Arts & Science 2D06

Quiz #7 2015 Mar 19 Name: *Solutions*

NB: Mark values are given in brackets [ ] beside each problem. Write all your answers on the quiz paper. No books or notes allowed. Time to write quiz: 50 minutes.

Photon energy: $E = hc/\lambda$

Energy levels of H atom: $E_n = -13.6\ \text{eV}/n^2$

Infinite square well: $E_n = (h^2/8mL^2)n^2$ $\psi(x) = A\sin(n\pi x/L)$

Wavelengths emitted by H atom: $\frac{1}{\lambda_n} = R(\frac{1}{n^2} - \frac{1}{m^2})$

de Broglie relation: $\lambda = h/p$

Speed of light $c = 3.00 \times 10^8$ m/sec
Planck's constant $h = 6.626 \times 10^{-34}$ J-sec and $\hbar = h/(2\pi)$
Rydberg constant $R = 1.097 \times 10^7\ \text{m}^{-1}$
Mass of electron $m_e = 9.11 \times 10^{-31}$ kg
Mass of proton (or neutron) $m_p = 1.67 \times 10^{-27}$ kg
$1\ MeV = 1.6 \times 10^{-13}$J

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1. [3] The probability of finding an electron somewhere in a hydrogen atom is directly proportional to its:

a) energy.
b) momentum.
c) wavefunction.
→ d) wavefunction, squared.
e) de Broglie wavelength.

2. [3] True or false: A neutron trapped in an infinitely deep square well potential (a.k.a. one-dimensional box) cannot have zero energy.

(Explain/derive your answer.)

→ a) TRUE.
b) FALSE.
c) More information is required.

$$E_n = \left(\frac{h^2}{8ML^2}\right)n^2 \quad , \quad n = 1, 2, 3 \ldots$$

$\therefore$ lowest energy : $n = 1$

$$\Rightarrow E_1 \neq 0$$

3. [4] If a hydrogen atom is in the $n = 4$ state, what is the largest number of photons that can be emitted as the atom goes back to the ground state? What is the smallest? (Explain/derive your answer.)

- <u>largest</u> : $4 \rightarrow 3$, $3 \rightarrow 2$, $2 \rightarrow 1$ } <u>3 photons</u>

- <u>Smallest</u> : $4 \rightarrow 1$ } <u>1 photon</u>

4. [2+2+2] An electron finds itself trapped in a one-dimensional box of width L.

a) Suppose the electron is in the ground state; if after absorbing a photon of 9 eV, the electron ends up in the first excited state, what is L?

- ground state : $n = 1 \Rightarrow E_1 = \frac{h^2}{8mL^2} = \frac{6.0 \times 10^{-38}}{L^2}$

- first excited state : $n = 2 \Rightarrow E_2 = 4E_1 = \frac{24 \times 10^{-38}}{L^2}$

$$E_2 - E_1 = \frac{18 \times 10^{-38}}{L^2} = E_{photon} = 9\,eV \left(\frac{1.6 \times 10^{-19} J}{1\,eV} \right)$$

$$\therefore \quad L^2 = 1.25 \times 10^{-19} \Rightarrow L = 3.5 \times 10^{-10}\ m \quad (0.35\ nm)$$

b) While in the first excited state, what is the electron's energy (in eV)?

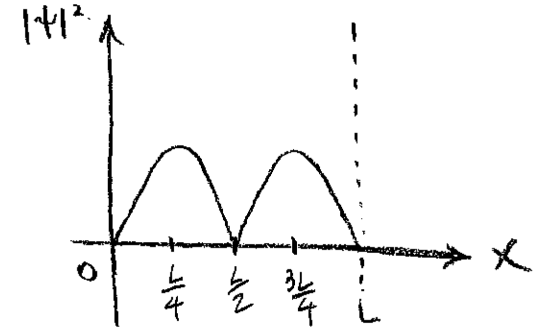
$$E_2 = \frac{24 \times 10^{-38}}{1.25 \times 10^{-19}} = 1.9 \times 10^{-18} J \left(\frac{1\,eV}{1.6 \times 10^{-19} J} \right)$$

$$= 12\ eV$$

c) How far from the left side of the box is the electron most likely to be found? (Justify your answer.)

- first excited state : $\psi = A \sin\left(\frac{2\pi x}{L}\right)$

- Plot $|\psi|^2$:



Most likely found at $x = \frac{L}{4}, \frac{3L}{4}$; or,

$x = 0.09$ nm and 0.26 nm

5. [4] What is the energy of a photon that has the same wavelength as a 100-MeV proton?

proton : $\lambda_p = \frac{h}{p_p} = \frac{h}{\sqrt{2 K_p m_p}}$

$\therefore \lambda_p = \frac{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}{\sqrt{2 (100 \text{ MeV} \cdot \frac{1.6 \times 10^{-13} \text{ J}}{\text{MeV}})(1.67 \times 10^{-27} \text{ kg})}}$

$= 2.9 \times 10^{-15} \text{ m}$

photon : $E = \frac{hc}{\lambda_p} = \frac{(6.626 \times 10^{-34})(3 \times 10^8)}{2.9 \times 10^{-15}}$

So, $\underline{E = 6.9 \times 10^{-11} \text{ J}}$

(or $\underline{E = 431 \text{ MeV}}$)

20/

[20] total marks