Arts & Science 2D06

Quiz #6 2019 Feb 27 **Name:** Solutions

NB: Mark values are given in brackets [ ] beside each problem. Write all your answers on the quiz paper. No books or notes allowed. Time to write quiz: 50 minutes.

Wave speed: $v = f\lambda$

Wave function: $y = A\sin(kx - \omega t + \varphi)$ where $k = 2\pi/\lambda$

Two-slit interference: $\sin\theta = m\lambda/d$ and $\sin\theta = (m + 1/2)\lambda/d$

$$I = I_0 \cdot \cos^2(\pi d sin\theta/\lambda)$$

Diffraction: $\sin\theta = m\lambda/a$

$$I = I_0 \frac{\sin^2(\alpha/2)}{(\alpha/2)^2} \text{ where } \alpha = 2\pi a sin\theta/\lambda$$

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1. [3] When two travelling waves whose displacements are given by $y_1(x,t)$ and $y_2(x,t)$ occupy the same region of space at the same time, what is the resulting wave function that one observes?

(You do not need to explain your answer, unless you would like to do so.)

(a) $y_1 \times y_2$
(b) $(y_1 \times y_2)^{1/2}$
→(c) $y_1 + y_2$
(d) $y_1 - y_2$
(e) $(y_1)^2 + (y_2)^2$

Principle of superposition for waves:

$$y = y_1 + y_2$$

2. [3] If a light ray from one slit of a Young's double slit experiment arrives at a point on the screen one-half wavelength behind the ray from the other slit, which of the following is observed at that point?

(Explain/derive your answer in the space below.)

(a) a bright fringe
→(b) a dark fringe
(c) a gray fringe
(d) a multi-coloured fringe
(e) none of the above.

- Rays are in phase when leaving their respective slits.

- Extra distance travelled by one ray $= \frac{\lambda}{2}$ ⇒ destructive interference between rays

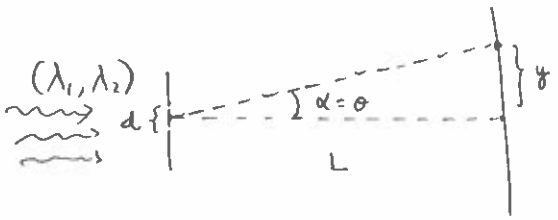
⇒ see dark fringe

3. [4] A wave is described by the function $y(x,t) = (3.0\ cm)\cos\left[(3.0\ m^{-1})x + (4.0\ s^{-1})t\right]$. What are this wave's wavelength and period?

- $\lambda = \frac{2\pi}{k} = \frac{2\pi}{3} = 2.1\ m$

- $T = \frac{2\pi}{\omega} = \frac{2\pi}{4} = 1.6\ s$

4. [5] In a double-slit experiment, the slit separation is 2.0 mm, and light comprising two wavelengths of 750 nm and 900 nm illuminates the slits. A screen is placed 2.0 m from the slits. At what distance from the central maximum on the screen will a bright fringe from one interference pattern first coincide with a bright fringe from the other? ($1\ nm = 10^{-9}\ m$)



① $\lambda_1 = 750\,nm$, angles θ, order m

bright fringes: $\sin\theta = \frac{m\lambda_1}{d}$

② $\lambda_2 = 900\,nm$, angles α, order M

bright fringes: $\sin\alpha = \frac{M\lambda_2}{d}$

"coincide": $\theta = \alpha \Rightarrow \sin\theta = \sin\alpha$

$$\Rightarrow \frac{m\lambda_1}{d} = \frac{M\lambda_2}{d}$$

$$m(750) = M(900)$$

$$m = 1.2\,M$$

- both m, M are integers: this first happens when

$$M = 5,\ m = 6$$

$$\therefore \sin\theta = \frac{6(750\times10^{-9})}{0.002} = 2.25\times10^{-3}$$

$$\Rightarrow \theta = 0.13^\circ$$

$$\therefore y = L\tan\theta = 2\tan(0.13^\circ) = 4.5\times10^{-3}m \quad (4.5\ mm)$$

5. [5] If the fifth-order minimum in a diffraction pattern is observed at 40°, at what angle is the pattern's second-order minimum?

$$\sin\theta_m = \frac{m\lambda}{a}$$

$$\sin 40^\circ = 5\left(\frac{\lambda}{a}\right) \Rightarrow \frac{\lambda}{a} = 0.13$$

$$\therefore \quad \sin\theta_2 = 2\left(\frac{\lambda}{a}\right) = 2 \cdot 0.13 = 0.26$$

$$\theta_2 = \sin^{-1} 0.26 = 15^\circ$$

20/

[20] total marks