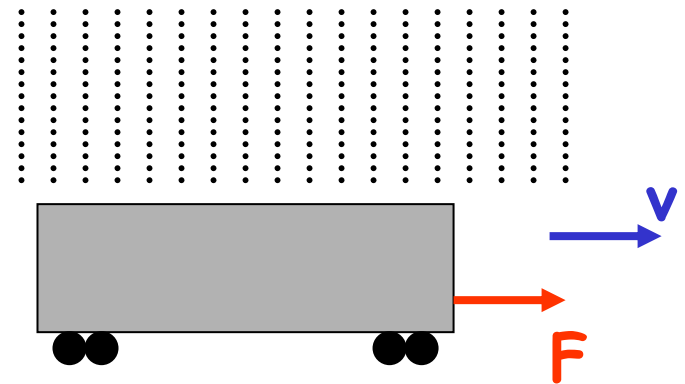


Momentum

- Newton's real 2nd law
- What is Linear Momentum?
- A new conservation law
- Rockets
- Impulse

Example

Rain is falling vertically into an open railroad car which moves along a horizontal track at a constant speed of 10 m/s. Ignoring friction, what force is required to keep the car moving at a constant speed if the water collects in the car at the rate of 1 kg/s?



Newton's 2nd law is hard to use if the mass is changing.

But we can put the 2nd law in another form:

$$\Sigma \vec{F} = m\vec{a} = m \frac{d\vec{v}}{dt} = \frac{d(m\vec{v})}{dt}$$

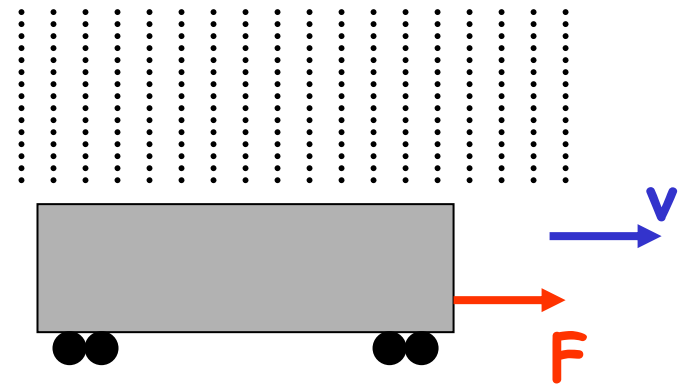
$$\mathbf{F} = \frac{d(m\mathbf{v})}{dt} = \frac{dm}{dt} \mathbf{v} + m \frac{d\mathbf{v}}{dt}$$



Product rule for
differentiation

Example - the other way

Rain is falling vertically into an open railroad car which moves along a horizontal track at a constant speed of 10 m/s. Ignoring friction, what force is required to keep the car moving at a constant speed if the water collects in the car at the rate of 1 kg/s?



Linear Momentum

Newton's Real 2nd law: $\Sigma \vec{F} = \frac{d(m\vec{v})}{dt}$

Linear Momentum: $\vec{p} \equiv m\vec{v}$

The 2nd law
then becomes:

$$\Sigma \vec{F} = \frac{d\vec{p}}{dt}$$

Quick Quiz

A projectile launcher is mounted on a cart. While the cart moves at a constant speed on an frictionless air track, the launcher fires a projectile up vertically. The speed of the cart after launching:

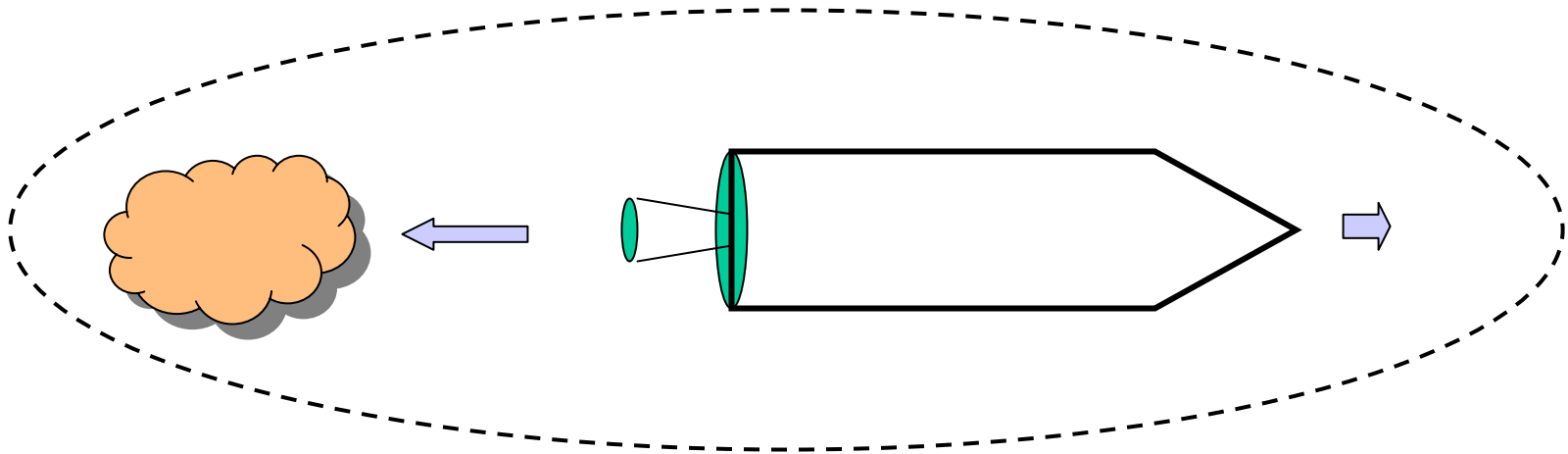
- a) remains the same as before launching
- b) is slower than its speed before launching
- c) is faster than its speed before launching

$$\text{if } \Sigma \vec{F} = 0 \Rightarrow \frac{d\vec{p}}{dt} = 0 \Rightarrow \vec{p} = \text{const}$$

Linear Momentum of an object is conserved if there are no external, unbalanced forces acting on the object.

This is the law of conservation of momentum.

Example: Rocket propulsion



Both the rocket and exhaust are part of a single "mechanical system" - the forces between them are *internal* forces

Collisions

A "collision" is an interaction between two or more objects which is *short and sudden*.

Quick Quiz

Two identical trucks traveling at equal speeds in opposite directions on a stormy night collide head-to-head. One is carrying a full load and the other is empty.

I. Which truck sustains the greatest damage?

- a) the full truck
- b) the empty truck
- c) the damages are equal

Two Particle System

If two objects interact with no outside forces then the total momentum is conserved.

$$\vec{P}_i = \vec{P}_f$$

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}$$

No External Forces?

- Momentum is only conserved if there is no external force
- If gravity is an external force, momentum isn't conserved
- This means that conservation of momentum is useless when applied to collisions happening on the Earth!

Impulse

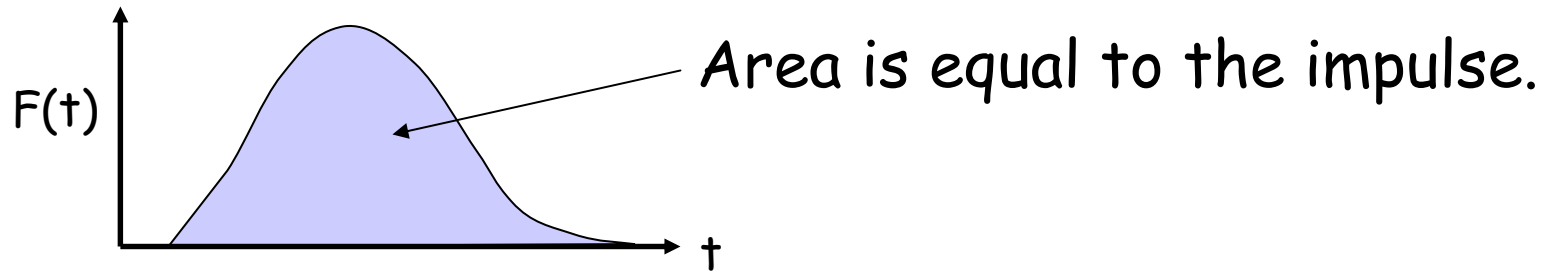
$$\Sigma \vec{F} = \frac{d\vec{p}}{dt} \quad \longrightarrow \quad d\vec{p} = \vec{F}_{\text{net}} dt$$

Integrate both sides $\rightarrow \int d\vec{p} = \int \vec{F}_{\text{net}} dt$

$$\vec{I} \equiv \Delta\vec{p} = \int \vec{F}_{\text{net}} dt$$

Impulse

The impulse is a vector equal to the change in momentum.



The Impulse Approximation

- During a collision, one force acts on a body that is much greater than any other force present.
- Small forces like Earth's gravity don't cause a large impulse over timescales typically associated with these collisions.
- Momentum is approximately conserved in short and sudden collisions.

Example

A 0.9 kg bat hits a 0.1 kg ball with a constant force of 5000 N, and is in contact with the ball for 0.01 s.

What is the impulse caused by the collision between the bat and the ball?

What is the impulse caused by gravity during the collision?