**Arts & Science 2D06**

**Quiz #5** **2018 Jan 24** **Name:** Solutions

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NB: Mark values are given in brackets [ ] beside each problem. Write all your answers on the quiz paper. No books or notes allowed. Time to write quiz: 50 minutes.

Surface area of sphere: $A = 4\pi r^2$ Volume of sphere: $V = \frac{4}{3}\pi r^3$

Hydrostatic law of pressure with depth: $\Delta P = \rho g \Delta y$

Archimedes' principle of buoyancy: $F_B = \rho_f V g$

Bernoulli's equation: $P + \rho g y + \frac{1}{2}\rho v^2 = const$

Air pressure at sea level $P_0 = 1.013 \times 10^5$ N/m$^2$

Density of air at sea level $\rho_{air} = 1.29$ kg/m$^3$

Density of water $\rho_{water} = 1000$ kg/m$^3$

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1. [3] At a given depth within a fluid at rest, the pressure pushing upward is:

(You do not need to explain your answer, unless you would like to do so.)

(a) less than the pressure pushing downward.

(b) zero, because the pressure only pushes in the horizontal direction.

(c) zero, because the fluid above does not support the weight of the fluid below.

(d) greater than the pressure pushing downward.

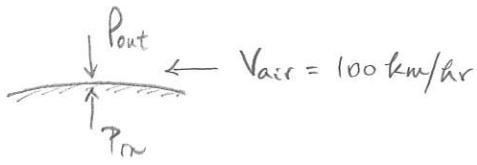
(e) equal to the pressure pushing downward.

At a point in a fluid, the pressure from all directions is the same.

2. [3] Suppose that you are driving a car with a (soft) convertible roof, at the speed limit of 100 km/h. The car's roof is up and the windows are closed. You observe that the roof:

(Explain/derive your answer in the space below.)

(a) bows inward.
(b) is not changed from when the car was at rest.
(c) bows outward.
(d) bows inward only when you are driving uphill.
(e) bows inward only when you are driving downhill.




- Bernoulli's equation: $P_{in} + \frac{1}{2}m\cancel{v_{in}^2}^{0} = P_{out} + \frac{1}{2}m v_{out}^2$

$\Rightarrow P_{in} > P_{out}$

$\therefore$ roof bows outward.

3. [4] An incompressible fluid flows steadily through a pipe whose diameter changes from place to place. The fluid speed at a location where the pipe diameter is 7.0 cm is 1.2 m/s. What is the fluid speed at a location where the diameter has narrowed to 4.0 cm?

Equation of continuity: $A_1 V_1 = A_2 V_2$

$\pi r_1^2 V_1 = \pi r_2^2 V_2$

$V_2 = \left(\frac{r_1}{r_2}\right)^2 V_1 = \left(\frac{7}{4}\right)^2 (1.2)$

$\therefore V_2 = 3.7$ m/s

4. [5] On planet $X$, the pressure at a depth of 2.00 m below the surface of a liquid-nitrogen lake is $5.00 \times 10^5$ N/m$^2$. At a depth of 5.00 m below the surface, the pressure is $8.00 \times 10^5$ N/m$^2$. (The density of liquid nitrogen is 808 kg/m$^3$.)

(a) What is the atmospheric pressure on the surface of the lake?

(b) What is the acceleration of gravity on planet $X$?

$$P = P_0 + \rho g h$$

(1) $5\times10^5 = P_0 + (808)g(2) = P_0 + 1616g$

$\therefore P_0 = (5\times10^5) - 1616g$ ——— (1)

(2) $8\times10^5 = P_0 + (808)g(5) = P_0 + 4040g$ ——— (2)

. Sub (1) in (2): $8\times10^5 = (5\times10^5) - 1616g + 4040g$

$2424g = 3\times10^5$

$g = 123.8$ m/s$^2$ (b)

. Sub this back in (1): $P_0 = (5\times10^5) - (1616)(123.8)$

$P_0 = 3\times10^5$ Pa (a)

5. [5] Consider a cargo boat whose bottom side is a rectangle of dimensions 10.0 m × 60.0 m, and whose sides are vertical. The boat is floating on fresh water, and a load of 620 tonnes (1 ton = 1000 kg) is placed on it. By how much has the boat sunken into the water after the load is placed?

• Before : $F_B = Mg$

$\rho_f V g = Mg$

$(1000)(y_1 \cdot 10 \cdot 60) = M$

$(6 \times 10^5) y_1 = M$ ——— (1)

• After : $F_B' = (M + 620 \times 10^3)\, g$

$\rho_f V' g = (M + 6.2 \times 10^5) g$

$1000\,(y_2 \cdot 600) = M + 6.2 \times 10^5$

$(6 \times 10^5) y_2 = M + 6.2 \times 10^5$ ——— (2)

• Eliminate "M" from (1) and (2) :

$(6 \times 10^5) y_2 - (6 \times 10^5) y_1 = (M + 6.2 \times 10^5) - M$

$(6 \times 10^5)(y_2 - y_1) = 6.2 \times 10^5$

$\therefore \underline{y_2 - y_1 = 1.03 \text{ m}}$

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[20] total marks