Arts & Science 2D06

Quiz #6 2020 Feb 26 **Name:** Solutions

NB: Mark values are given in brackets [ ] beside each problem. Write all your answers on the quiz paper. No books or notes allowed. Time to write quiz: 50 minutes.

Wave speed: $v = f\lambda$

Wave function: $y = A\sin(kx - \omega t + \varphi)$ where $k = 2\pi/\lambda$

Two-slit interference: $\sin\theta = m\lambda/d$ and $\sin\theta = (m + 1/2)\lambda/d$

$$I = I_0 \cdot \cos^2(\pi d sin\theta/\lambda)$$

Diffraction: $\sin\theta = m\lambda/a$

$$I = I_0 \frac{\sin^2(\alpha/2)}{(\alpha/2)^2} \text{ where } \alpha = 2\pi a sin\theta/\lambda$$

Photon energy: $E = hc/\lambda$

Planck's constant $h = 6.626 \times 10^{-34}$ J-sec

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1. [3] The phenomenon of light interference gives evidence supporting

(You do not need to explain your answer, unless you would like to do so.)

(a) the particle theory of light.
(b) the particle theory of light, except in the case of constructive interference.
(c) both the particle and wave theories of light.
→(d) the wave theory of light.
(e) the wave theory of light, except in the case of destructive interference.

Interference: due to phase difference between light waves (e.g., Young's double-slit experiment)

2. [3] In order for a photon to eject an electron from a metal's surface in the photoelectric effect, the photon's

(Explain/derive your answer in the space below.)

→ (a) frequency must be greater than a certain minimum value.

(b) speed must oscillate in time.

(c) wavelength must be greater than a certain minimum value.

(d) momentum must be identically zero.

(e) mass must be finite.

Photon transfers energy to electron in a collision.

A minimum transferred energy, and therefore photon energy, is needed to eject the electron.

∴ Since $E_{ph} = hf$, the frequency must also be greater than a minimum value.

3. [4] A slit of width 0.01 mm has light of wavelength 600 nm passing through it onto a screen 60 cm away. How wide (in cm) is the central maximum observed on the screen? ($1\ nm = 10^{-9}\ m$)

Diffraction minima:

$$\sin\theta = \frac{m\lambda}{a} \approx \frac{y_m}{L}$$

Width of central maximum:

$$y_1 - y_{-1} = 2y_1 = 2\left[\frac{(0.6)(1)(600\times10^{-9})}{0.01\times10^{-3}}\right]$$

$$= 0.072\ m = 7.2\ cm.$$

4. [5] Light from a monochromatic source (i.e., of a single wavelength) shines through a double slit setup onto a screen placed 5.0 m away. The slits are 0.18 mm apart. If the dark bands/fringes on the screen are measured to be 1.7 cm apart, find the wavelength of the incident light.

Consider 2 consecutive interference minima of orders $m, m+1$:

- $y_{m+1} \cong L \cdot \left[(m+1) + \frac{1}{2}\right] \frac{\lambda}{d} = (m+3/2)\left(\frac{L\lambda}{d}\right)$
- $y_m \cong (m+\frac{1}{2})\left(\frac{L\lambda}{d}\right)$

$$\therefore \quad y_{m+1} - y_m = \left[(m+3/2) - (m+1/2)\right]\left(\frac{L\lambda}{d}\right)$$

$$= \frac{L\lambda}{d} = 1.7 \times 10^{-2}\ m$$

$$\Rightarrow \quad \lambda = \frac{(1.7\times10^{-2})(0.18\times10^{-3})}{5}$$

$$= 6.12 \times 10^{-7}\ m$$

5. [5] A travelling wave is described by the function

$$y(x,t) = (2.0\ m)\cos\left[(3.0\ m^{-1})x + (5.0\ s^{-1})t\right].$$

(a) Find the speed with which the wave propagates through its medium.

$k = \frac{2\pi}{\lambda} \Rightarrow \lambda = \frac{2\pi}{3}$

$\omega = \frac{2\pi}{T} \Rightarrow T = \frac{2\pi}{5}$

$\therefore\ V = \frac{\lambda}{T} = \frac{2\pi/3}{2\pi/5} = \frac{5}{3} = 1.7\ m/s$

(b) What is the velocity of the *medium* at $x = 0$ and $t = 1$ second?

$V_p = \frac{dy}{dt} = -(2.0)(5.0)\sin[3x+5t]$

$\therefore\ V_p(0,1) = -10\sin[5\ rad] = +9.6\ m/s$ [4p]

20/

[20] total marks