Arts & Science 2D06

Quiz #7 **2017 Mar 17** **Name:** Solutions

NB: Mark values are given in brackets [ ] beside each problem. Write all your answers on the quiz paper. No books or notes allowed. Time to write quiz: 50 minutes.

Photon energy: $E = hc/\lambda$

Energy levels of H atom: $E_n = -13.6 \text{ eV}/n^2$

Infinite square well: $E_n = (h^2/8mL^2)n^2 \quad \psi(x) = A \sin(n\pi x/L)$

Wavelengths emitted by H atom: $\frac{1}{\lambda_n} = R(\frac{1}{n^2} - \frac{1}{m^2})$

de Broglie relation: $\lambda = h/p$

Speed of light $c = 3.00 \times 10^8$ m/sec
Planck's constant $h = 6.626 \times 10^{-34}$ J-sec and $\hbar = h/(2\pi)$
Rydberg constant $R = 1.097 \times 10^7 \text{ m}^{-1}$
Mass of electron $m_e = 9.11 \times 10^{-31}$ kg
Mass of proton (or neutron) $m_p = 1.67 \times 10^{-27}$ kg
$1\ eV = 1.6 \times 10^{-19}$J

---

1. [2] Suppose that a particle is trapped in an infinitely deep square-well potential (a.k.a. one-dimensional box). How does the energy difference between adjacent energy levels change, as the quantum number n increases?

(No explanation required.)

a) The spacing decreases.

b) The spacing alternately increases and decreases, depending on whether the wave function is a sine or a cosine function.

c) The spacing remains constant.

→ d) The spacing increases.

e) None of the above.

2. [3] A proton and an electron are both accelerated to the same final (non-relativistic) kinetic energy. If $\lambda_{\cancel{e}\,p}$ is the de Broglie wavelength of the proton and $\lambda_{\cancel{p}\,e}$ is the de Broglie wavelength of the electron, then

(Explain/derive your answer.)

a) $\lambda_p > \lambda_e$.

b) $\lambda_p = \lambda_e$.

→ c) $\lambda_p < \lambda_e$.

d) more information is required to answer the question.

$$K_p = K_e \Rightarrow \frac{P_p^2}{\cancel{2}m_p} = \frac{P_e^2}{\cancel{2}m_e}$$

Since $p = \frac{h}{\lambda}$:

$$\frac{\cancel{h^2}/\lambda_p^2}{m_p} = \frac{\cancel{h^2}/\lambda_e^2}{m_e}$$

$$\therefore \lambda_p = \sqrt{\frac{m_e}{m_p}}\,\lambda_e$$

$$\Rightarrow \underline{\lambda_p < \lambda_e}$$

3. [3] A certain particle's energy is known to within 10^{-18} J. What is the minimum uncertainty in its arrival time at a detector?

$$\Delta E \cdot \Delta t \lesssim \hbar \quad , \quad \Delta E = 10^{-18}\,J$$

$$\therefore (\Delta t)_{min} \sim \frac{\hbar}{\Delta E}$$

$$\sim \frac{6.626 \times 10^{-34}\,J\cdot s}{(2\pi)\,10^{-18}\,J}$$

$$\sim 1.05 \times 10^{-16}\,s$$

4. [4] A beam of red light and a beam of violet light each deliver the same power (i.e., energy per second) onto a surface. Which beam has the most photons hitting the surface per second?

(Explain/derive your answer; *hint*: the wavelength of red light is longer than that of violet.)

$$E_{photon} = h \cdot f = \frac{hc}{\lambda}$$

Since $\lambda_{red} > \lambda_{violet}$, we have

$$E_{red} < E_{violet}$$

$\therefore$ the <u>red beam</u> has the most photons/second.

5. [4] If the ground state energy level of an electron in a one-dimensional box is 5.0 eV, what is the width of the box? Where is the electron most likely to be found (explain your answer)?

$$5\,eV = 5\,eV \cdot \frac{1.6\times10^{-19}\,J}{1\,eV} = 8\times10^{-19}\,J$$

ground state : $n = 1 \Rightarrow E = \dfrac{h^2}{8mL^2}$

$$\therefore \frac{h^2}{8mL^2} = 8\times10^{-19}$$

$$L = \sqrt{\frac{(6.626\times10^{-34})^2}{8(9.11\times10^{-31})(8\times10^{-19})}}$$

$$\Rightarrow \underline{L = 2.7\times10^{-10}\,m = 0.27\,nm}$$

6. [4] The energy of the ground state in the Bohr model is -13.6 eV. In a transition from the $n = 2$ state to the $n = 4$ state, a photon of energy:

(Explain/derive your answer.)

a) 3.40 eV is emitted.

b) 3.40 eV is absorbed.

→ c) 2.55 eV is absorbed.

d) 2.55 eV is emitted.

e) 0.85 eV is emitted.

For hydrogen atom:

$$E_n = -\frac{13.6}{n^2} \text{ eV}$$

- for $n=1$: $E_1 = -13.6 \text{ eV}$

$\therefore$ this Bohr model is for hydrogen.

- for $n=2$: $E_2 = -\frac{13.6}{4} = -3.4 \text{ eV}$
- for $n=4$: $E_4 = -\frac{13.6}{16} = -0.85 \text{ eV}$

$\therefore \Delta E = E_f - E_i = E_4 - E_2 = -0.85 - (-3.4) = 2.55 \text{eV}$

⇒ a photon of energy 2.55eV is required for the transition.

Since $E_4 > E_2$ → photon must be absorbed.

20

[20] total marks