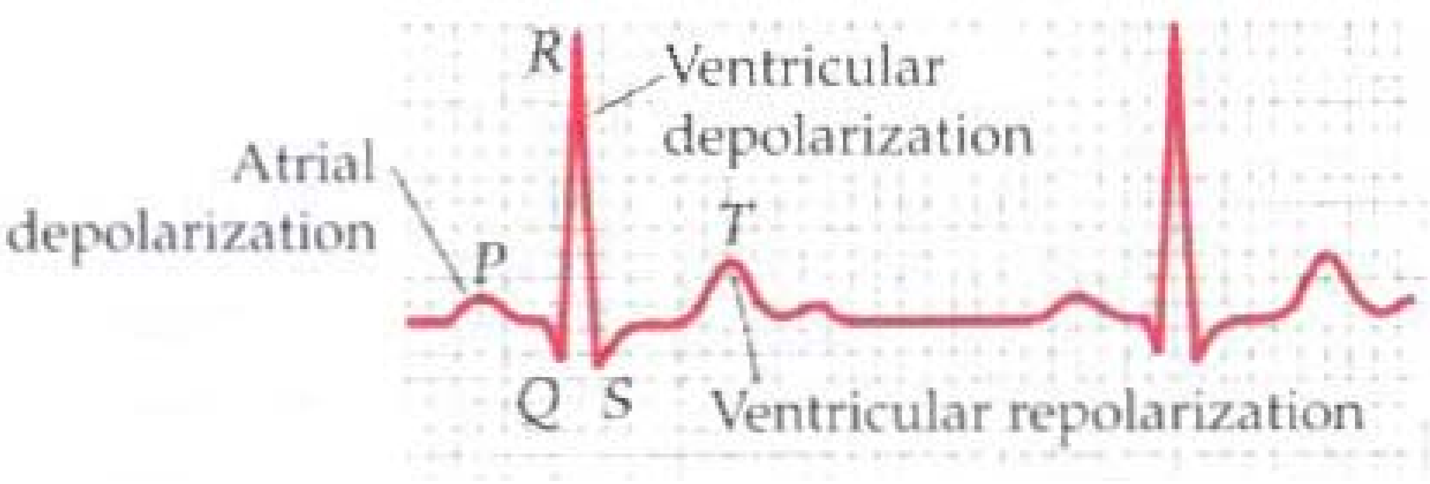
- Information is transmitted in body in the form of electrical signals (electric impulse) by the nerves.
- Electric potential difference plays a key role in this process.
- There is a potential difference between inside and outside of a biological cell across the cell membrane.
- This potential difference is due to different concentration of ions in the fluid inside and outside the cell.

Potential Differences in the Human Body

- Human body is not an equipotential surface. There are potential difference between different points on the body.
- Muscle activity, the beating of the heart, and nerve impulses in the brain cause an electrical potential difference between two points on the skin.

Electrocardiogram

- An electrocardiogram (EKG) shows the potential difference between points on the body as a function of time.



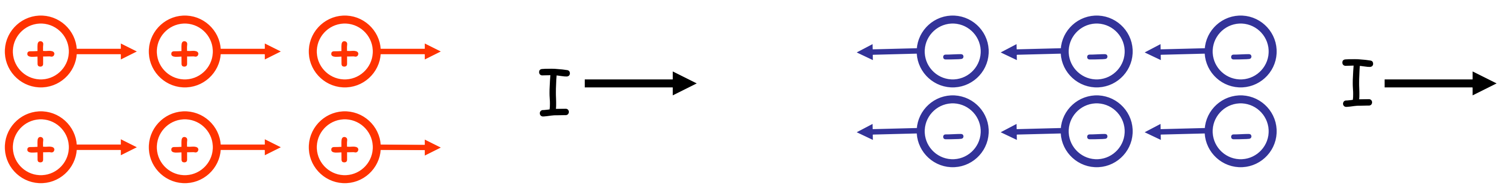
Current & Resistance   (Chap 28)

CURRENT exists wherever there is a net flow of charge.
Defined as *charge per unit time* flowing past a given point

$$I \equiv dQ/dt$$

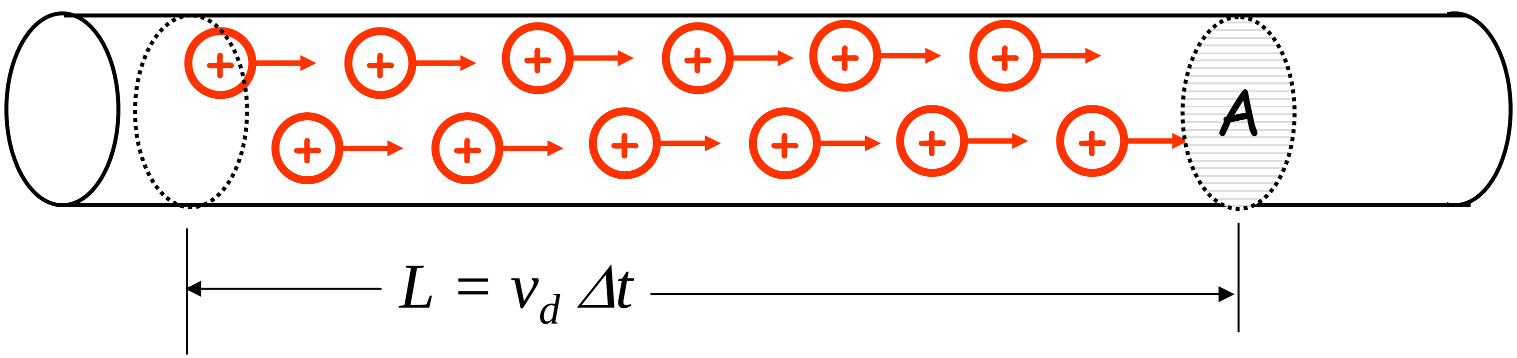
Units: 1 ampere (A) = 1 C/s

Direction: current goes in the same direction as **positive** charge (electrons move in the opposite direction)

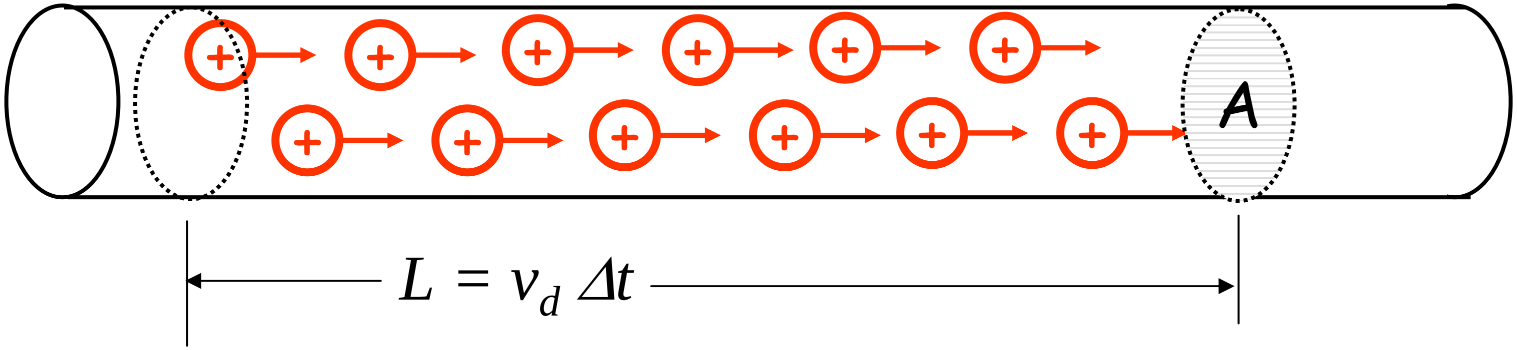


How fast are charges moving in a wire?

Assume: n = number of mobile charges/unit volume
q = charge on each one
A = area (cross-section) of wire
v_d = average ("drift") velocity of each charge



After time Δt, all the charges within a cylinder of length $L = v_d\,\Delta t$ will have passed through the shaded circle.



Charge passing through the shaded circle in time Δt :

Q = (number of charges/volume) x (charge on each one) x volume

$Q = n\,q\,(AL) = nqAv_d\,\Delta t$

Current: $I = Q/\Delta t = nqAv_d\,\Delta t/\Delta t$

So, $$I = nqAv_d$$

Example 25:

The mobile charges in metals are electrons, with about one or two electrons per atom being free to move. There are about 10^{29} mobile charges per cubic metre of copper.

For a 14 gauge copper wire which has about 1 square millimetre of cross-sectional area and carries a current of 15 amperes:

What is the approximate drift velocity of the electrons?

Pachinko demo