Arts & Science 2D06

Quiz #5 2015 Jan 29 Name: *Solutions*

NB: Mark values are given in brackets [ ] beside each problem. Write all your answers on the quiz paper. No books or notes allowed. Time to write quiz: 50 minutes.

Surface area of sphere: $A = 4\pi r^2$ Volume of sphere: $V = \frac{4}{3}\pi r^3$

Hydrostatic law of pressure with depth: $\Delta P = \rho g \Delta y$

Archimedes' principle of buoyancy: $F_B = \rho_f V g$

Bernoulli's equation: $P + \rho g y + \frac{1}{2}\rho v^2 = const$

Period of simple pendulum: $T = 2\pi\sqrt{\frac{L}{g}}$ Wave speed: $v = f\lambda$

SHM equation of motion: $x = A\cos(\omega t + \varphi)$ where $\omega = \sqrt{k/m} = 2\pi/T$

Air pressure at sea level $P_0 = 1.013 \times 10^5\ \mathrm{N/m^2}$

Density of air at sea level $\rho_{air} = 1.29\ \mathrm{kg/m^3}$

Density of water $\rho_{H2O} = 1000\ \mathrm{kg/m^3}$

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1. [3] When you blow air along one side of a paper strip, the paper moves toward that side. This is because

(a) the air on that side moves faster, and thus the pressure there is higher.

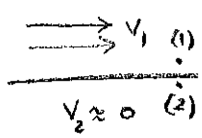
→ (b) the air on that side moves faster, and thus the pressure there is lower.

(c) the air on that side moves faster, but the pressure doesn't change.

(d) the air on that side moves slower, and thus the pressure there is higher.

(e) the air on that side moves slower, and thus the pressure there is lower.

Explain/derive your choice in the space below.



Apply Bernouilli's equation to points (1) and (2)

$$P_1 + \cancel{\rho g y_1}^{\,0} + \frac{1}{2}\rho v_1^2 = P_2 + \cancel{\rho g y_2}^{\,0} + \cancel{\frac{1}{2}\rho v_2^2}^{\,0}$$

$$\therefore \quad P_1 = P_2 - \frac{1}{2}\rho v_1^2$$

$$\therefore \quad \underline{P_1 < P_2}$$

2. [4] A sphere with a radius of 2.5 cm is (completely) immersed in a fluid. The pressure at the top point of the sphere is measured to be 594 Pa, while the pressure at the bottom point is 1133 Pa. What is the density of the fluid?

Hydrostatic law: $\Delta P = \rho_f g \Delta y$

$$1133 - 594 = \rho_f (9.81)(0.05)$$

$$\therefore \underline{\rho_f = 1100 \text{ kg/m}^3}$$

3. [3] A student designs a clock using a pendulum (i.e., a mass on a string). Each full oscillation of the pendulum is supposed to advance the clock by one second. However, after the student builds the clock, a full oscillation is found to take two seconds instead. What change can be made to fix this clock?

(a) Double the amplitude of the oscillations.

(b) Quadruple the length of the string.

→(c) Reduce the string's length by a factor of four.

(d) Quadruple the mass.

(e) None of the above.

Explain/derive your choice in the space below.

Pendulum period: $T = 2\pi\sqrt{\frac{L}{g}}$

$$\therefore L \propto T^2$$

So if T is to decrease from 2 s to 1 s, L should be shortened by a factor of $2^2 = \underline{4}$.

4. [5] A 0.50-kg mass, attached to a spring, moves along a horizontal frictionless surface. The spring constant is measured to be 20 N/m. The mass oscillates in simple harmonic motion and has a speed of 1.5 m/s at the equilibrium position.

(a) What is the amplitude of the oscillation?

$$SHM: \quad x = A\cos(\omega t + \phi) \quad , \quad \omega = \sqrt{\tfrac{k}{m}}$$

$$V = \frac{dx}{dt} = -\omega A \sin(\omega t + \phi) \Rightarrow |V_{max}| = \omega A$$

V_{max} occurs at eq. position $\therefore$ $A = \frac{|V_{max}|}{\omega} = |V_{max}| \cdot \sqrt{\frac{m}{k}} = \underline{0.24\ m}$

(b) Where is the object's kinetic energy and the potential energy equal to each other?

$$\left.\begin{array}{l} U = \frac{1}{2}kx^2 \\ K = \frac{1}{2}mv^2 \end{array}\right\} \quad E = K + U$$

if $K = U$, then $E = 2U = kx^2$

Now, the total energy is also given by: $E = \frac{1}{2}kA^2$

$$\therefore \cancel{k}x^2 = \frac{1}{2}\cancel{k}A^2$$

$$x = \frac{A}{\sqrt{2}} = \frac{0.24}{\sqrt{2}} = \underline{0.17\ m}$$

($\underline{\text{Another way}}$: $\frac{1}{2}kx^2 = \frac{1}{2}mv^2$

$$\cancel{kA^2}\cos^2(\omega t + \phi) = \underbrace{m\omega^2}_{=k}\cancel{A^2}\sin^2(\omega t + \phi)$$

$$\therefore \cos(\omega t + \phi) = \sin(\omega t + \phi)$$

$$\Rightarrow \omega t + \phi = 45°$$

$$\therefore x = (0.24\,m)\cos 45° = \underline{0.17\,m}$$)

5. [5] A board that is 20.0 cm wide, 5.00 cm thick, and 3.00 m long has a density of 400 kg/m^3. The board is floating, partially submerged in water. What is the minimum mass that would need to be set on the board in order to submerge the board completely?

Let the minimum mass $= M$; $m_b =$ mass of board

Then, when board is completely submerged:

$$F_B = F_g$$

$$\rho_f V_f \cancel{g} = (m_b + M)\cancel{g}$$

$$\therefore \quad M = \rho_f V_f - m_b$$

$$= (1000\ kg/m^3)(0.2 \times 0.05 \times 3\ m^3) - (400\ kg/m^3)(0.2 \times 0.05 \times 3\ m^3)$$

$$= (600)(0.2 \times 0.05 \times 3)$$

$$= \underline{18\ kg}$$

20

[20] total marks