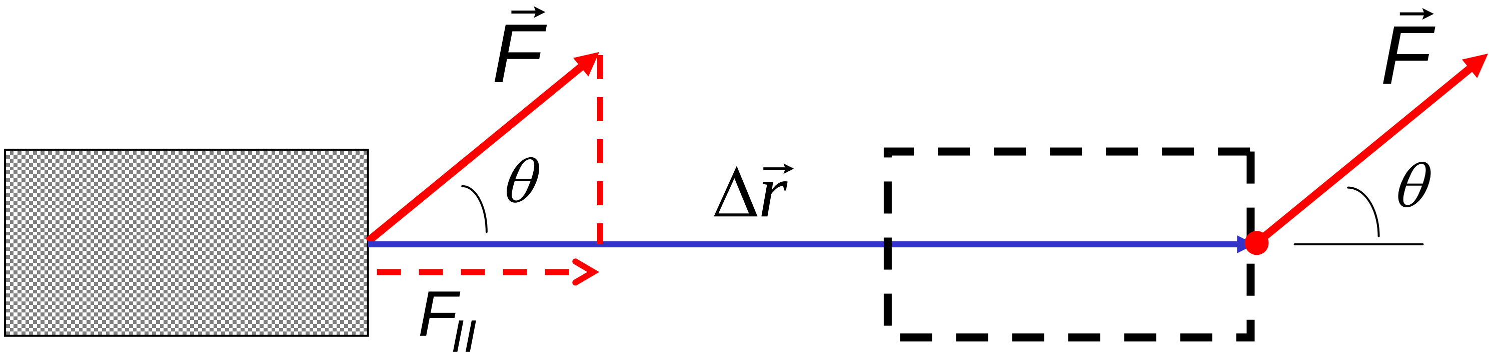
Mechanical Work

- How physicists define work?
- Work as a scalar/dot product
- How is work related to Kinetic Energy?
- Springs

Work by a Constant Force



Work by a force $\vec{F}$ during a displacement $\vec{S}$:

$$W = \vec{F} \bullet \Delta\vec{r}$$

A Scalar!

(constant force $\vec{F}$)

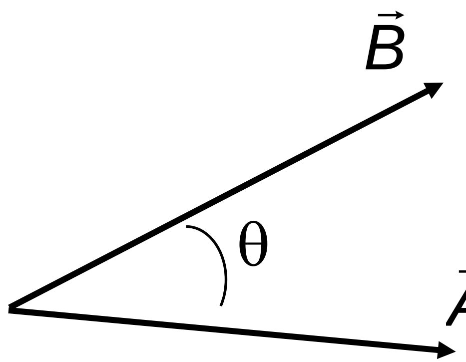
Units : $N \cdot m \equiv$ joule (J)

Review : The Scalar Product

The **scalar product** or **dot product** of two vectors gives a scalar result: vector • vector = scalar

$$\vec{A} \bullet \vec{B} = AB\cos\theta$$

Where A means $|\vec{A}|$, etc.



- the result is a **scalar** (no direction)
- for perpendicular vectors, $\vec{A} \bullet \vec{B} = 0$
- $\vec{A} \bullet \vec{B}$ is positive if $\theta < 90°$, negative for $\theta > 90°$
- $\vec{A} \bullet \vec{A} = |\vec{A}|^2$

In components:

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$
$$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

Then, $\vec{A} \bullet \vec{B} = A_x B_x + A_y B_y + A_z B_z$ = single scalar

To prove this, expand $\vec{A} \bullet \vec{B}$ using the laws of arithmetic (distributive, commutative), and notice that

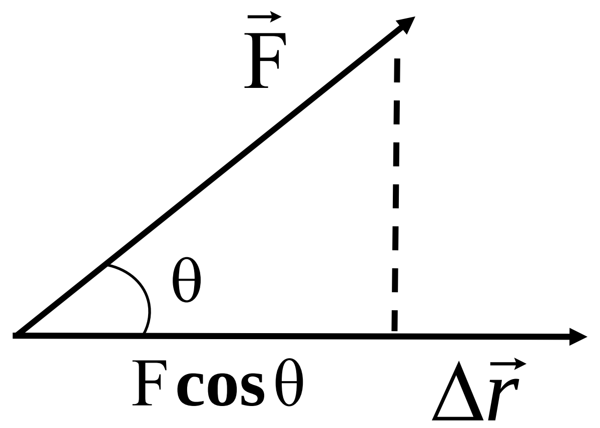
$\hat{i} \bullet \hat{j} = \hat{j} \bullet \hat{k} = \hat{i} \bullet \hat{k} = 0$ since, i, j, k are mutually perpendicular

and $\hat{i} \bullet \hat{i} = \hat{j} \bullet \hat{j} = \hat{k} \bullet \hat{k} = 1$ since they are unit vectors

Example

What does $\vec{v} \bullet \hat{i}$ mean?

So, for Work:



So, for example, $W = \vec{F} \bullet \Delta\vec{r}$ means:

Work = (component of force parallel to motion) $\times$ (distance)

MATH QUIZ

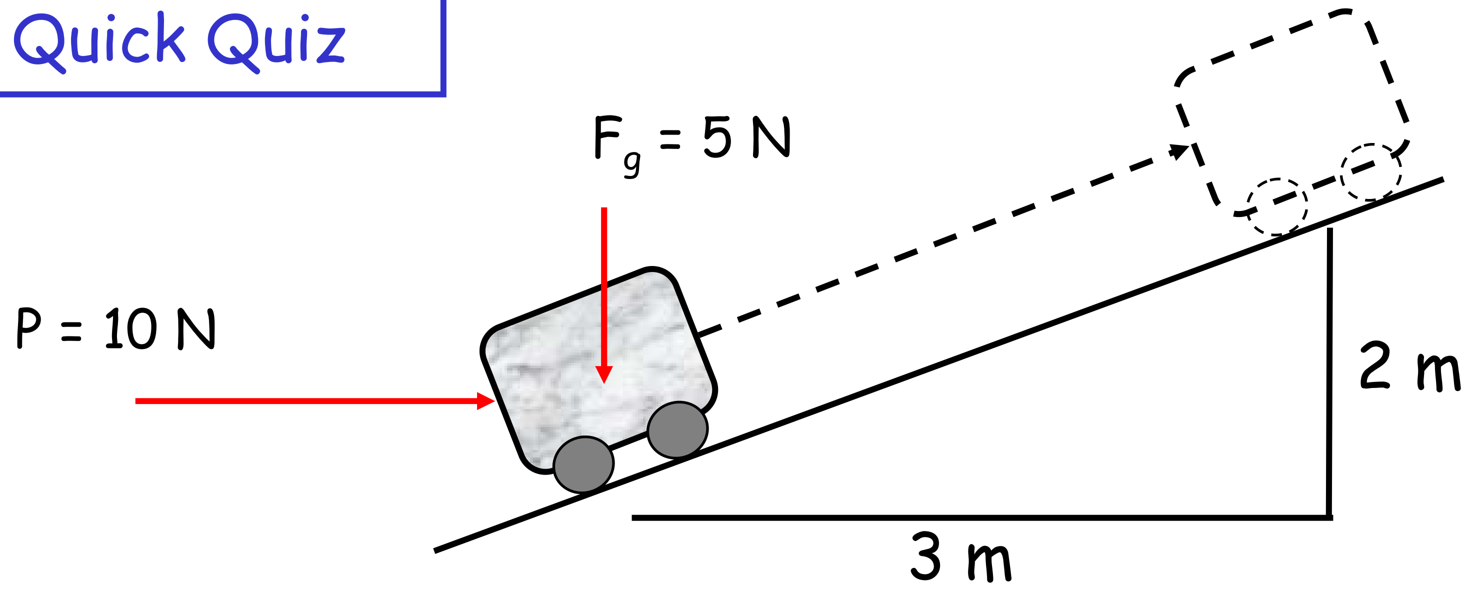
A constant force $\vec{F} = (1\hat{i} + 2\hat{j} + 3\hat{k})\,\text{N}$ is applied to an object while it undergoes a displacement $\vec{s} = (2\hat{i} + 2\hat{j} + 2\hat{k})\,\text{m}$. The work done by $\vec{F}$ is :

a) $(2\hat{i} + 4\hat{j} + 6\hat{k})\,\text{J}$

b) $\sqrt{2^2 + 4^2 + 6^2}\,\text{J}$

c) 12 J

Quick Quiz



The two forces, **P** and **F**$_g$ are constant as the block moves up the ramp. The total work done by these two forces combined is:

a) 20 J

b) $\sqrt{30^2 + 10^2}$ J $\approx$ 32 J

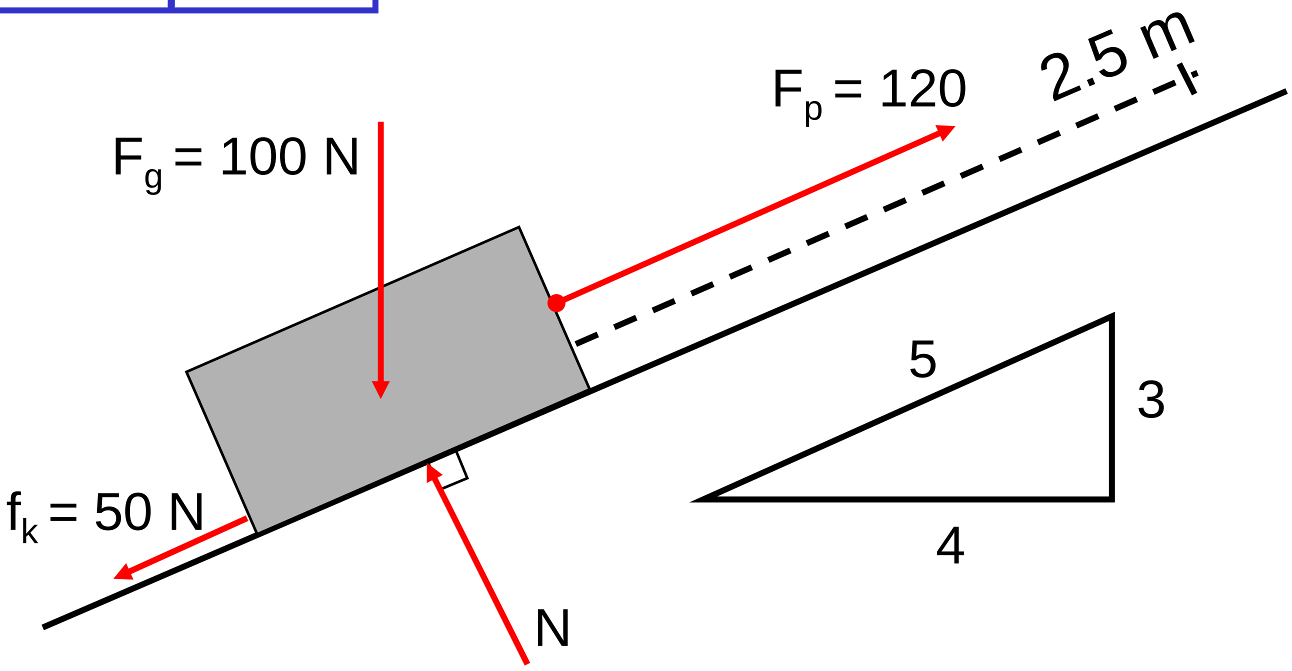
c) 40 J

Quick Quiz

In a simple pendulum, the tension in the string:

a) Cannot do work on the ball.
b) Must do work because the speed of the ball changes.
c) Does work equal and opposite to gravity.

Example



Block is dragged 2.5 m along slope. **Find :**

a) work done by F_p

b) work done by f_k

c) work done by gravity

d) work done by normal force

e) Total work done on the block

Example

If the block starts out going up the ramp at 2 m/s, how much faster ($v_f - v_i$) is the block moving after the 2.5 m?

Quick Quiz

If the block starts out going up the ramp at 100 m/s, how much faster (v_f-v_i) is the block moving after the 2.5 m?

a) Same as in the last question.
b) More than in the last question.
c) Less than in the last question.
d) The block isn't moving any faster.

How much work is done by a spring?

A spring pulls or pushes with a force proportional to the amount it is stretched or compressed.

$$F = -\ k\, x$$

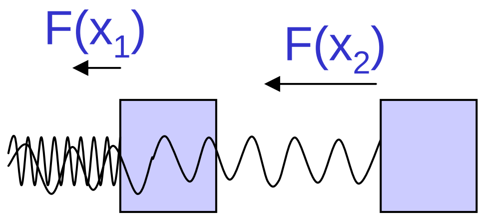
displacement from equilibrium position (x)

spring constant (k)

negative ($-$), since increasing length along x axis makes spring pulls back

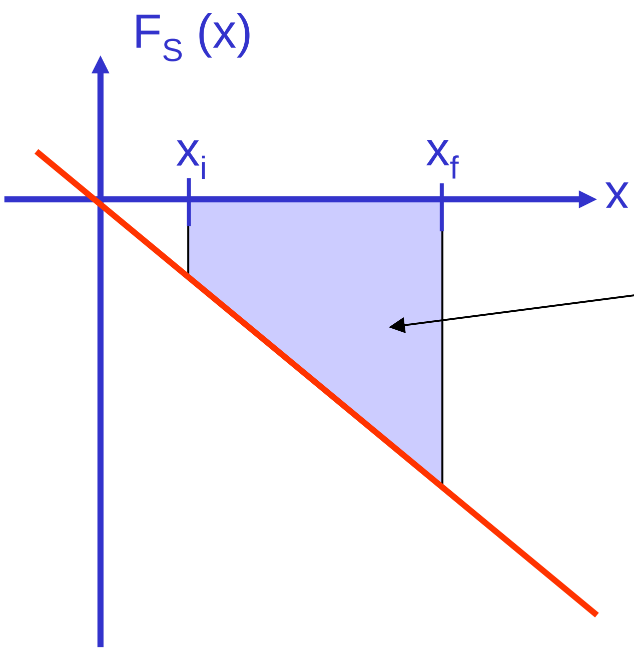
Stretch a spring

Stretch a spring: x=0 is equilibrium position



$| F(x_2) | > | F(x_1) |$

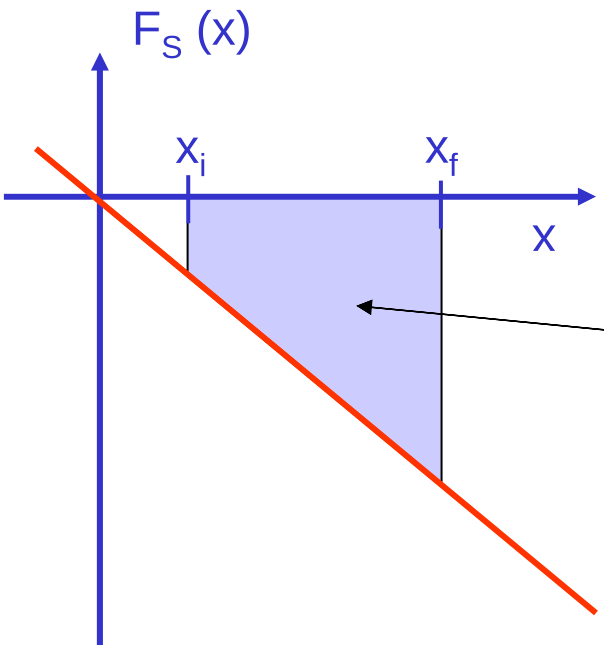
But both forces act to bring the spring back to its equilibrium position.



The work done **BY** the spring is the area under this curve.

The work done **BY** this spring is negative.

How do we get the area under a curve?



$$W = \int_{x_i}^{x_f} F_x \, dx$$

$$W_s = \int_{x_i}^{x_f} (-k\,x)\,dx$$

$$W_s = \frac{1}{2}k\,x_i^2 - \frac{1}{2}k\,x_f^2$$

Muzzle velocity demo

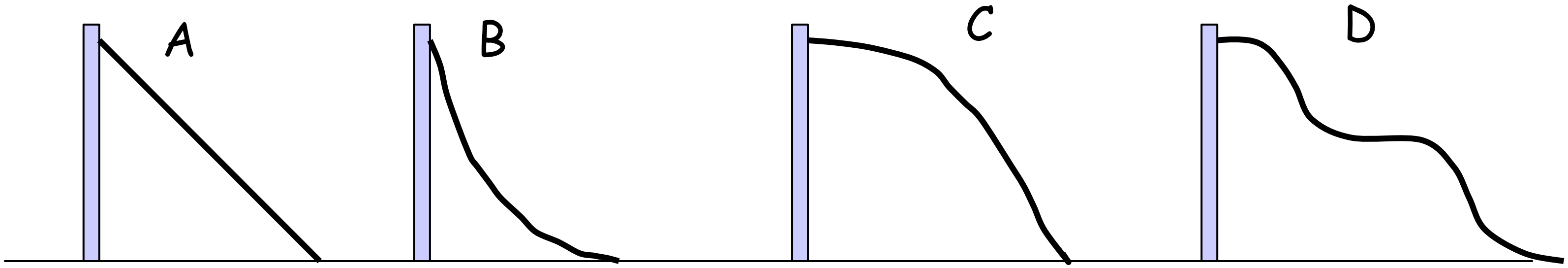
Quick Quiz

You walk around a circular path of radius 20 m while pushing a stroller with a force of 100 N. If you end up exactly where you started, how much work did you do?

a) None, because the displacement is zero.
b) None, because the force is always tangential.
c) About 10 000 J (about two cookies worth).

Quick Quiz

The four playground slides shown below have the same height but different shapes.



i) If friction is negligible, on which slide will a child attain the highest speed?

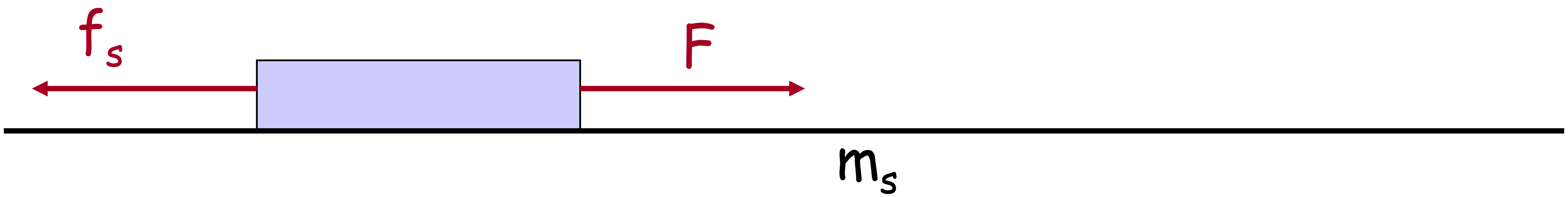
Example

A curler throws a curling rock toward the "button", which is 50 m away, and 30 cm in diameter. What percentage error can she make in the delivery speed and still have part of the rock on the button? (Choose the closest answer.)

a) 10%　　b) 1%　　c) 0.1%　　d) 0.01%

Handling frictional work

Static friction will usually do no work



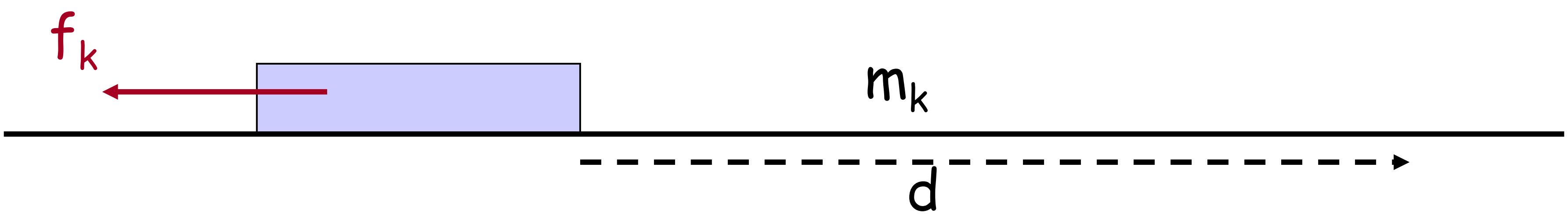
For this example: $W = -f_s d = 0$ (because $d = 0$)

Include this in the work-energy theorem:

$$\Delta K = K_f - K_i = 0$$

Handling frictional work

Kinetic (sliding) friction will usually remove kinetic energy
i.e. it does *negative work* on a moving object



For this example, $W = -f_k d = -\mu_k N d$

Include this in the work-energy theorem:

$$\Delta K = K_f - K_i = - f_k d$$

Can frictional forces ever *increase* an object's kinetic energy?