Arts & Science 2D06

Quiz #7 2019 Mar 20 **Name:** *Solutions*

NB: Mark values are given in brackets [ ] beside each problem. Write all your answers on the quiz paper. No books or notes allowed. Time to write quiz: 50 minutes.

Photon energy: $E = hc/\lambda$

Energy levels of H atom: $E_n = -13.6\ \mathrm{eV}/n^2$

Infinite square well: $E_n = (h^2/8mL^2)n^2 \qquad \psi(x) = A\sin(n\pi x/L)$

Wavelengths emitted by H atom: $\frac{1}{\lambda_n} = R(\frac{1}{n^2} - \frac{1}{m^2})$

de Broglie relation: $\lambda = h/p$

Speed of light $c = 3.00 \times 10^8$ m/sec
Planck's constant $h = 6.626 \times 10^{-34}$ J-sec and $\hbar = h/(2\pi)$
Rydberg constant $R = 1.097 \times 10^7\ \mathrm{m}^{-1}$
Mass of electron $m_e = 9.11 \times 10^{-31}$ kg
Mass of proton (or neutron) $m_p = 1.67 \times 10^{-27}$ kg
$1\ eV = 1.6 \times 10^{-19}$J

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1. [3] The ratio of a photon's energy to its frequency gives

(No explanation required.)

$E = hf \Rightarrow \frac{E}{f} = h$

a) its amplitude.
b) its velocity.
c) its wavelength.
→ d) Planck's constant.
e) its interference pattern.

2. [3] For everyday objects like baseballs, cars, and airplanes, an uncertainty in momentum of 1%, say, yields simultaneously a limiting uncertainty in position that

(Explain/derive your answer.)

a) is also about 1%.

→ b) is negligibly small.

c) is infinite.

d) does not exist.

e) is exactly 50%.

$\Delta x \cdot \Delta p \sim \hbar$

$\Delta p = m \Delta v \Rightarrow \Delta x \sim \frac{\hbar}{m \Delta v}$

- $\hbar$ is a very small number ($\sim 10^{-34}$)
- everyday objects : m is a relatively big number, compared to electron/proton.

$\therefore$ even with $\frac{\Delta v}{v} \sim 1\%$ (assume "m" is very precisely known),

Δx will be a very small number.

3. [4] Suppose that the de Broglie wavelength of an electron is measured to be 380 nm. What is this electron's speed? (1 nm $= 10^{-9}$ m)

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

$$\Rightarrow \quad v = \frac{h}{m\lambda} = \frac{6.626 \times 10^{-34}}{(9.11 \times 10^{-31})(380 \times 10^{-9})}$$

$$\therefore \quad v = 1914 \text{ m/s}$$

4. [5] Find how many photons per second are given off by a 100-Watts light bulb, operating at an efficiency of 4.80%. Assume the wavelength of the light to be 600 nm. (1 Watt = 1 J/s)

$$E_{ph} = \frac{hc}{\lambda} = \frac{(6.626 \times 10^{-34})(3 \times 10^{8})}{600 \times 10^{-9}} = 3.3 \times 10^{-19}\ J$$

$$\therefore \text{\# of photons per second} = (0.048)\left(\frac{100\ J}{s}\right)\left(\frac{1}{3.3 \times 10^{-19}\ J}\right)$$

$$= 1.5 \times 10^{19}$$

5. [5] A particle is trapped in an infinitely deep square well potential (i.e., a one-dimensional box). If the particle's ground state energy is 4 eV, what energy must be absorbed by the system for the particle to jump from the $n = 2$ state to the $n = 4$ state?

$$\cdot \quad E_1 = \frac{h^2}{8mL^2} = 4\,\text{eV} \Rightarrow \frac{h^2}{mL^2} = 32\,\text{eV} \qquad (1)$$

$$\cdot \quad \Delta E = E_4 - E_2 = \frac{h^2}{8mL^2}4^2 - \frac{h^2}{8mL^2}2^2$$

$$= \frac{h^2}{mL^2}\left(2 - \frac{1}{2}\right) = \frac{3}{2}\left(\frac{h^2}{mL^2}\right) \qquad (2)$$

Sub (1) in (2) : $\Delta E = \frac{3}{2} \cdot 32 = 48\,\text{eV}$

20/[20] total marks