Kinematics in Two Dimensions

- Remember Motion Diagrams?
- Going from parallel-perpendicular to x-y
- Dynamics (just add forces)

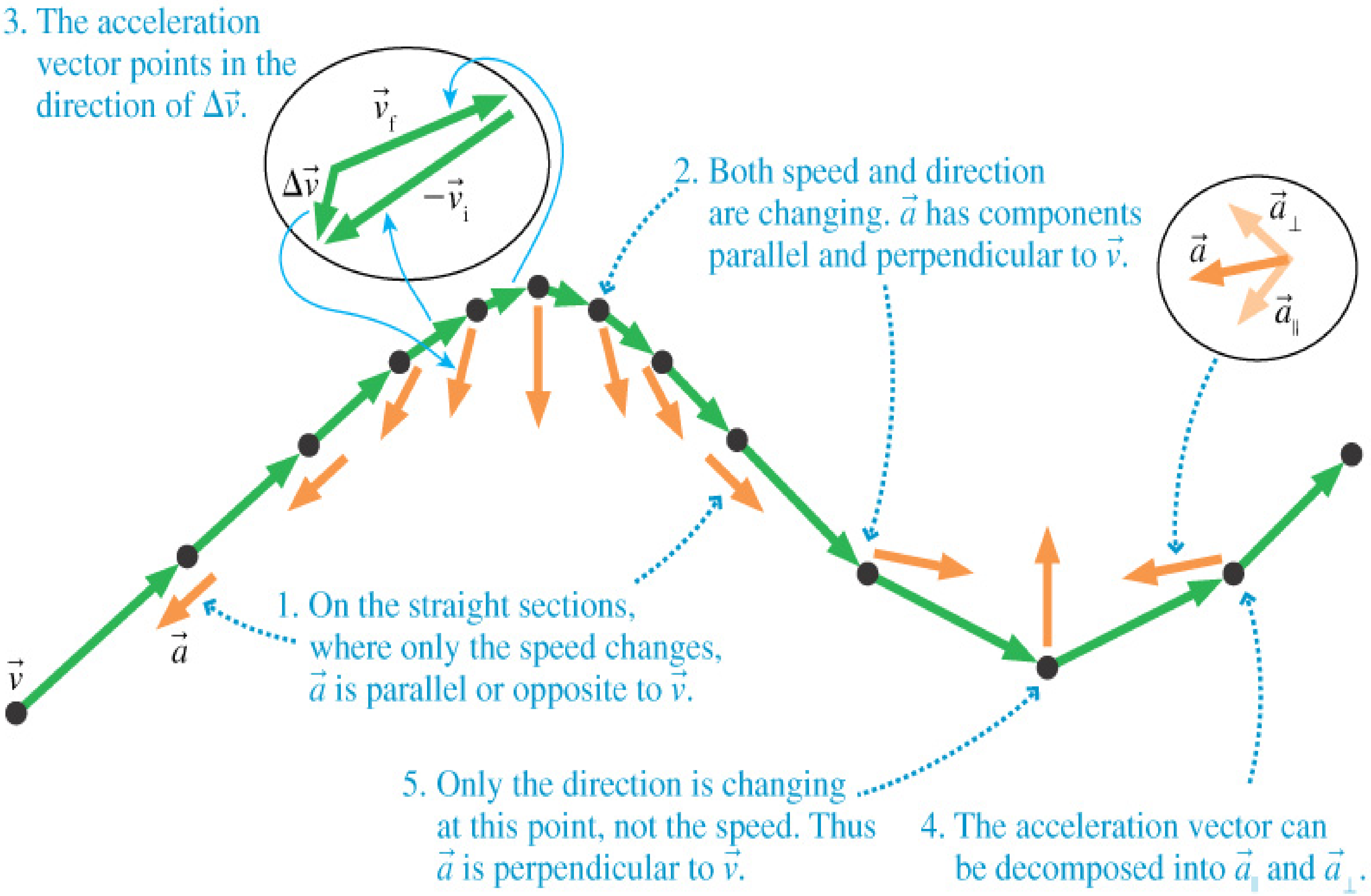
Two Dimensional Motion Diagram

- A two dimensional motion diagram is drawn on x-y axes

- Be sure to put enough dots in to represent the motion

- Recall that the velocity vectors simply connect one dot to the next dot

- The acceleration vectors are found by the vector subtraction of adjacent velocities



3. The acceleration vector points in the direction of $\Delta\vec{v}$.
$\vec{v}_f$
$\Delta\vec{v}$
$-\vec{v}_i$
2. Both speed and direction are changing. $\vec{a}$ has components parallel and perpendicular to $\vec{v}$.
$\vec{a}_\perp$
$\vec{a}$
$\vec{a}_\parallel$
$\vec{v}$
$\vec{a}$
1. On the straight sections, where only the speed changes, $\vec{a}$ is parallel or opposite to $\vec{v}$.
5. Only the direction is changing at this point, not the speed. Thus $\vec{a}$ is perpendicular to $\vec{v}$.
4. The acceleration vector can be decomposed into $\vec{a}_\parallel$ and $\vec{a}_\perp$.

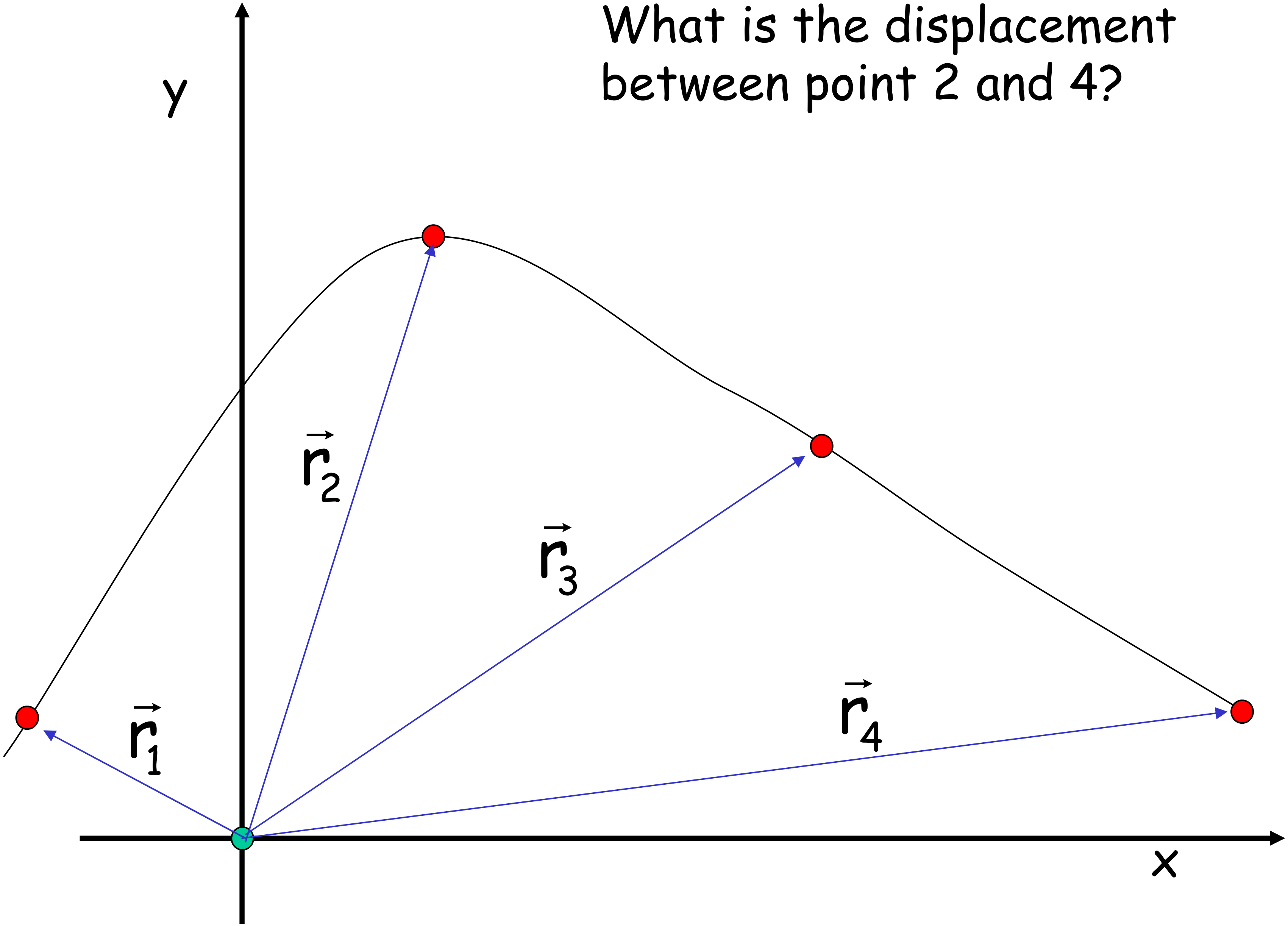
Project onto x-y plane

- Our Newton's laws of motion have worked well so far because we've always analysed motion in an x-y plane

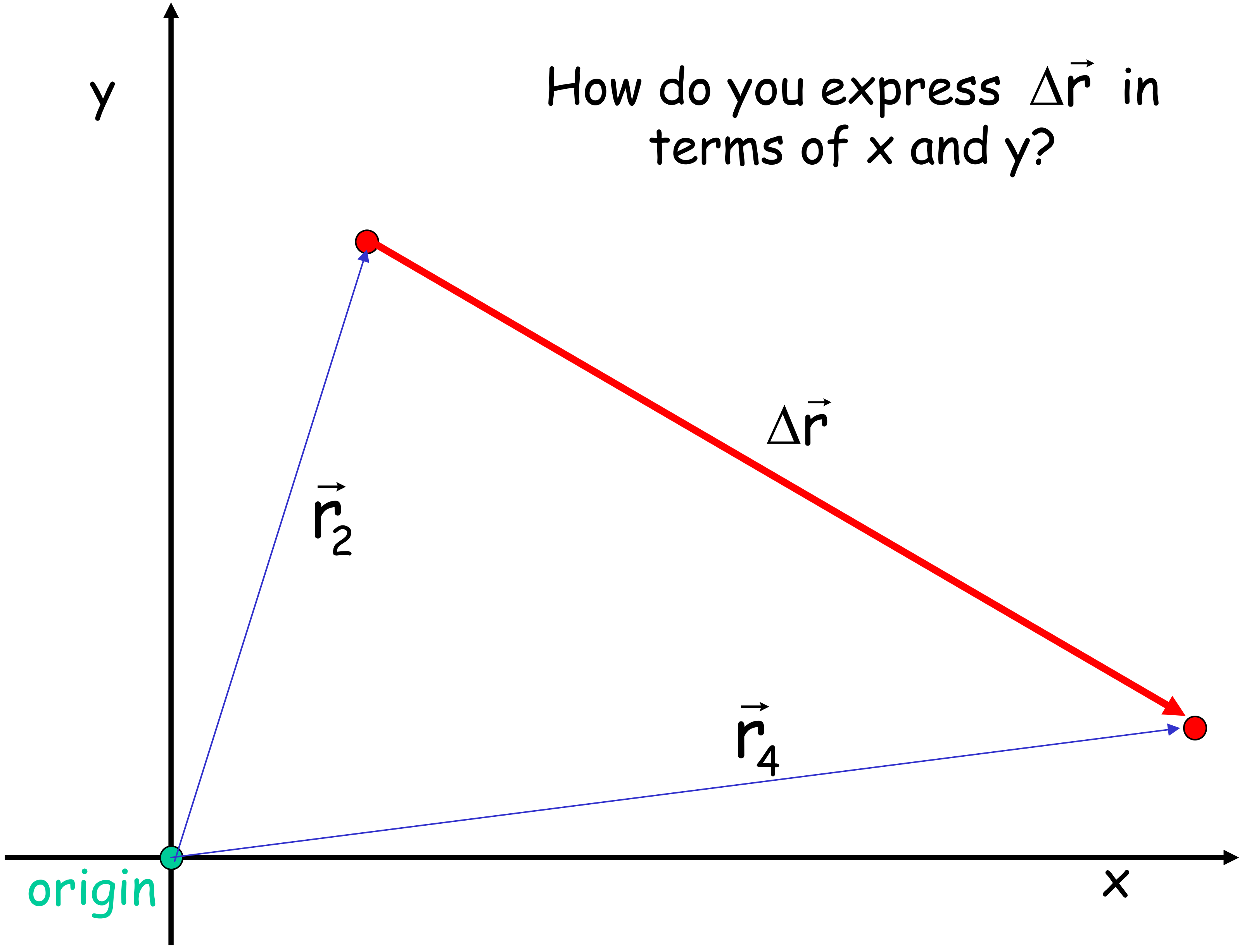
- Our motion diagrams have been using axes which are parallel and perpendicular to the **curve** at a given location

- Have to project those axes onto an x-y coordinate system

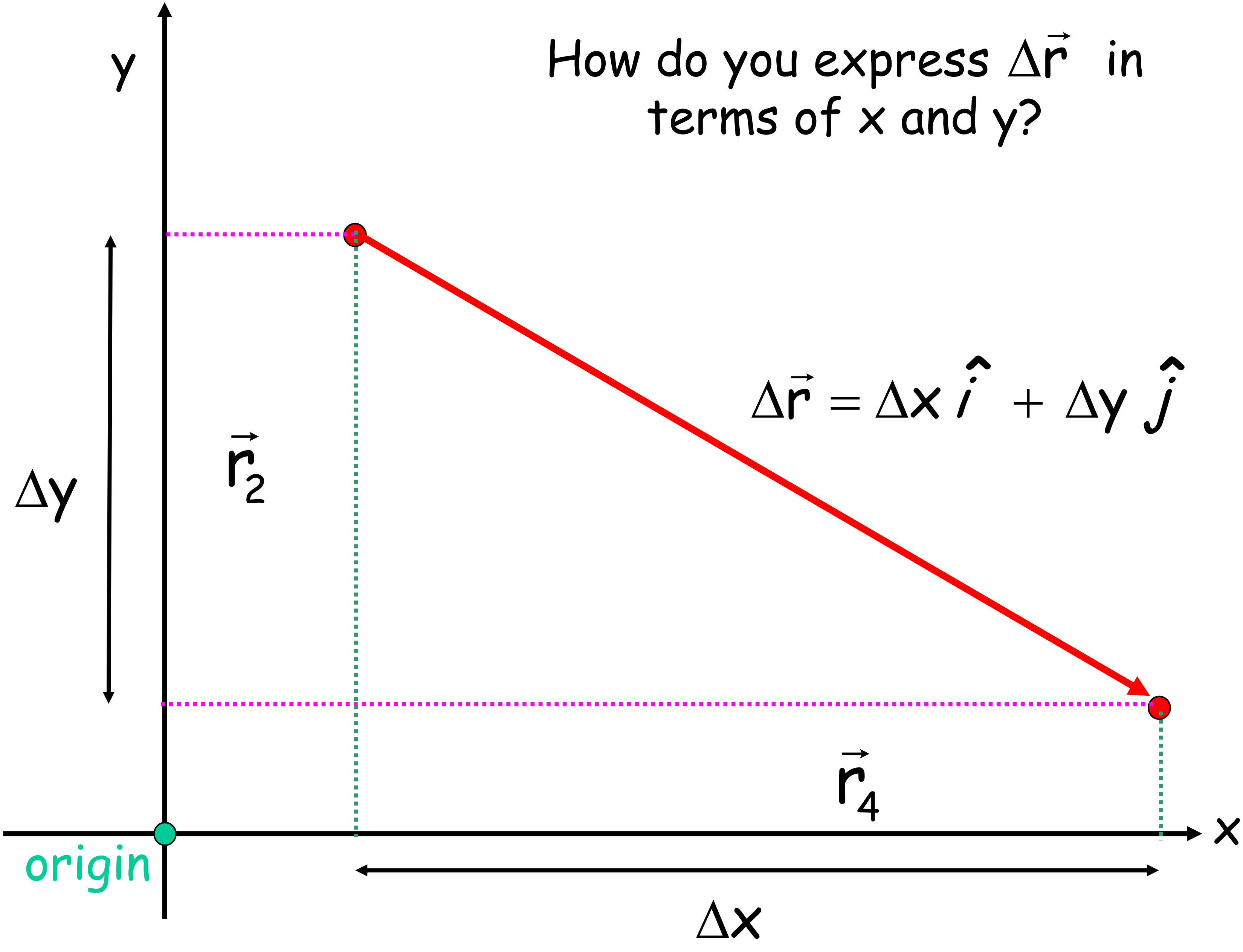
What is the displacement between point 2 and 4?



How do you express $\Delta\vec{r}$ in terms of x and y?



How do you express $\Delta\vec{r}$ in terms of x and y?



How About Velocity?

$$\Delta\vec{r} = \Delta x\,\hat{i} + \Delta y\,\hat{j}$$

$$\frac{\Delta\vec{r}}{\Delta t} = \frac{\Delta x}{\Delta t}\,\hat{i} + \frac{\Delta y}{\Delta t}\,\hat{j}$$

$$\vec{v} = v_x\,\hat{i} + v_y\,\hat{j}$$

How About Acceleration?

$$\vec{v} = v_x \hat{i} + v_y \hat{j}$$

$$\frac{\Delta \vec{v}}{\Delta t} = \frac{\Delta v_x}{\Delta t} \hat{i} + \frac{\Delta v_y}{\Delta t} \hat{j}$$

$$\vec{a} = a_x \hat{i} + a_y \hat{j}$$

Why is that interesting?

$$\vec{a} = a_x \hat{i} + a_y \hat{j}$$

plus

$$\vec{F}_{net} = m\vec{a}$$

$$\vec{F} = F_x \hat{i} + F_y \hat{j}$$

This means that all of the work that we do breaking up forces into their x and y components is useful for determining two-dimensional kinematics.

Common Problems!

- A force in x-direction only changes the motion along the x-direction (this means that a bouncing ball that is thrown sideways and bounces off of the floor does not have its x-direction motion changed by the normal force that made the ball bounce)

- After a force momentarily acts on an object, its acceleration drops off to zero, but the velocity that it imparted to the object doesn't disappear when the force stops acting on the object.

Quick Quiz:

A warship is traveling at 10 m/s parallel to the coastline as it fires its guns directly sideways at the shore. When the shell explodes on the shore, the crew sees the explosion:

(A) a little ahead of the ship
(B) directly beside the ship
(C) a little behind the ship
(D) not enough information is given

Quick Quiz:

A space ship starts from rest at the space station*, and fires its forward thrusters to give an acceleration of $\vec{a} = 1\hat{i}$ m/s² for 10 seconds. Then it turns off the forward thrusters and fires the side thrusters to give an acceleration of $\vec{a} = 1\hat{j}$ m/s² for 10 seconds. At the end of this 20 seconds:

(A) How far has the spaceship traveled along the x axis at t=20 seconds?

* take the origin to be the space station

Example:

A space ship starts from rest at the space station*, and fires its forward thrusters to give an acceleration of $\vec{a} = 1\hat{i}$ m/s^2 for 10 seconds. Then it turns off the forward thrusters and fires the side thrusters to give an acceleration of $\vec{a} = 1\hat{j}$ m/s^2 for 10 seconds. At the end of this 20 seconds:

(A) How far from the space station is the spaceship?
(B) Draw the path the spaceship traversed over the 20 seconds.

* take the origin to be the space station