Collisions

- Kinetic energy
- Inelastic collisions
- Elastic collisions

Kinetic Energy

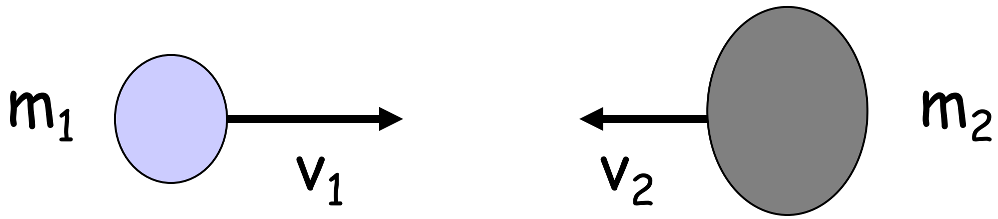
Another way of describing how much motion an object has:

$$K = \frac{1}{2} m v^2$$

Types of Collisions

- Elastic collisions: p and K are conserved
- Completely inelastic collisions: K not conserved, just p
- Inelastic collisions: This is what happens in the real world

Collisions (one-dimensional)



Two objects which just collide and stick: this is a ***totally inelastic*** collision:

$$p(\text{initial}) = p(\text{final})$$

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2)\, V_{both}$$

Example

A 50 g bullet is fired into block of wood of mass 5 kg. The collision is completely inelastic in that the bullet goes into the wood but does not go through it. If the speed of the wood (with the bullet in it) is 10 m/s right after the collision, what was the speed of the bullet right before the collision?

Elastic Collisions

1) Momentum is conserved

2) Initial kinetic energy equals final kinetic energy

Quick Quiz

When two stars moving through the Galaxy pass close to each other, their mutual gravitational pull causes them to change speed and direction. We can call this a collision, even if there was no contact between the stars. Will this be an elastic or inelastic collision?

A. elastic
B. inelastic
C. totally inelastic
D. could be any of the above
E. could be B or C, but not A

Elastic Collisions

Object 1 and 2 collide along the x-axis:

1) Momentum is conserved:

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

In one dimension, the velocities are represented by positive or negative numbers to indicate direction.

2) Total Kinetic Energy doesn't change:

$$\tfrac{1}{2} m_1 v_{1i}^2 + \tfrac{1}{2} m_2 v_{2i}^2 = \tfrac{1}{2} m_1 v_{1f}^2 + \tfrac{1}{2} m_2 v_{2f}^2$$

We can solve for two variables if the other four are known.

$$\frac{1}{2}m_1 v_{1i}^2 + \frac{1}{2}m_2 v_{2i}^2 = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2$$

$$m_1 v_{1i}^2 + m_2 v_{2i}^2 = m_1 v_{1f}^2 + m_2 v_{2f}^2$$

$$m_1 v_{1i}^2 - m_1 v_{1f}^2 = m_2 v_{2f}^2 - m_2 v_{2i}^2$$

$$m_1 (v_{1i}^2 - v_{1f}^2) = m_2 (v_{2f}^2 - v_{2i}^2)$$

Divide these equations:

$$\frac{m_1 (v_{1i}^2 - v_{1f}^2)}{m_1 (v_{1i} - v_{1f})} = \frac{m_2 (v_{2f}^2 - v_{2i}^2)}{m_2 (v_{2f} - v_{2i})}$$

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$m_1 v_{1i} - m_1 v_{1f} = m_2 v_{2f} - m_2 v_{2i}$$

$$m_1 (v_{1i} - v_{1f}) = m_2 (v_{2f} - v_{2i})$$

$$\frac{m_1(v_{1i}^2 - v_{1f}^2)}{m_1(v_{1i} - v_{1f})} = \frac{m_2(v_{2f}^2 - v_{2i}^2)}{m_2(v_{2f} - v_{2i})} \quad \Rightarrow \quad \frac{(v_{1i}^2 - v_{1f}^2)}{(v_{1i} - v_{1f})} = \frac{(v_{2f}^2 - v_{2i}^2)}{(v_{2f} - v_{2i})}$$

$$\Rightarrow \quad \frac{(v_{1i} - v_{1f})(v_{1i} + v_{1f})}{(v_{1i} - v_{1f})} = \frac{(v_{2f} - v_{2i})(v_{2f} + v_{2i})}{(v_{2f} - v_{2i})}$$

$$\rightarrow \quad v_{1i} + v_{1f} = v_{2f} + v_{2i}$$

$$\rightarrow \quad \boxed{v_{1i} - v_{2i} = -(v_{1f} - v_{2f})}$$

Initial relative speed = - Final relative speed

(for ALL elastic collisions)

Quick Quiz

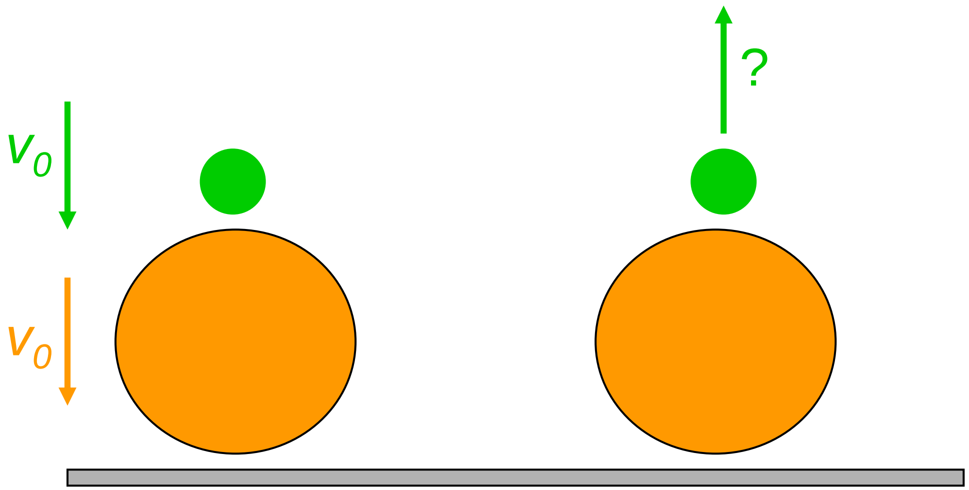
A tiny ball bearing is dropped off of a bridge over a highway. It falls straight down and is hit by the windshield of a large truck traveling at 30 m/s. After the collision, the ball bearing is traveling in the same direction as the truck. **Relative to the truck**, what is the speed of the ball bearing right after the collision?

A. 0 m/s
B. 30 m/s
C. 60 m/s
D. 90 m/s

*treat collision as elastic

Example

A tennis ball is placed on top of a basketball and both are dropped. The basketball hits the ground at speed v_0. What is the maximum speed at which the tennis ball can bounce upward from the basketball? (assume the basketball is much more massive than the tennis ball, and the collision is elastic).



If you know the initial velocities of two bodies colliding elastically, how do you predict the velocities after the collision?

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$\Rightarrow \quad m_1 v_{1f} = m_1 v_{1i} + m_2 v_{2i} - m_2 v_{2f}$$

$$v_{1i} - v_{2i} = -\left(v_{1f} - v_{2f}\right) \quad \Rightarrow \quad v_{2f} = v_{1i} + v_{1f} - v_{2i}$$

$$m_1 v_{1f} = m_1 v_{1i} + m_2 v_{2i} - m_2\left(v_{1i} + v_{1f} - v_{2i}\right)$$

$$m_1 v_{1f} = m_1 v_{1i} + m_2 v_{2i} - m_2 v_{1i} - m_2 v_{1f} + m_2 v_{2i}$$

$$m_1 v_{1f} + m_2 v_{1f} = m_1 v_{1i} + m_2 v_{2i} - m_2 v_{1i} + m_2 v_{2i}$$

$$v_{1f}\left(m_1 + m_2\right) = \left(m_1 - m_2\right) v_{1i} + \left(m_2 + m_2\right) v_{2i}$$

$$v_{1f} = \frac{(m_1 - m_2)}{(m_1 + m_2)} v_{1i} + \frac{(2m_2)}{(m_1 + m_2)} v_{2i}$$

Formulae for Elastic Collisions

$$v_{1f} = \frac{(m_1 - m_2)}{(m_1 + m_2)} v_{1i} + \frac{(2m_2)}{(m_1 + m_2)} v_{2i}$$

Similarly:

$$v_{2f} = \frac{(m_2 - m_1)}{(m_1 + m_2)} v_{2i} + \frac{(2m_1)}{(m_1 + m_2)} v_{1i}$$

Summary

- Momentum is conserved in all collisions that the impulse approximation holds true.

- Kinetic energy is conserved in *elastic* collisions

- The *relative* speeds before and after an elastic collision are equal.