Arts & Science 2D06

Quiz #5 2017 Jan 25 **Name:** *Solutions*

NB: Mark values are given in brackets [ ] beside each problem. Write all your answers on the quiz paper. No books or notes allowed. Time to write quiz: 50 minutes.

Surface area of sphere: $A = 4\pi r^2$ Volume of sphere: $V = \frac{4}{3}\pi r^3$

Hydrostatic law of pressure with depth: $\Delta P = \rho g \Delta y$

Archimedes' principle of buoyancy: $F_B = \rho_f V g$

Bernoulli's equation: $P + \rho g y + \frac{1}{2}\rho v^2 = const$

Period of simple pendulum: $T = 2\pi\sqrt{\frac{L}{g}}$ Wave speed: $v = f\lambda$

SHM equation of motion: $x = A\cos(\omega t + \varphi)$ where $\omega = \sqrt{k/m} = 2\pi/T$

Air pressure at sea level $P_0 = 1.013 \times 10^5\ \mathrm{N/m^2}$

Density of air at sea level $\rho_{air} = 1.29\ \mathrm{kg/m^3}$

Density of water $\rho_{water} = 1000\ \mathrm{kg/m^3}$

---

1. [3] As a rock sinks deeper and deeper into water of constant density, what happens to the buoyant force on the rock? (You do not need to explain your answer unless you would like to.)

(a) It increases.

→ (b) It remains constant.

(c) It decreases.

(d) It may increase or decrease, depending on the shape of the rock.

(e) It may increase or decrease, depending on the weight of the rock.

$F_B = \rho_f V g$

↳ all constant as the rock sinks (starting submerged)

2. [3] A mass M is attached to a spring with spring constant k. When this system is set in motion with amplitude A, it moves with a period T. What is the period if the amplitude of the motion is increased to $2A$?

(Explain/derive your answer in the space below.)

(a) $2T$

(b) $T/2$

→(c) T

(d) $4T$

→(e) T

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} \rightarrow \text{independent of amplitude}$$

3. [4] The deepest point of the Pacific Ocean is at 11,033 m, in the Mariana Trench. What is the water pressure at that point? (The density of seawater is 1025 kg/m^3.)

$$P_{H_2O} = \rho g h$$

$$= (1025\ kg/m^3)(9.81\ m/s^2)(11{,}033\ m)$$

$$= \underline{1.109 \times 10^8\ Pa}$$

4. [5] A horizontal pipe contains an incompressible fluid with a density of 1200 kg/m^3 flowing through it. At one position within the pipe, the pressure is measured to be 300 kPa and the speed of the flow is 20.0 m/s. At another position, the pressure is 200 kPa. What is the speed of the flow at this second position?

$$P_1 + \cancel{\rho g y_1}^{0} + \frac{1}{2}\rho V_1^2 = P_2 + \cancel{\rho g y_2}^{0} + \frac{1}{2}\rho V_2^2$$

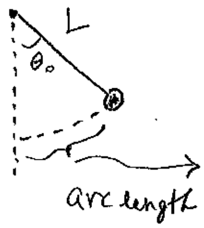
$$V_2^2 = \frac{2}{\rho}\left(P_1 - P_2 + \frac{1}{2}\rho V_1^2\right)$$

$$V_2 = \sqrt{\frac{2(P_1 - P_2)}{\rho} + V_1^2}$$

$$= \sqrt{\frac{2(300-200)\times 10^3}{1200} + (20)^2}$$

$$= 23.8 \text{ m/s}$$

5. [5] A pendulum that was originally built by Foucault (French scientist) at the Pantheon in Paris for the Paris Exhibition in 1851, was restored in 1995. It has a 28.0 kg sphere suspended from a 67.0-m cable. If the amplitude of the swing is 5.00 m, what is the maximum speed of the sphere? (Ignore the mass of the cable.)



$S = 5m = L \cdot \theta_0 \Rightarrow \theta_0 = 0.075$ radians

$\theta = \theta_0 \cos(\omega t + \phi)$, let $\phi = 0$

$$\frac{d\theta}{dt} = -\theta_0 \omega \sin \omega t$$

$$\left(\frac{d\theta}{dt}\right)_{max} = \theta_0 \omega = (0.075)\sqrt{\frac{9.81}{67}} = 0.029 \text{ rad/s}$$

$$\Rightarrow V_{max} = \left(\frac{ds}{dt}\right)_{max} = L\left(\frac{d\theta}{dt}\right)_{max} = (67)(0.029) = 1.92 \text{ m/s}$$

[20] total marks 20