

Fluid Mechanics

- Pressure
- Pascal's Law
- Pressure and Gravity
- Buoyancy

Fluids

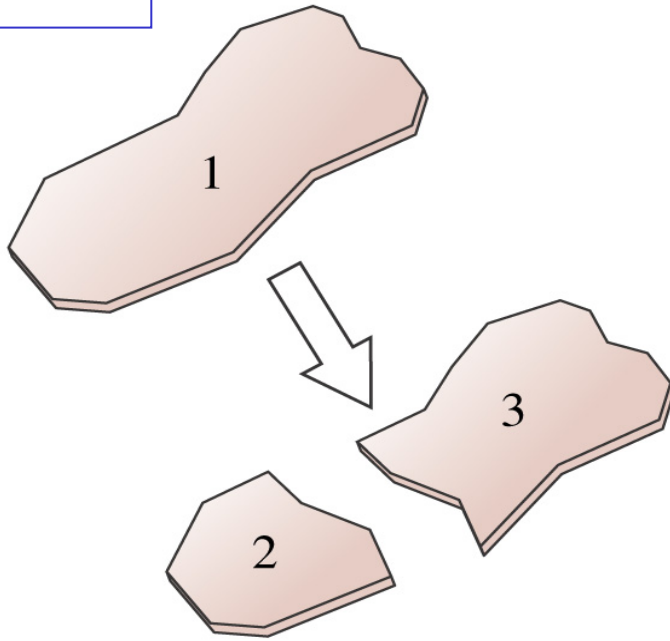
- Includes liquids and gases. No resistance to “shear” (changes in shape), in equilibrium.
- To describe mechanics of a continuous fluid (instead of a discrete object), we use density, pressure instead of mass and force.

Density

Density, ρ ("rho"), is mass per unit volume (kg/m^3).

Substance	ρ
<i>water</i>	1000 kg/m^3
<i>mercury</i>	13600 kg/m^3
<i>air</i>	1.29 kg/m^3
<i>helium</i>	0.18 kg/m^3

Quick Quiz



A piece of glass is broken into two pieces of different size. Rank order, from largest to smallest, the mass densities of pieces 1, 2, and 3.

- A. $\rho_1 > \rho_3 > \rho_2$
- B. $\rho_2 > \rho_3 > \rho_1$
- C. $\rho_1 > \rho_2 = \rho_3$
- D. $\rho_2 = \rho_3 > \rho_1$
- E. $\rho_1 = \rho_2 = \rho_3$

Pressure

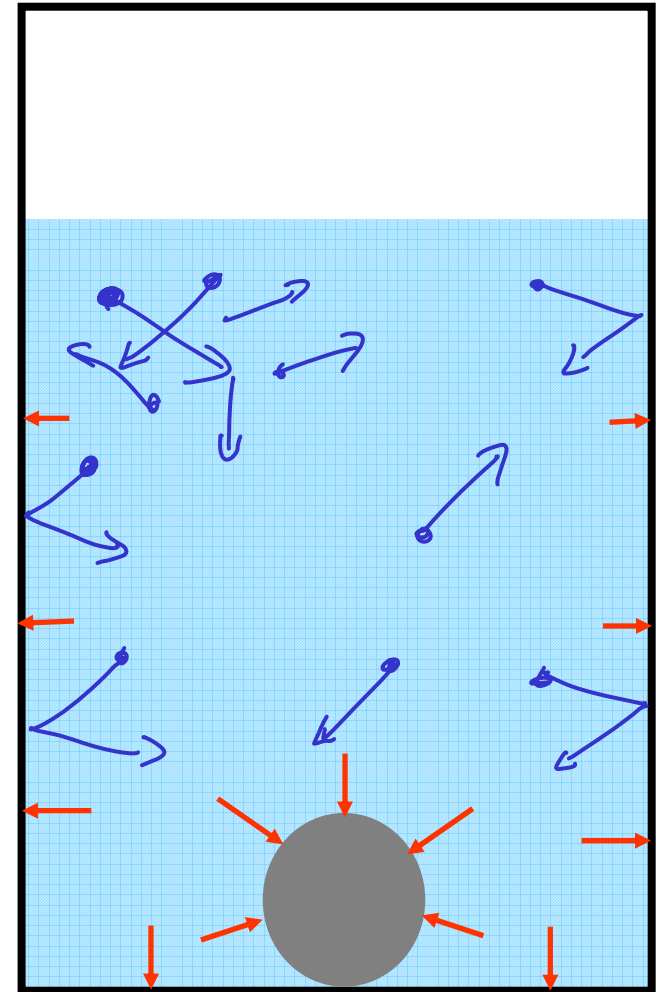
$P \equiv$ force per unit area

unit: $1 \text{ N/m}^2 = 1 \text{ pascal (Pa)}$

Also, 1 atmosphere (atm) = 101.3 kPa

Pressure is a scalar property of the fluid;
the force is always exerted perpendicular to the
surface in contact with the fluid.

*Forces exerted by
the fluid*

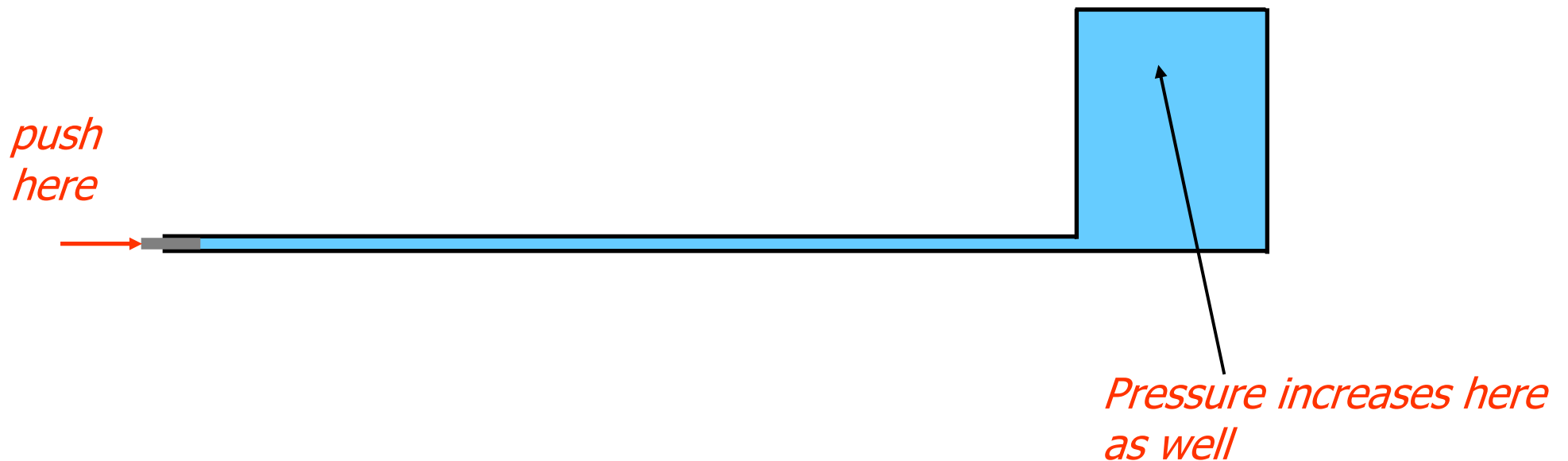


Pressure Units

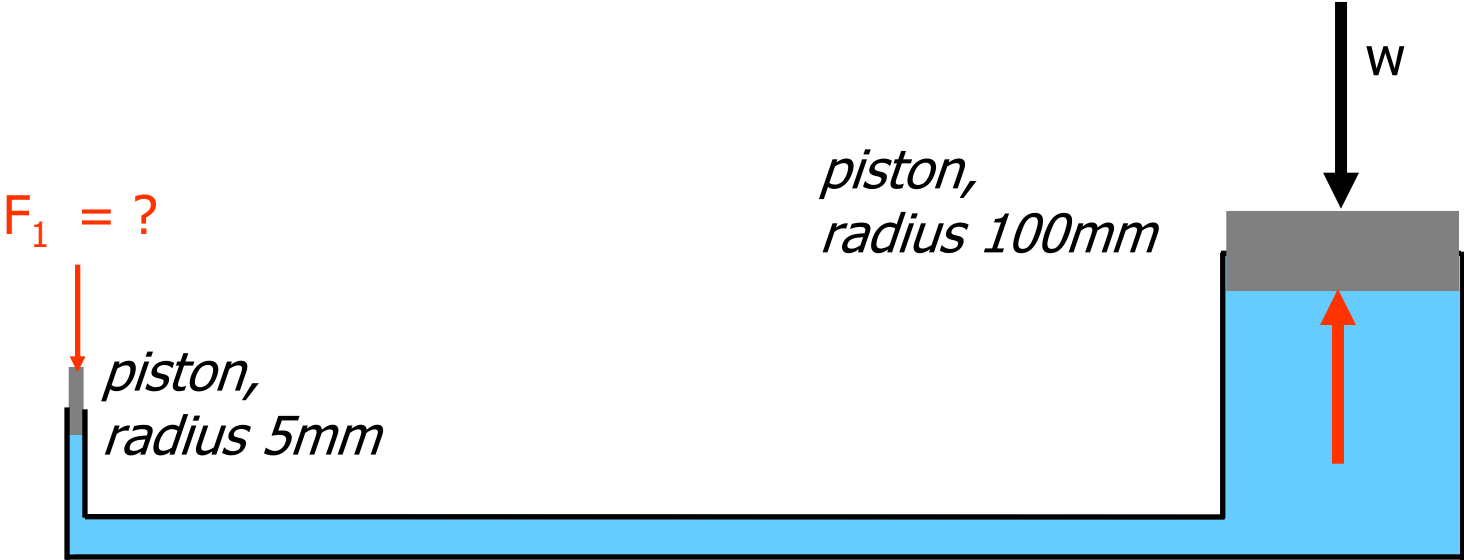
	pascal (Pa)	bar (bar)	atmosphere (atm)	torr (Torr)	pound-force per square inch (psi)
1 Pa	$\equiv 1 \text{ N/m}^2$	10^{-5}	9.8692×10^{-6}	7.5006×10^{-3}	145.04×10^{-6}
1 bar	100,000	$\equiv 10^6 \text{ dyn/cm}^2$	0.98692	750.06	14.5037744
1 atm	101,325	1.01325	$\equiv 1 \text{ atm}$	760	14.696
1 torr	133.322	1.3332×10^{-3}	1.3158×10^{-3}	$\equiv 1 \text{ Torr}; \approx 1 \text{ mmHg}$	19.337×10^{-3}
1 psi	6,894.76	68.948×10^{-3}	68.046×10^{-3}	51.715	$\equiv 1 \text{ lbf/in}^2$

Pascal's Law: Pressure in an enclosed fluid in equilibrium is the same everywhere, *except* for differences due to gravity.

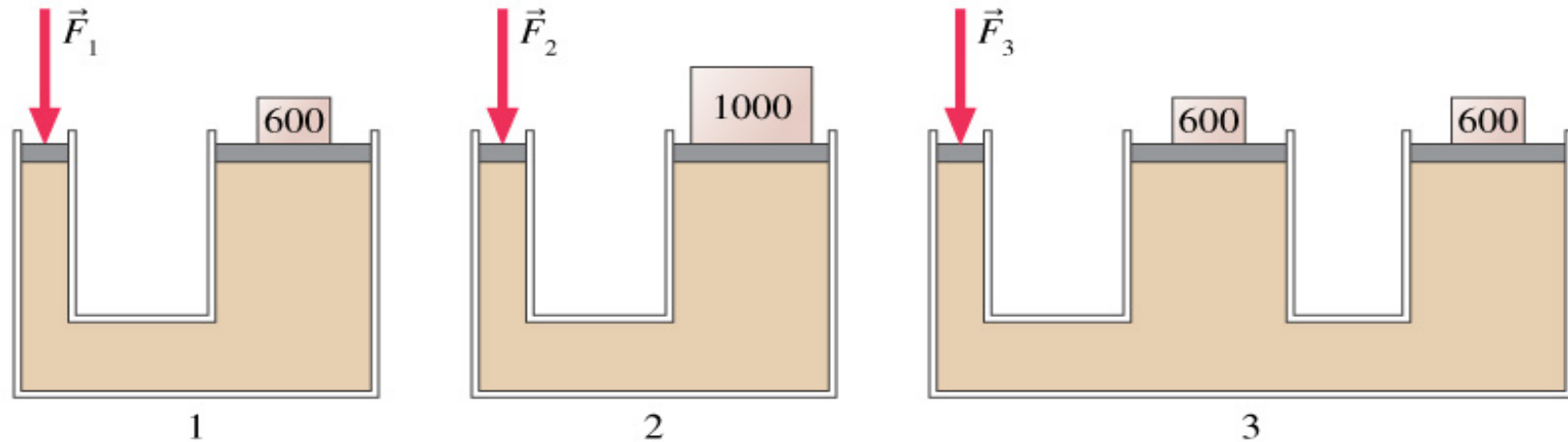
Or, pressure changes are transmitted throughout a fluid in equilibrium without loss; there is no static friction in fluids.



Example: How hard do you need to push to lift a cement truck (weight $w = 200 \text{ kN}$)?



Quick Quiz



Rank in order, from largest to smallest, the magnitudes of the forces \vec{F}_1 , \vec{F}_2 and \vec{F}_3 required to balance the masses. The masses are in kilograms.

- A. $F_2 > F_1 > F_3$
- B. $F_2 > F_1 = F_3$
- C. $F_3 > F_2 > F_1$
- D. $F_3 > F_1 > F_2$
- E. $F_1 = F_2 = F_3$

Pressure variation with depth

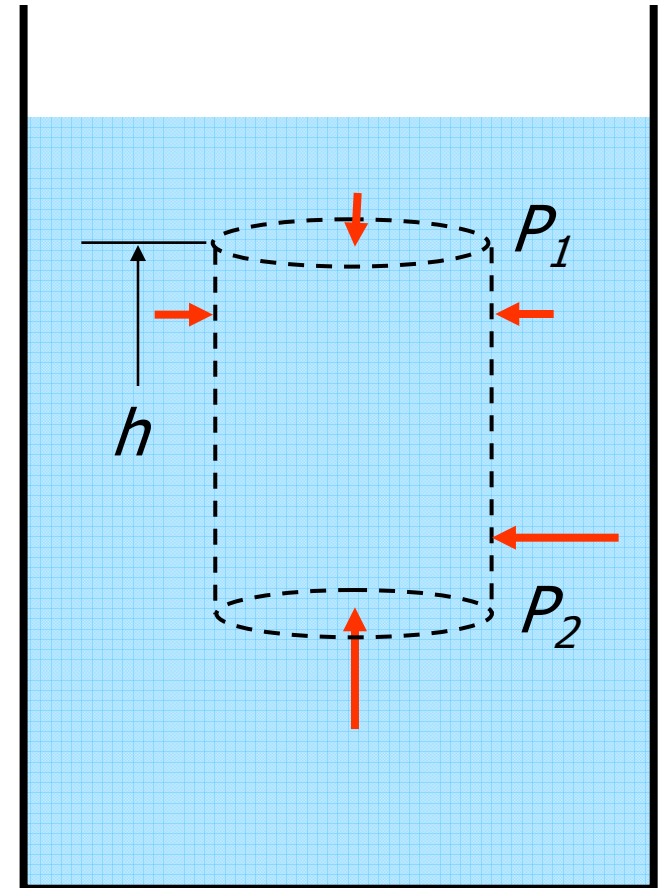
Pressure increases with depth, by an amount

$$P_2 - P_1 = \rho gh$$

(if ρ and g are uniform).

Proof:

Consider forces on a cylinder of fluid



“Gauge Pressure” : pressure **difference** between a fluid and the surrounding atmosphere. It is equal to $P_2 - P_1$.

Example: a tire gauge measures gauge pressure, and reads zero when the air inside the tire is at atmospheric pressure.

“Absolute Pressure” is the pressure compared to vacuum. Zero absolute pressure means a vacuum.

Example: the pressure on the surface of the earth.

Example

At what depth in water is the pressure 1 atm higher than the pressure on the surface? That is, where is $P=2\text{atms}$?

Example

What is the difference in air pressure between the floor and the ceiling?

Example

What is the total mass of air directly above a 1-metre square, from ground level all the way to outer space?

Example

Approximately how thick is the atmosphere, assuming (incorrectly) that the air density is uniform?

15. Fluid Mechanics

- Pressure
- Buoyancy

Buoyancy

The fluid exerts an **upward** force on the object equal to the weight of fluid displaced.

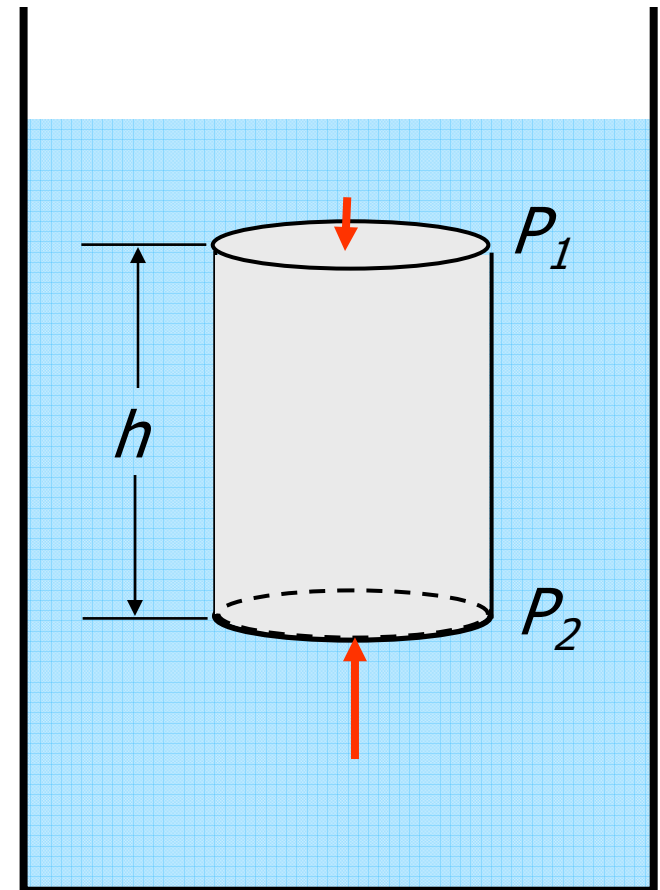
This force is result of **pressure differences** in the fluid; the pressure is greater at the bottom than at the top.

This is why objects can 'float': the magnitude of the buoyant force equals that of the force of gravity:

$$F_B = F_g$$

so:

$$\rho V g = m g$$



Archimedes's Principle: the magnitude of the buoyant force always equals the weight of the fluid displaced by the object.

Totally Submerged Object

For a submerged object in fluid with ρ_f the upward force is $B = \rho_f V_o g$ (=weight of fluid displaced) and so the net force is:

$$\begin{aligned} B - F_g &= \rho_f V_o g - \rho_o V_o g \\ &= (\rho_f - \rho_o) V_o g \end{aligned}$$

Floating Object

For a floating object, it displaces a volume of fluid equal to its own volume, and so:

$$\begin{aligned} B &= F_g \\ \rho_f V_f g &= \rho_o V_o g \end{aligned}$$

Example

A ice cube ($\rho = 917 \text{ kg/m}^3$) floats in a pail of cold water. If the cube is 100.0 mm on each side, how far is the top surface of the ice above the water?

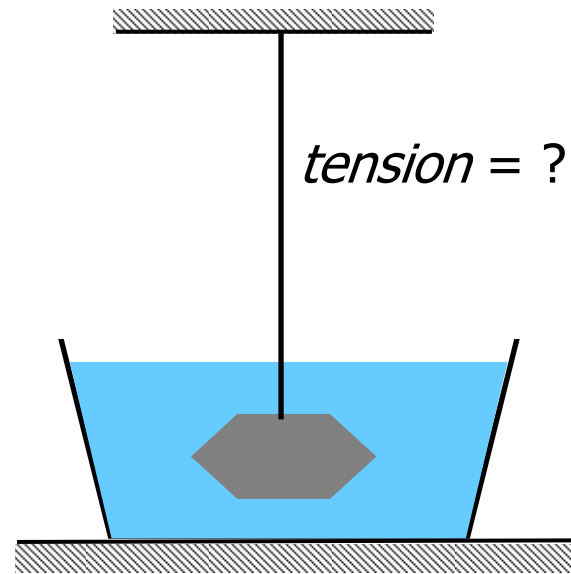
Quick Quiz

An ice cube is floating in a glass of water that is filled entirely to the brim. When the ice cube melts, the water level will

- A. stay the same, right at the brim.
- B. rise, causing the water to spill.
- C. fall.

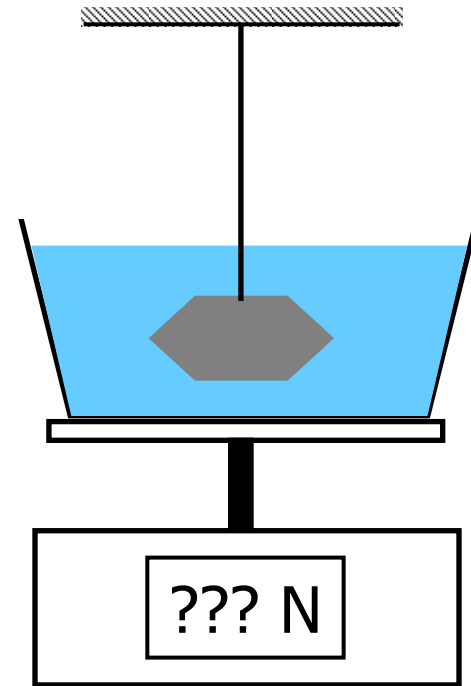
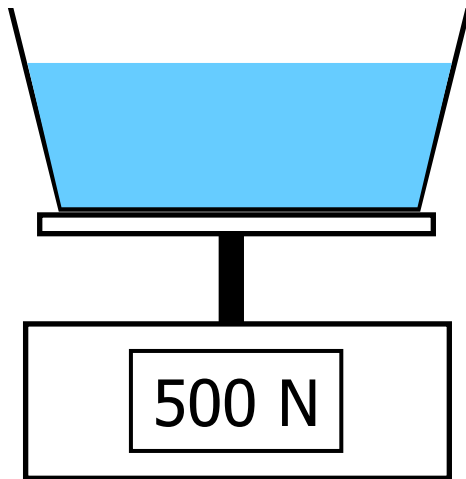
Example

A 10-kg rock (density 2500 kg/m^3) is suspended in a large bucket of water by a cord. What is the tension in the cord?



Quick Quiz

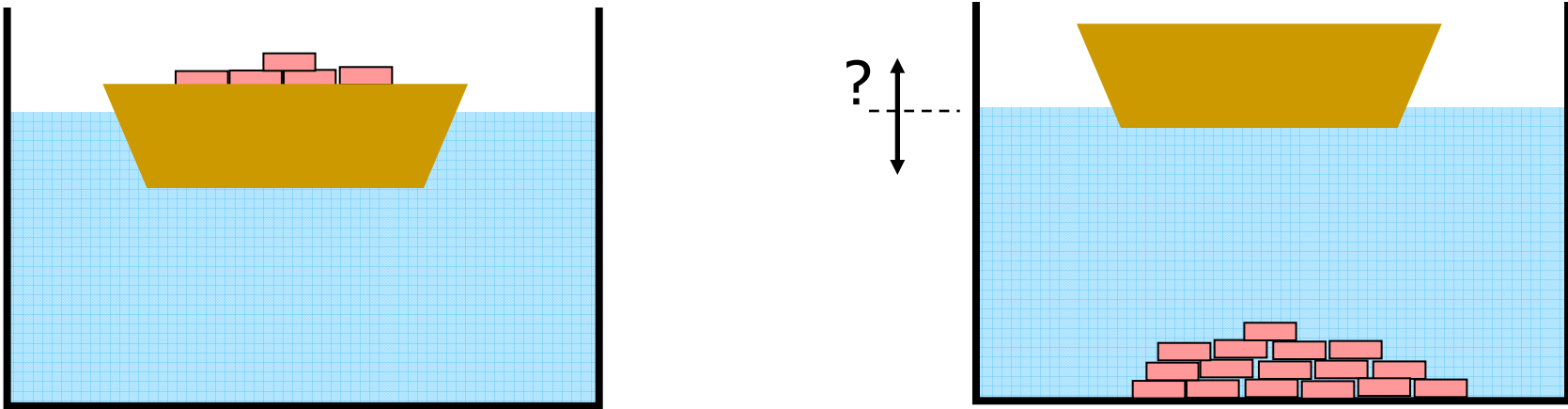
A large bucket half full of water is placed on a scale; the scale reads 500 N. A 10-kg rock (density 2500 kg/m^3) is lowered into the bucket by a cord. When the rock is completely underwater, but not touching the sides or bottom of the bucket, what does the scale supporting the bucket read?



- A) More than 500 N
- B) Less than 500 N
- C) 500 N

Quick Quiz

A boat filled with bricks floats in a swimming pool. If the bricks are dumped overboard, what happens to the water level?



- a) rises
- b) falls
- c) stays the same
- d) depends on the density of the bricks

How heavy a balloon can 100 litres of helium lift?

Hydrogen has half the density of helium. How heavy a balloon can be lifted by 100 litres of hydrogen?