Arts & Science 2D06

Quiz #5 2019 Jan 30 **Name:** Solutions

NB: Mark values are given in brackets [ ] beside each problem. Write all your answers on the quiz paper. No books or notes allowed. Time to write quiz: 50 minutes.

Surface area of sphere: $A = 4\pi r^2$ Volume of sphere: $V = \frac{4}{3}\pi r^3$

Hydrostatic law of pressure with depth: $\Delta P = \rho g \Delta y$

Archimedes' principle of buoyancy: $F_B = \rho_f V g$

Bernoulli's equation: $P + \rho g y + \frac{1}{2}\rho v^2 = const$

Period of simple pendulum: $T = 2\pi\sqrt{\frac{L}{g}}$

SHM equation of motion: $x = A\cos(\omega t + \varphi)$ where $\omega = \sqrt{k/m} = 2\pi/T$

Air pressure at sea level $P_0 = 1.013 \times 10^5$ N/m^2

Density of air at sea level $\rho_{air} = 1.29$ kg/m^3

Density of water $\rho_{water} = 1000$ kg/m^3

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1. [3] A mass is attached to a vertical spring and moves up and down between points A and B. Where is the mass located when its kinetic energy is a maximum?

(You do not need to explain your answer, unless you would like to do so.)

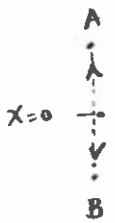
(a) at either A or B

→ (b) midway between A and B

(c) one-third of the way between A and B

(d) one-fourth of the way between A and B

(e) none of the above



SHM: $V = \frac{dx}{dt} = -\omega A \sin(\omega t + \phi)$

$\Rightarrow |V_{max}| = \omega A$ when $x = 0$ ($\cos(\omega t + \phi) = 0$, $\sin(\omega t + \phi) = 1$)

$K \propto V^2 \Rightarrow K_{max}$ @ $x = 0$

(midway b/w A and B)

2. [3] A fluid within a pipe is stationary. When a valve on the pipe is opened, fluid flows through the pipe. What happens to the pressure anywhere in the pipe when the valve is opened?

(Explain/derive your answer in the space below.)

(a) Whether the pressure increases or decreases will depend on the direction of the fluid flow.

(b) Whether the pressure increases or decreases will depend on the density of the fluid.

→(c) The pressure decreases.

(d) The pressure increases.

(e) The pressure stays the same.

Bernoulli's equation: if $v \uparrow$, then $P \downarrow$ (and vice-versa) for any point in the fluid.

- At first, $V_0 = 0$; $P = P_0$, say
- After valve is opened: $V_1 > 0 \Rightarrow P_1 < P_0$

(everywhere, even if ρ changes from place to place)

3. [4] Air is flowing through a rocket nozzle. Inside the rocket the air has a density of 5.25 kg/m^3 and a speed of 1.20 m/s. The interior diameter of the rocket is 15.0 cm. At the nozzle exit, the diameter is 2.50 cm and the density is 1.29 kg/m^3. What is the speed of the air when it leaves the nozzle?

Use Equation of continuity : $\rho_1 A_1 V_1 = \rho_2 A_2 V_2$

for points 1 and 2 in a flowing fluid.

Let $\rho_1 = 5.25\ kg/m^3$ $\quad$ $\rho_2 = 1.29\ kg/m^3$

$V_1 = 1.2\ m/s$ $\quad$ $V_2 = ?$

$A_1 = \pi\left(\frac{0.15}{2}\right)^2 = 0.018\ m^2$ $\quad$ $A_2 = \pi\left(\frac{0.025}{2}\right)^2 = 4.9 \times 10^{-4}\ m^2$

Solve for V_2 : $\quad V_2 = \dfrac{\rho_1 A_1 V_1}{\rho_2 A_2}$

$$= \frac{(5.25)(0.018)(1.2)}{(1.29)(4.9\times10^{-4})}$$

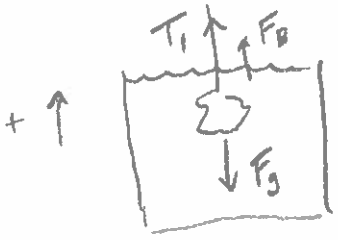
$$= 179.4\ m/s.$$

4. [5] A rock (in air) is suspended from a scale that reads 20.0 N. A jar of water is raised up so that the rock is totally submerged in the water. The scale now reads 12.5 N. What is the density of the rock?

- In air:

$$T = F_g = 20\,N \Rightarrow m = \frac{20}{g} = 2.04\,kg$$

- In water:



$$T_1 + F_B - F_g = 0$$

$$F_B = F_g - T_1$$

$$= 20 - 12.5$$

$$= 7.5N$$

$$F_B = \rho_f V g = 7.5$$

$$(1000)\left(\frac{m}{\rho}\right)(9.8) = 7.5$$

$$\rho = \frac{(1000)(2.04)(9.8)}{7.5} = 2667\ kg/m^3$$

5. [5] A pendulum 3.0 m long is oscillating through a maximum angle of 4.8°. What is its speed when the angle is 2.4°?

$\theta = \theta_0 \cos(\omega t + \phi)$, $\omega = \sqrt{\frac{g}{L}} = \sqrt{\frac{9.8}{3}} = 1.8\ s^{-1}$

$L = 3m$, $\theta_0 = 4.8°$

let $\phi = 0$ (pendulum released from $\theta = \theta_0$ at $t=0$)

- Speed : $V = \left|\frac{ds}{dt}\right| = \left|\frac{d}{dt}(L\theta)\right| = L\left|\frac{d\theta}{dt}\right| = L\omega\theta_0 \sin(\omega t)$ (radians)

- when $\theta = 2.4°$: $2.4 = 4.8\cos(1.8t)$

degrees ok, as long as both sides have same units

$\cos(1.8t) = 0.5$

$1.8t = \frac{\pi}{3}$ radians

$t = 0.58$ second

$V = (3)(1.8)(4.8° \cdot \frac{\pi\ rad}{180°}) \sin(\underbrace{1.8 \cdot 0.58}_{\text{radians}})$

$= 0.39\ m/s$

20

[20] total marks