Projectile Motion

- What is a projectile?
- Strategy for solving Projectile Motion problems
- Galileo's Theorem
- Examples
- Galilean Relativity

What is a Projectile?

- moves in two dimensions
- only force is gravity

Quick Quiz:

A basketball is thrown into the air at an angle of 45° to the horizontal. If we can think of the basketball as a projectile, which of the following statements are true?

1. the x-position is constant
2. the y-position is constant
3. the position vector is constant
4. the x-component of the velocity is constant
5. the y-component of the velocity is constant
6. the velocity is constant
7. the x-component of the acceleration is constant
8. the y-component of the acceleration is constant

A) 4 and 7   B) 4, 7 and 8   C) 6, 7 and 8   D) 3, 5 and 7

Strategy for Solving

- Draw a picture with a set of coordinates…always make the y-axis parallel (or anti-parallel) to g

- These are kinematics problems, so write down the kinematics equations for each axis, and for each object involved in the problem

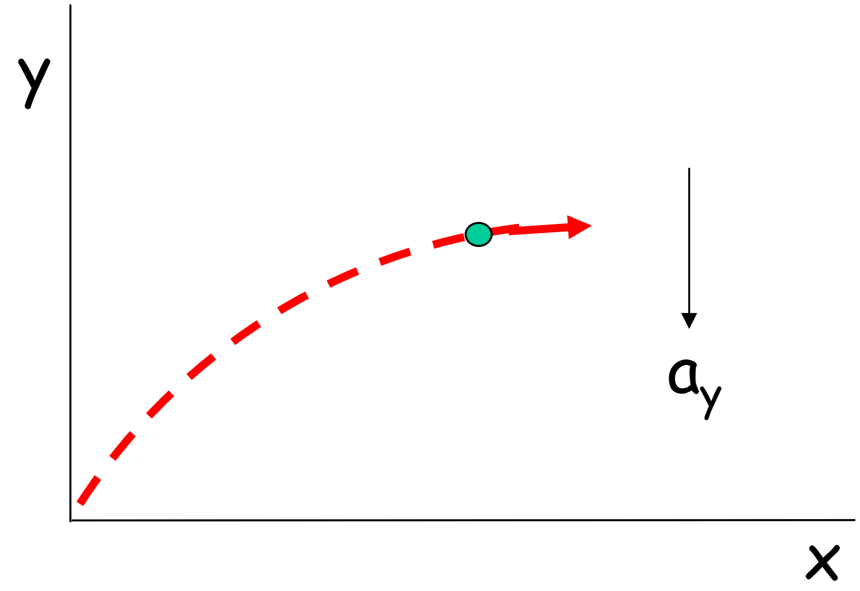
- Decide which of the symbols in the kinematics equations have values that you know, which are constants, and which are what you are looking to solve for (write these down in an orderly fashion)

- Remember that the time connects the kinematic equations

Projectile motion

2-D motion in "free fall" is called *projectile motion.*

$a_x = 0, \quad a_y = -g = -9.8\ m/s^2$



So the equations of motion become

$$v_{x,f} = v_{x,i} + a_x t = v_{x,i}$$

$$v_{y,f} = v_{y,i} - g t$$

$$x_f = x_i + v_{x,i} t$$

$$y_f = y_i + v_{y,i} t - \frac{1}{2} g t^2$$

Common Problems!

- When air resistance is ignored, all things fall at the same rate, regardless of mass

- x-components and y-components are independent. Motion in the x-direction doesn't affect freefall in the y-direction

- Because the axes are orthogonal, you can describe motion in both axes simultaneously by using unit vectors.

Quick Quiz:

What set of circumstances would cause something to follow this path?

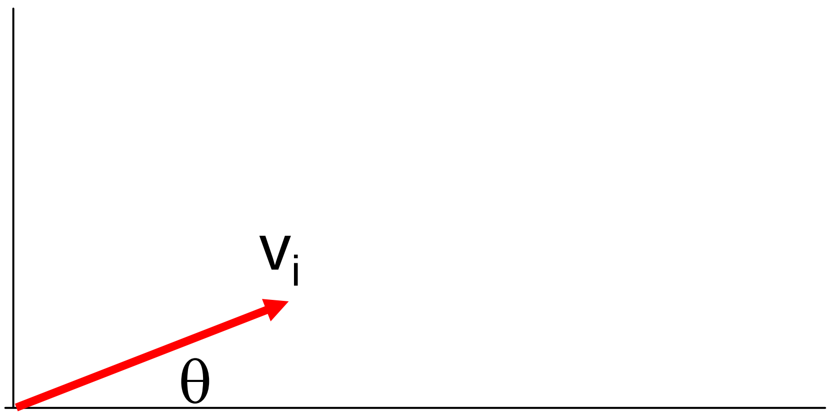
$$\vec{r} = (20t)\hat{i} + (30 - 4.9t^2)\hat{j}$$

(a) A ball thrown straight up into the air
(b) A penny dropped off of the CN tower
(c) A car driven off of a cliff
(d) Something far more complicated

Hint: use derivatives to find velocity and acceleration, then identify initial positions, initial velocities, … by setting t=0.

Worked Example:

If you throw a ball on level ground at speed v_i and angle θ, how far away will it land?



Demo: Galileo's theorem

Quick Quiz:

A sailor climbs to the top of a ship's mast, 20 m above the deck. The ship is moving forward at a speed of 1 m/s parallel to the coastline. When he gets to the top of the mast, he points his spyglass perpendicular to the motion of the boat. When he spots a person on the coast, he drops the spyglass and it falls to the deck below.

Where does it land with respect to the base of the mast?

A) Ahead of where it was released
B) Behind where it was released
C) Directly below where it was released

* there is no wind

This is projectile

Quick Quiz:

A sailor climbs to the top of a ship's mast, 20 m above the deck. The ship is moving forward at a speed of 1 m/s parallel to the coastline. When he gets to the top of the mast, he points his spyglass perpendicular to the motion of the boat. When he spots a person on the coast, he drops the spyglass and it falls to the deck below.

Where does it land with respect to the person that he saw on the coast?

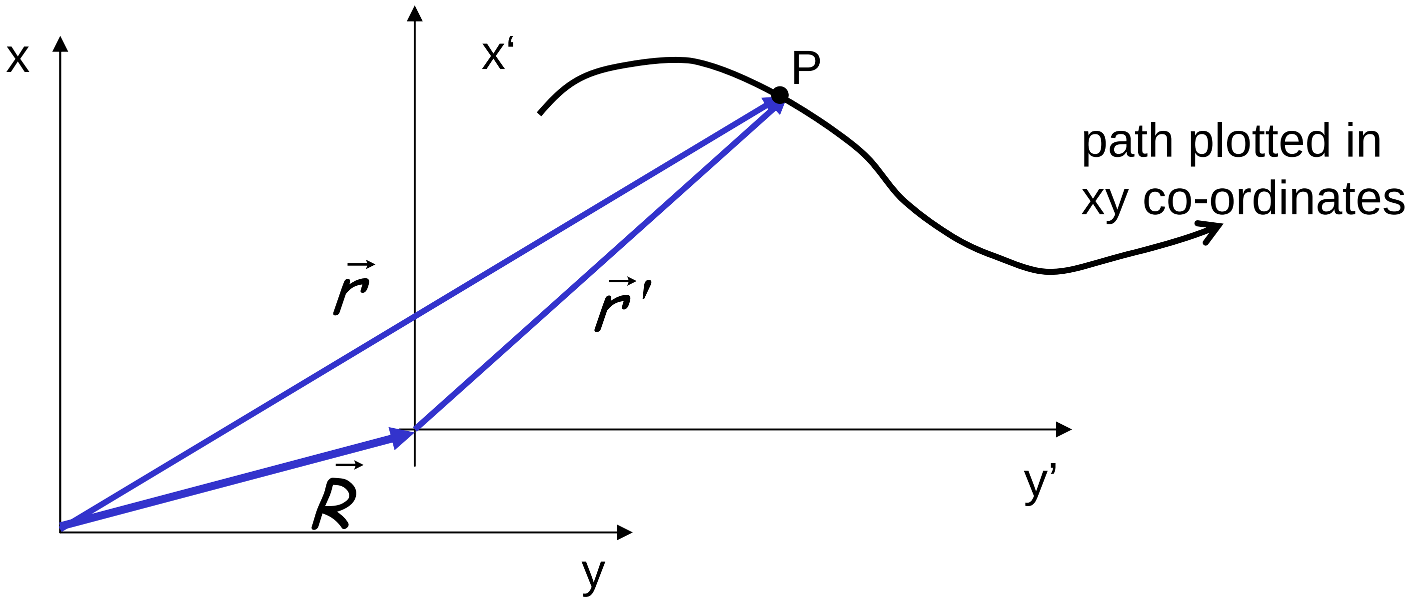
A) Ahead of where it was released
B) Behind where it was released
C) Directly below where it was released

* there is no wind

This is projectile

Galileo's Principle of Relativity

- A coordinate system specifies direction vectors
- The coordinate system may be moving
- Inertial coordinate systems are not accelerating
- An inertial coordinate system is called an inertial reference frame
- Newton's laws hold true in an inertial reference frame



- The position a particle P is described by $\vec{r}$ in (x,y)
- The same particle is described by $\vec{r}\,'$ in (x',y')
- $\vec{R}$ connects the origins of the two coordinate systems.

$$\vec{r} = \vec{r}\,' + \vec{R}$$

Quick Quiz:

In your reference frame, you see a student at position vector:

$$\vec{r} = 10\,\hat{i} + 1\,\hat{j} \quad [m]$$

In your reference frame, I am at a position vector:

$$\vec{R} = 5\,\hat{i} + 10\,\hat{j} \quad [m]$$

What is the position vector of the student in a reference frame where I am at the origin?

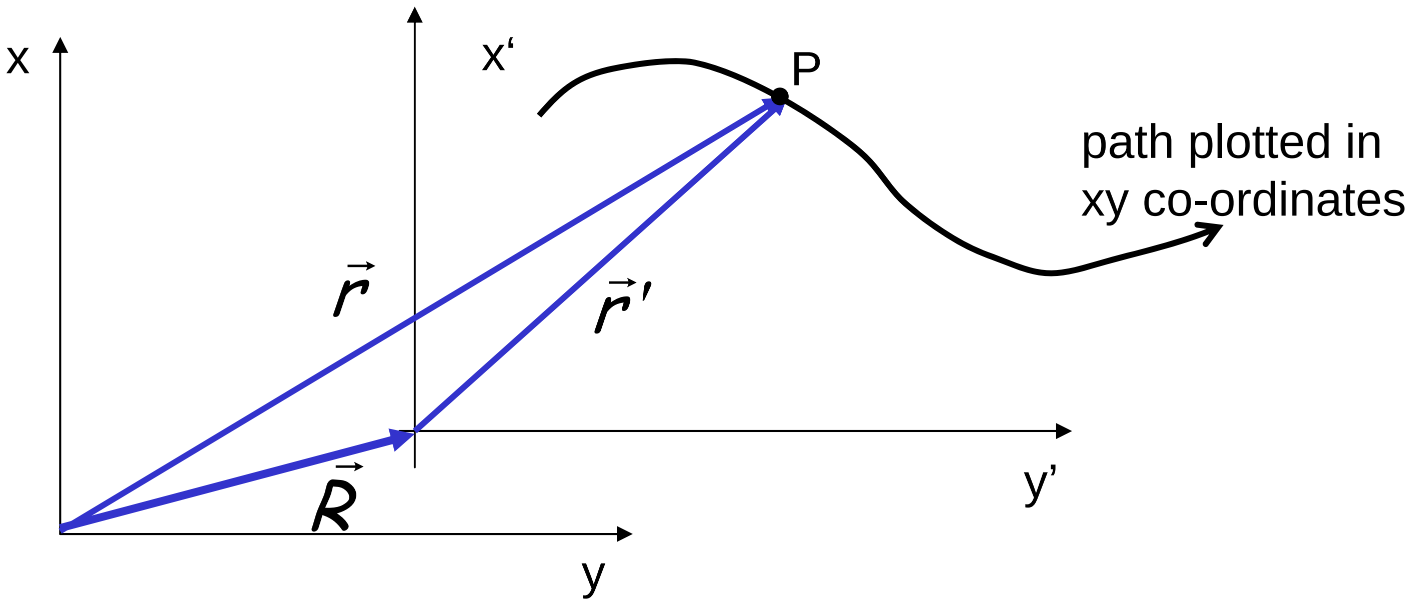
A) $\vec{r}' = 5\,\hat{i} - 9\,\hat{j} \quad [m]$

C) $\vec{r}' = 15\,\hat{i} - 11\,\hat{j} \quad [m]$

B) $\vec{r}' = -5\,\hat{i} + 9\,\hat{j} \quad [m]$

D) $\vec{r}' = 15\,\hat{i} - 9\,\hat{j} \quad [m]$

What if the reference frames are moving?



$$\vec{r} = \vec{r}\,' + \vec{R} \rightarrow \frac{d\vec{r}}{dt} = \frac{d\vec{r}\,'}{dt} + \frac{d\vec{R}}{dt} \rightarrow \vec{v} = \vec{v}\,' + \vec{V}$$

Quick Quiz:

In your reference frame, you see a student moving with a velocity given by:

$$\vec{v} = 10\,\hat{i} + 1\,\hat{j} \quad [\text{m/s}]$$

In my reference frame, I see the same student moving with a velocity given by:

$$\vec{v}\,' = 5\,\hat{i} + 10\,\hat{j} \quad [\text{m/s}]$$

What is my velocity relative to you?

A) $\vec{V} = 5\,\hat{i} - 9\,\hat{j} \quad [\text{m/s}]$

B) $\vec{V} = -5\,\hat{i} + 9\,\hat{j} \quad [\text{m/s}]$

C) $\vec{V} = 15\,\hat{i} - 11\,\hat{j} \quad [\text{m/s}]$

D) $\vec{V} = 15\,\hat{i} - 9\,\hat{j} \quad [\text{m/s}]$

Galilean Principle of Relativity

$$\frac{d\vec{v}}{dt} = \frac{d\vec{v}\,'}{dt} + \frac{d\vec{V}}{dt} \quad \Rightarrow \quad \vec{a} = \vec{a}\,' + \frac{d\vec{V}}{dt}$$

An Inertial Reference Frame is one in which $\vec{V}$ is a constant.

$$\vec{a} = \vec{a}\,' + \frac{d\vec{V}}{dt} \quad \Rightarrow \quad \vec{a} = \vec{a}\,' \quad \Rightarrow \quad \vec{F} = \vec{F}\,'$$