Arts & Science 2D06

Quiz #6 2014 Mar 6 Name: Solutions

NB: Mark values are given in brackets [ ] beside each problem. Write all your answers on the quiz paper. No books or notes allowed. Time to write quiz: 50 minutes.

Wave function: $y = A\sin(kx - \omega t + \varphi)$ where $k = 2\pi/\lambda$

Two-slit interference: $\sin\theta = m\lambda/d$ and $\sin\theta = (m+1/2)\lambda/d$

Diffraction: $\sin\theta = m\lambda/a$

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1. [3] Two wave pulses pass each other on a string. One is traveling to the right with a positive amplitude, while the other is traveling to the left with a negative amplitude. The magnitudes of their amplitudes are the same. What happens when the pulses occupy the same region of space at the same time?

(a) Constructive interference occurs.
→(b) Destructive interference occurs. (no explanation needed)
(c) A standing wave is produced.
(d) A traveling wave is produced.
(e) None of the above.

2. [3] In a double-slit experiment, if the distance between the slits and the screen is increased, which of the following happen(s) to the interference pattern appearing on the screen?

Explain/derive your choice(s) in the space below.

(a) The minima get closer together.
(b) The maxima stay at the same position.
(c) The minima and maxima stay at the same position.
→(d) The maxima get further apart.
(e) None of the above.

- max: $\sin\theta = m\lambda/d$
- min: $\sin\theta = (m+1/2)\lambda/d$

} RHS's unchanged

$\therefore$ θ_{min}, θ_{max} are unchanged

$\Rightarrow$ since $\sin\theta \sim \tan\theta = \frac{y}{L}$ (can be assumed)

if $L\uparrow$, then y also $\uparrow$. $\therefore$ maxima get further apart.

3. [4] A single-slit diffraction pattern is formed on a distant screen. Assuming that the angles involved are small, by what factor will the width of the central bright spot on the screen change, if the slit width is doubled?

diffraction minima : $\sin\theta = \frac{m\lambda}{a}$, $m = 1, 2, 3 \ldots$

Small-angle approximation : $\sin\theta \sim \tan\theta = \frac{y_m}{L}$

$\therefore \frac{y_1}{L} = \frac{\lambda}{a}$

and $y_1 = \frac{\lambda L}{a}$

$\Rightarrow$ Width of central bright spot $= 2 \cdot y_1 = \frac{2\lambda L}{a}$

Now, if "a" $\rightarrow$ "$2a$" :

new width $= \frac{2\lambda L}{2a} = \frac{1}{2}$ (old width)

$\therefore$ width decreases by a factor of $\underline{2}$.

4. [2+2+2] The vertical displacement of a string is given by

$$y(x,t) = (6.00\ mm)\ \sin\ [(3.25\ m^{-1})x - (7.22\ s^{-1})t].$$

(a) What is the amplitude of this wave?

from $y = A\sin(kx - \omega t + \phi) \rightarrow A = 6\ mm$

(b) Determine the wave's period.

from $y(x,t)$: $\omega = 7.22\ s^{-1}$

$$\therefore \quad T = \frac{2\pi}{\omega} = 0.87\ s$$

(c) Lastly, find the wave's speed.

from $y(x,t)$: $k = 3.25\ m^{-1} = \frac{2\pi}{\lambda}$

$$\therefore \quad \lambda = 1.93\ m$$

$$\therefore \quad V = \frac{\lambda}{T} = \frac{1.93\ m}{0.87\ s} = 2.22\ m/s$$

5. [4] In a Young's double-slit experiment, the slit separation is made to be exactly 4 λ, where λ is the wavelength of the light used in the experiment. The interference pattern is observed on a screen: At what angle will the pattern's third-order dark fringe occur?

3rd order dark fringe :

$$\sin\theta = (m + 1/2)\lambda/d \quad , \; m = 3$$

$$= \frac{3.5\lambda}{d}$$

since $d = 4\lambda$: $\sin\theta = \frac{3.5\lambda}{4\lambda} = 0.875$

$\therefore \quad \theta = 61°$

≈ 1.065 rad

[20] total marks