Arts & Science 2D06

Quiz #6 2017 Feb 15 **Name:** Solutions

NB: Mark values are given in brackets [ ] beside each problem. Write all your answers on the quiz paper. No books or notes allowed. Time to write quiz: 50 minutes.

Wave function: $y = A\sin(kx - \omega t + \varphi)$ where $k = 2\pi/\lambda$

Two-slit interference: $\sin\theta = m\lambda/d$ and $\sin\theta = (m + 1/2)\lambda/d$

$$I = I_0 \cdot \cos^2(\pi d sin\theta/\lambda)$$

Diffraction: $\sin\theta = m\lambda/a$

$$I = I_0 \frac{\sin^2(\alpha/2)}{(\alpha/2)^2} \text{ where } \alpha = 2\pi a sin\theta/\lambda$$

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1. [3] A standing wave along a string is formed by the superposition of:

(*No explanation required.*)

(a) two equal amplitude, equal wavelength waves traveling in the same direction.
(b) two waves of different amplitude, but equal wavelength traveling in the same direction.
(c) two waves of different frequency, but equal amplitude traveling in the same direction.
(d) two waves of different frequency, but equal amplitude traveling in opposite directions.
→ (e) two equal amplitude, equal wavelength waves traveling in opposite directions.

Standing wave:

$$y = y_1 + y_2 = A\sin(kx + \omega t) + A\sin(kx - \omega t)$$

2. [3] Suppose that in a double-slit experiment the distance from the slits to the screen is increased. Which of the following happens to the interference fringe pattern shown on the screen?

(Explain/derive your answer in the space below.)

(a) The minima move closer together.

(b) The maxima stay at the same position.

(c) The minima and maxima stay at the same position.

→(d) The maxima move farther apart.

(e) The minima stay at the same position.

- maxima: $\sin\theta = \frac{m\lambda}{d}$
- minima: $\sin\theta = (m+\frac{1}{2})\lambda/d$

} if $L\uparrow$, for maxima: let $\sin\theta_m \cong \frac{y_m}{L}$

$\therefore y_{m+1} - y_m \propto L$

$\Rightarrow$ as $L\uparrow$, $y_{m+1} - y_m$ increases

3. [4] An interference fringe pattern is formed with two slits separated by 0.1 mm and light of wavelength 580 nm. At what angle from the centre of the pattern does the light intensity first fall to half of its central maximum? (Ignore diffraction effects.)

$$I = I_0 \cos^2(\pi d \sin\theta/\lambda)$$

$$\frac{I_0}{2} = I_0 \cos^2(\pi d \sin\theta/\lambda)$$

$$\cos(\pi d \sin\theta/\lambda) = 0.71$$

$$\frac{\pi d \sin\theta}{\lambda} = \frac{\pi}{4}$$

$$\sin\theta = \frac{\lambda}{4d} = \frac{580\times10^{-9}}{4\times0.1\times10^{-3}} = 1.45\times10^{-3}$$

$$\Rightarrow \underline{\theta = 0.083^\circ}$$

4. [5] Light of wavelength 687 nm is incident on a single slit 0.75 mm wide. At what distance from the slit should a screen be placed if the second dark fringe in the diffraction pattern is to be 1.7 mm from the centre of the screen?

$$\sin\theta = \frac{m\lambda}{a} \quad \text{with } m = 2$$

$$\text{So, } \sin\theta = \frac{(2)(687\times10^{-9})}{0.75\times10^{-3}} = 1.8\times10^{-3}$$

$$\therefore \theta = 0.105^\circ\left(\frac{\pi}{180^\circ}\right) = 1.8\times10^{-3} \text{ radians}$$

$$\therefore \tan\theta = \frac{y_2}{L} \Rightarrow L = \frac{y_2}{\tan\theta} = \frac{1.7\times10^{-3}}{\tan(1.8\times10^{-3})}$$

$$\Rightarrow \underline{L = 0.94\text{m}}$$

5. [5] At $t = 0$, a traveling wave is described by the wave function $y(x) = a \sin(bx)$, where $a = 0.085$ m and $b = 3.25$ m^{-1}.

(a) What is its wavelength?

(b) If the wave has a speed of 18.0 m/s, calculate its frequency f.

(c) What is the angular frequency of the wave's oscillation?

(a) b = wave number "k" = $\frac{2\pi}{\lambda}$

$$\therefore \quad \lambda = \frac{2\pi}{b} = \frac{2\pi}{3.25} = 1.93 \text{ m}$$

(b) $v = \lambda f \Rightarrow f = \frac{v}{\lambda} = \frac{18}{1.93} = 9.3 \text{ s}^{-1} \text{ (or Hz)}$

(c) $\omega = 2\pi f = 58.4 \text{ s}^{-1} \text{ (or rad/s)}$

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[20] total marks