Arts & Science 2D06

Quiz #7 2018 Mar 16 **Name:** *Solutions*

NB: Mark values are given in brackets [ ] beside each problem. Write all your answers on the quiz paper. No books or notes allowed. Time to write quiz: 50 minutes.

Photon energy: $E = hc/\lambda$

Energy levels of H atom: $E_n = -13.6 \text{ eV}/n^2$

Infinite square well: $E_n = (h^2/8mL^2)n^2$ $\quad \psi(x) = A \sin(n\pi x/L)$

Wavelengths emitted by H atom: $\frac{1}{\lambda_n} = R(\frac{1}{n^2} - \frac{1}{m^2})$

de Broglie relation: $\lambda = h/p$

Speed of light $c = 3.00 \times 10^8$ m/sec

Planck's constant $h = 6.626 \times 10^{-34}$ J-sec and $\hbar = h/(2\pi)$

Rydberg constant $R = 1.097 \times 10^7 \text{ m}^{-1}$

Mass of electron $m_e = 9.11 \times 10^{-31}$ kg

Mass of proton (or neutron) $m_p = 1.67 \times 10^{-27}$ kg

$1 \; eV = 1.6 \times 10^{-19}$J

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1. [3] In a hydrogen atom, when an electron jumps from an orbit where $n = 5$ to one where $n = 4$,

(No explanation required.)

→ a) a photon is emitted.

b) a photon is absorbed.

c) two photons are emitted.

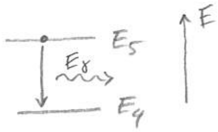
d) two photons are absorbed.

e) a photon is neither emitted nor absorbed.

$E_5 = -\frac{13.6}{5^2} = -0.54 \text{ eV}$

$E_4 = -\frac{13.6}{4^2} = -0.85 \text{ eV}$

$\therefore \; E_5 > E_4$:



So, for $E_5 \rightarrow E_4$: Photon is emitted.

2. [3] Which of the following is NOT true of Bohr's atomic model?

(Explain/derive your answer.)

a) It successfully predicted the electron energies in atoms with one electron.

b) It explained the emission line spectrum of hydrogen.

c) It proposed a mechanism for preventing the electron from collapsing onto the nucleus.

d) It introduced the new concept of energy quantization into atomic physics.

→ e) It successfully predicted the electron wavefunctions for the neutral helium atom (which has 2 protons and 2 electrons).

Bohr's model represented the electron as a particle, and not by a wave function.

It also only worked for atoms with one electron.

3. [4] The McMaster Reactor produces a beam of neutrons with an energy of 0.025 eV. Suppose that, following in the footsteps of Prof. B. Brockhouse (our local Nobelist), you're planning an experiment to study the interference of these neutrons after they pass through an exotic crystal. As a first step, calculate the wavelength of the neutrons in your beam.

$$K = 0.025\ \text{eV} \cdot \frac{1.6\times10^{-19}\ \text{J}}{1\ \text{eV}} = 4\times10^{-21}\ \text{J}$$

momentum : $p = \sqrt{2mK} = \sqrt{2\cdot1.67\times10^{-27}\cdot4\cdot10^{-21}}$

$$= 3.6\times10^{-24}\ \text{kg}\cdot\text{m/s}$$

$$\therefore\ \lambda = \frac{h}{p} = \frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{3.6\times10^{-24}\ \text{kg}\cdot\text{m/s}} = 1.8\times10^{-10}\ \text{m}\ \ (1.8\ \text{Å})$$

(Note: "$E = \frac{hc}{\lambda}$" only applies to photons.)

4. [5] An electron is trapped in an infinitely deep square well potential of width L (i.e., a one-dimensional box), and is initially in the ground state. Suppose that the system absorbs a 9-eV photon, thereby putting the electron in the $n = 2$ state.

(a) What is the width of the box?

(b) After this quantum jump, what is the probability that the electron will be found right in the middle of the box? Explain/derive your answer.

(a) $E_n = \left(\frac{h^2}{8mL^2}\right)n^2 \rightarrow E_2 = \frac{h^2}{2mL^2}, \quad E_1 = \frac{h^2}{8mL^2}$; $E_{ph} = 9eV\left(\frac{1.6\times10^{-19}J}{1eV}\right)$

$= 1.4\times10^{-18} J$

$\therefore \quad \Delta E = E_2 - E_1 = E_{ph}$

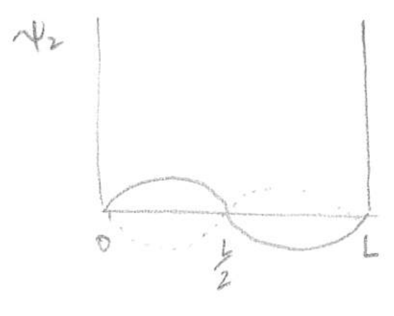
$$\frac{h^2}{2mL^2} - \frac{h^2}{8mL^2} = E_{ph}$$

$$\frac{3h^2}{8mL^2} = E_{ph}$$

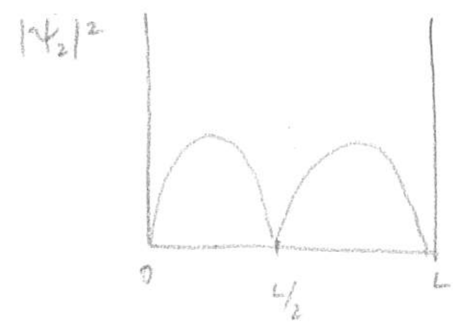
$$L = \left(\frac{3h^2}{8mE_{ph}}\right)^{1/2} = \left[\frac{3(6.626\times10^{-34})^2}{8(9.11\times10^{-31})(1.4\times10^{-18})}\right]^{1/2}$$

$\Rightarrow$ $\underline{L = 3.6\times10^{-10} m}$

(b) $n = 2$: $\psi_2 = A\sin(2\pi x/L)$



$\Rightarrow$



@ $x = \frac{L}{2}$: $|\psi_2|^2 = 0$ $\therefore$ $\underline{\text{Probability} = 0}$

5. [5] Suppose that a light-emitting diode (LED) emits one microwatt of 640-nm photons. What is the energy of each photon? How many photons are emitted each second? (1 Watt = 1 J/s; 1 microwatt = 10^{-6} watts)

$\lambda = 640 \times 10^{-9} m$: $E = \frac{hc}{\lambda} = \frac{(6.626 \times 10^{-34})(3 \times 10^8)}{640 \times 10^{-9}}$

$\rightarrow$ $E = 3.1 \times 10^{-19} J$ ($\approx 1.9 eV$)

Energy emitted per second : $10^{-6} J$

$\therefore \frac{\text{\# of photons}}{\text{sec}} = 10^{-6} \frac{J}{s} \cdot \frac{1 \text{ photon}}{3.1 \times 10^{-19} J}$

$= 3.2 \times 10^{12}$

20

[20] total marks