Arts & Science 2D06

Quiz #5 2020 Jan 29 **Name:** Solutions

NB: Mark values are given in brackets [ ] beside each problem. Write all your answers on the quiz paper. No books or notes allowed. Time to write quiz: 50 minutes.

Surface area of sphere: $A = 4\pi r^2$ Volume of sphere: $V = \frac{4}{3}\pi r^3$

Hydrostatic law of pressure with depth: $\Delta P = \rho g \Delta y$

Archimedes' principle of buoyancy: $F_B = \rho_f V g$

Bernoulli's equation: $P + \rho g y + \frac{1}{2}\rho v^2 = const$

Period of simple pendulum: $T = 2\pi\sqrt{\frac{L}{g}}$

SHM equation of motion: $x = A\cos(\omega t + \varphi)$ where $\omega = \sqrt{k/m} = 2\pi/T$

Air pressure at sea level $P_0 = 1.013 \times 10^5\ \mathrm{N/m^2}$

Density of air at sea level $\rho_{air} = 1.29\ \mathrm{kg/m^3}$

Density of water $\rho_{water} = 1000\ \mathrm{kg/m^3}$ Density of aluminum $\rho_{Al} = 2700\ \mathrm{kg/m^3}$

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1. [3] A piece of iron rests on top of a piece of wood that is floating in a bathtub. If the iron is removed from the wood, what happens to the water level in the tub?

(You do not need to explain your answer, unless you would like to do so.)

(a) It goes up.
→(b) It goes down.
(c) It does not change.
(d) It cannot be determined from the information given.
(e) It oscillates up and down.

remove iron → Wood by itself displaces less water

∴ Water level drops.

2. [3] A mass on a spring undergoes Simple Harmonic Motion (SHM). When the mass is at maximum displacement from equilibrium, its instantaneous acceleration

(Explain/derive your answer in the space below.)

→ (a) is a maximum.

(b) is less than maximum, but not zero.

(c) is zero.

(d) cannot be determined without knowing the mass.

(e) cannot be determined without knowing the spring constant.

SHM : $x = A\cos(\omega t + \phi)$

→ $v = -A\omega \sin(\omega t + \phi)$

$a = -A\omega^2 \cos(\omega t + \phi)$

when $x = A$, $\cos(\omega t + \phi) = 1$ → acceleration is max.

$(|a_{max}| = A\omega^2)$

3. [4] A pendulum of length 0.5 meters is released from an angle of 5°. How long does it take the pendulum to reach its highest speed? What if it is released from 3° instead?

$$\theta(t) = \theta_0 \cos(\omega t + \phi)$$

@ $t=0,\ \theta = \theta_0 \Rightarrow \cos\phi = 1 \Rightarrow \phi = 0°$

$$\omega = \frac{2\pi}{T} = 2\pi\left(\frac{1}{2\pi}\sqrt{\frac{g}{L}}\right) = \sqrt{\frac{g}{L}} \qquad (L = 0.5\ m)$$

$$\therefore \quad \theta = \theta_0 \cos\left(\sqrt{\frac{g}{L}}\,t\right) = \theta_0 \cos(4.43\,t)$$

also, highest speed occurs at lowest point, i.e., $\theta = 0°$.

$\therefore$ for $\theta_0 = 5°$:

$$0° = 5° \cos(4.43t)$$

$$\cos(4.43t) = 0$$

$$4.43t = \frac{\pi}{2} \quad (\text{radians, since } \omega\ [\text{rad/s}])$$

$$t = 0.35 \text{ sec.}$$

for $\theta = 3°$:

$$0° = 3° \cos(4.43t)$$

$$\cos(4.43t) = 0 \Rightarrow t = 0.35 \text{ sec.,}$$

still.

4. [5] A 3.8-kg chunk of aluminum is measured to have an apparent mass of 2.1 kg when completely immersed in an unknown liquid. What is the density of this liquid?

In the liquid:

Apparent weight $W = F_g - F_B$

$$(2.1\,kg)(9.81) = (3.8\,kg)(9.81) - \rho_f V (9.81)$$

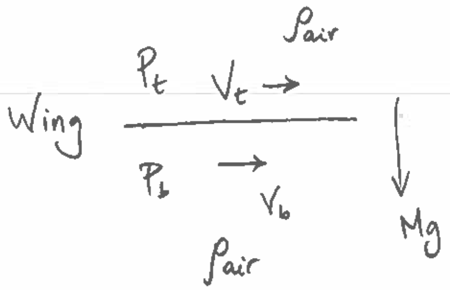
$$\rho_f V = 1.7$$

$$\rho_f = \frac{1.7}{V} = \frac{1.7}{\frac{M}{\rho_{Al}}} = \frac{1.7}{\left(\frac{3.8}{2700}\right)}$$

$$\Rightarrow \rho_f = 1208\ kg/m^3$$

5. [5] Consider an airplane that has a mass of 2.0×10^6 kg. Suppose that while in flight, the air flows past the bottom surface of the plane's wings at 105 m/s. The wings have a combined area of 1000 m^2. Calculate how fast the air should flow over the wings' top surfaces if the plane is to maintain its flight altitude.

(Ignore the thickness of the wings by imagining them as two-dimensional flat surfaces. Also, you may assume $\rho_{air} = 1.29$ kg/m^3.)



Maintain altitude : $P_t \cdot A + Mg = P_b \cdot A$

$$\Rightarrow P_b - P_t = \frac{Mg}{A}$$

$$P_t + \frac{1}{2}\rho_{air} V_t^2 = P_b + \frac{1}{2}\rho_{air} V_b^2 \quad (\Delta y \approx 0)$$

$$\frac{1}{2}\rho_{air} V_t^2 = (P_b - P_t) + \frac{1}{2}\rho_{air} V_b^2$$

$$V_t^2 = \frac{2}{\rho_{air}}\left(\frac{Mg}{A}\right) + V_b^2$$

$$V_t = \sqrt{\frac{2}{1.29}\left(\frac{2 \times 10^6 \times 9.81}{1000}\right) + (105)^2}$$

$$\Rightarrow \quad V_t = 204 \text{ m/s}$$

20

[20] total marks