Arts & Science 2D06

Quiz #7 2014 Mar 25 Name: *Solutions*

NB: Mark values are given in brackets [ ] beside each problem. Write all your answers on the quiz paper. No books or notes allowed. Time to write quiz: 50 minutes.

Photon energy: $E = hc/\lambda$

Energy levels of H atom: $E_n = -13.6 \text{ eV}/n^2$

Infinite square well: $E_n = (h^2/8mL^2)n^2$ $\psi(x) = A \sin(n\pi x/L)$

Wavelengths emitted by H atom: $\frac{1}{\lambda_n} = R(\frac{1}{n^2} - \frac{1}{m^2})$

de Broglie relation: $\lambda = h/p$

Speed of light $c = 3.00 \times 10^8$ m/sec

Planck's constant $h = 6.626 \times 10^{-34}$ J-sec and $\hbar = h/(2\pi)$

Rydberg constant $R = 1.097 \times 10^7 \text{ m}^{-1}$

Mass of electron $m_e = 9.11 \times 10^{-31}$ kg

Mass of proton (or neutron) $m_p = 1.67 \times 10^{-27}$ kg

1 $MeV = 1.6 \times 10^{-13}$J

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1. [3] As a particle travels faster and faster, its de Broglie wavelength:

(Explain/derive your answer.)

a) increases.

→ b) decreases.

c) remains constant.

d) could be any of the above; it depends on other factors.

$\lambda = \frac{h}{p} = \frac{h}{mv}$

$\therefore v\uparrow, \lambda\downarrow$

2. [3] The energy difference between adjacent orbit radii in a hydrogen atom:

(Explain/derive your answer.)

a) increases with increasing values of n.

→ b) decreases with increasing values of n.

c) remains constant for all values of n.

d) varies randomly with increasing values of n.

$$E_n = -\frac{13.6}{n^2}$$

$$\therefore \left.\begin{aligned} E_2 - E_1 &\propto \left(-\tfrac{1}{4}+1\right) = 0.75 \\ E_3 - E_2 &\propto \left(-\tfrac{1}{9}+\tfrac{1}{4}\right) = 0.14 \\ E_4 - E_3 &\propto \left(-\tfrac{1}{16}+\tfrac{1}{9}\right) = -0.05 \end{aligned}\right\} \Delta E_{n,n+1} \downarrow \text{ as } n \uparrow$$

3. [4] An electron inside a hydrogen atom is confined to a region of space of 0.11 nm wide. Under these conditions, what is the uncertainty in the electron's velocity?

let $\Delta x = 0.11 \times 10^{-9}\ m$

H.U.P. : $\Delta x \cdot \Delta p \gtrsim \hbar$; $\Delta p = m \Delta v$

$$\therefore\ m\Delta v \gtrsim \frac{\hbar}{\Delta x}$$

$$\Delta v \gtrsim \frac{\hbar}{m\Delta x}$$

$$\Delta v \gtrsim \frac{6.626 \times 10^{-34}/2\pi}{(9.11 \times 10^{-31}\ kg)(0.11 \times 10^{-9}\ m)}$$

so, $\Delta v \approx 1.05 \times 10^{6}\ m/s$

4. [2+2+2] A proton finds itself trapped in an infinitely deep square well potential (a.k.a. one-dimensional box), of width L.

a) If the ground state energy is 4 MeV, what is the smallest amount of energy that the proton can absorb?

ground state : $n = 1$

$$E_1 = \left(\frac{h^2}{8mL^2}\right)(1)^2 = 4\,\text{MeV} \Rightarrow \frac{h^2}{8mL^2} = 4$$

So $E_n = 4n^2$

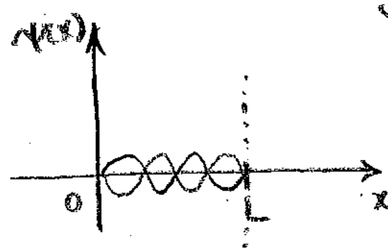
Smallest ΔE_{min} : $n=1 \rightarrow n=2$

$E_2 = 4(4) = 16$ MeV , so $\Delta E_{min} = 16 - 4 = 12$ MeV

Suppose now that the proton is in the third excited state:

b) Sketch the proton's wavefunction. Where inside the well/box will the particle never be found?

3rd excited state : $n = 4$

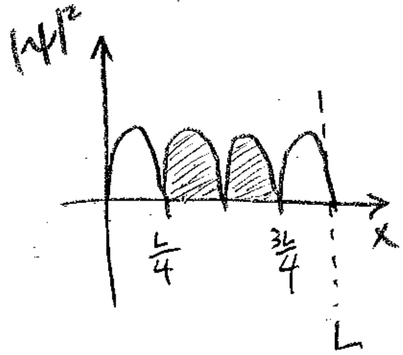


particle will never be found at nodes of ψ :

$x = \frac{L}{4}, \frac{L}{2}, \frac{3L}{4}$

$\psi_4(x) = A \sin\left(\frac{4\pi x}{L}\right)$

c) Suppose you measure the particle's position. What is the probability that the proton will be found in the region between $x > L/4$ and $x < 3L/4$? Justify your answer.



Probability $\rightarrow |\psi(x)|^2$

$\text{Prob}\left(L/4 < x < \frac{3L}{4}\right) = 50\%$,

by inspection of figure

(or, integrate $\int_{L/4}^{3L/4} \left(\frac{2}{L}\right) \sin^2 \frac{4\pi x}{L}\, dx$)

5. [4] Suppose a 60-W light-bulb converts 6.2% of its input energy into visible light of wavelength 580 nm. How many (visible) photons per second does the bulb emit? (1 W = 1 J/sec)

- bulb's energy output (per second):

$$E_{total} = 0.062 \times 60\,J = 3.72\,J$$

- energy of a photon: $E = \frac{hc}{\lambda}$

$$= \frac{(6.626 \times 10^{-34})(3 \times 10^{8})}{580 \times 10^{-9}}$$

$$= 3.4 \times 10^{-19}\,J$$

$$\therefore \#\text{ of photons/sec} = \frac{3.72\,J}{3.4 \times 10^{-19}\,J} \cong 10^{19}$$

[20] total marks