Arts & Science 2D06

Quiz #6 2018 Feb 14 Name: Solutions

NB: Mark values are given in brackets [ ] beside each problem. Write all your answers on the quiz paper. No books or notes allowed. Time to write quiz: 50 minutes.

Period of simple pendulum: $T = 2\pi\sqrt{\frac{L}{g}}$ Wave speed: $v = f\lambda$

SHM equation of motion: $x = A\cos(\omega t + \varphi)$ where $\omega = \sqrt{k/m} = 2\pi/T$

Wave function: $y = A\sin(kx - \omega t + \varphi)$ where $k = 2\pi/\lambda$

Two-slit interference: $\sin\theta = m\lambda/d$ and $\sin\theta = (m + 1/2)\lambda/d$

$$I = I_0 \cdot \cos^2(\pi d sin\theta/\lambda)$$

Diffraction: $\sin\theta = m\lambda/a$

$$I = I_0 \frac{\sin^2(\alpha/2)}{(\alpha/2)^2} \text{ where } \alpha = 2\pi a sin\theta/\lambda$$

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1. [3] Choose the best answer: In a single slit diffraction pattern the intensity of the maxima is:

(You can explain your choice if you would like, but you do not have to.)

(a) the same everywhere.
(b) directly proportional to the sine of the angle from the central maximum.
(c) directly proportional to the square of the sine of the angle from the central maximum.
→(d) dependent on the wavelength, the slit width, and the angle from the central maximum.
(e) none of the above.

$$I = I_0 \frac{\sin^2(\alpha/2)}{(\alpha/2)^2}$$

$$= \frac{I_0 \sin^2(\pi a \sin\theta/\lambda)}{\left(\frac{\pi a \sin\theta}{\lambda}\right)^2}$$

$\therefore$ I depends on λ, a and θ

2. [3] A pendulum undergoes simple harmonic oscillations, with a period of 1.46 seconds. What is the pendulum's length?

$$T = 2\pi\sqrt{\frac{L}{g}} \rightarrow L = \left(\frac{T}{2\pi}\right)^2 g = \left(\frac{1.46}{2\pi}\right)^2 (9.81)$$

$$\therefore L = 0.53\ m$$

3. [4] A mass is attached to a vertical spring and oscillates up and down between points A and B. Where is the mass located when its kinetic energy is a minimum?

(Explain/derive your answer in the space below.)

→ (a) At either A or B.

(b) Midway between A and B.

(c) One-third of the way between A and B.

(d) One-fourth of the way between A and B.

(e) None of the above.

$K = \frac{1}{2}mv^2$

So $K = 0$ when $v = 0$

Simple harmonic motion:

$y = G\cos(\omega t + \phi)$, G = amplitude

$$v = \frac{dy}{dt} = -G\omega\sin(\omega t + \phi)$$

→ $v = 0$ when $\sin(\omega t + \phi) = 0$

$\sin(\omega t + \phi) = 0 \Rightarrow \cos(\omega t + \phi) = \pm 1$

$\therefore$ $v = 0$ when $y = \pm G$, i.e., at either A or B.

4. [5] A rope with a length of 2.0 m is stretched between two fixed supports, such that transverse, traveling waves propagate along it with speeds of 40 m/s. Find the wavelengths and frequencies of this string's (a) fundamental (also known as the first harmonic); and (b) third harmonic.

(a) fundamental :

$\lambda_1 = 2 \cdot 2m = 4m$

$f_1 = \frac{v}{\lambda_1} = \frac{40}{4} = 10\ s^{-1}$

(b) 3rd harmonic :

$\lambda_3 = \frac{2}{3}(2m) = 1.3m$

$f_3 = \frac{v}{\lambda_3} = \frac{40}{1.3} = 30.8\ s^{-1}$

5. [5] Light of wavelength 575 nm shines on a double-slit system and the third order bright fringe is seen at an angle of 6.5°. What is the separation between the slits? If this separation is gradually increased, does the fringe pattern spread out or get compressed? Explain.

$\lambda = 575\,nm = 575 \times 10^{-9}\,m = 5.75 \times 10^{-7}\,m$

"third order" $\Rightarrow$ $m = 3$; "bright fringe" $\Rightarrow$ interference maximum

$\theta_3 = 6.5°$

$\therefore \quad \sin\theta = \frac{m\lambda}{d} \rightarrow d = \frac{m\lambda}{\sin\theta} = \frac{3 \cdot 5.75 \times 10^{-7}}{\sin(6.5°)} = 1.5 \times 10^{-5}\,m$

and if "d" is increased, then $\sin\theta$ decreases:

$\rightarrow$ the angle of each fringe becomes smaller.

$\therefore$ the whole pattern gets <u>compressed.</u>

20/20

[20] total marks