Arts & Science 2D06

Quiz #5 2014 Jan 30 **Name:** Solutions

NB: Mark values are given in brackets [ ] beside each problem. Write all your answers on the quiz paper. No books or notes allowed. Time to write quiz: 50 minutes.

Solution for quadratic equation: $x = (-b \pm \sqrt{b^2 - 4ac})/2a$

Surface area of sphere: $A = 4\pi r^2$ Volume of sphere: $V = \frac{4}{3}\pi r^3$

Bernoulli's equation: $P + \rho g y + \frac{1}{2}\rho v^2 = const$

Period of simple pendulum: $T = 2\pi\sqrt{\frac{L}{g}}$ Wave speed: $v = f\lambda$

SHM equation of motion: $x = A\cos(\omega t + \varphi)$ where $\omega = \sqrt{k/m} = 2\pi/T$

Air pressure at sea level $P_0 = 1.013 \times 10^5$ N/m^2

Density of air at sea level $\rho_{air} = 1.29$ kg/m^3

Density of water $\rho_{H2O} = 1000$ kg/m^3

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1. [3] In simple harmonic motion of a mass+spring system, the mass's acceleration is proportional to its:

(a) velocity.

(b) frequency.

(c) amplitude.

→(d) displacement.

(e) all of the above.

$\vec{F} = m\vec{a}$

$\Rightarrow ma = -kx$

$a = -\frac{k}{m}x$ ∴ (d)

Explain/derive your choice in the space below.

OR SHM : $x = A\cos(\omega t + \phi)$

$a = \frac{d^2x}{dt^2} = -\omega^2 A\cos(\omega t + \phi)$ $(= -\omega^2 x)$

∴ $a \propto A$ as well (c)

either (d) or (c) (or both) is fine.

2. [4] At position A within a tube containing a compressible fluid that is moving with steady laminar flow, the speed of the fluid is 12.0 m/s and the tube has a diameter 12.00 cm. At position B, the speed of the fluid is 18.0 m/s and the tube has a diameter 6.00 cm. What is the ratio of the density of the fluid at position A to the density of the fluid at position B?

Use equation of continuity:

$$\rho_A V_A A_A = \rho_B V_B A_B$$

$$\therefore \frac{\rho_A}{\rho_B} = \frac{V_B}{V_A}\frac{A_B}{A_A} = \frac{V_B}{V_A}\left(\frac{r_B}{r_A}\right)^2 = \left(\frac{18}{12}\right)\left(\frac{3}{6}\right)^2$$

$$= 0.38$$

3. [3] You are originally 1.0 m beneath the surface of a pool. If you dive from there to 2.0 m beneath the surface, what happens to the absolute pressure that you will feel?

(a) It increases by a factor of 4.

(b) It increases by a factor of 2.

→ (c) It increases, but by a factor smaller than 2.

(d) It decreases by a factor of 2.

(e) It does not change.

Explain/derive your choice in the space below.

$$P = P_0 + \rho g h$$

$h = 1\,m \longrightarrow h = 2m$

$$P_1 = P_0 + \rho g \longrightarrow P_2 = P_0 + 2\rho g$$

$\therefore P_2 > P_1$ but only by a bit.

4. [5] A piece of aluminum with a mass of 1.0 kg and density of 2700 kg/m^3 is suspended from a string and then completely immersed in a container of water.

(a) Determine the volume of the piece of aluminum.

$$V = \frac{m}{\rho_{H_2O}} = \frac{1\ kg}{2700\ kg/m^3} = 3.7 \times 10^{-4}\ m^3$$

(b) Determine the tension in the string after the metal is immersed in the container of water.

$\uparrow y^+$

$$\Sigma F_y = ma_y = 0$$

$$T - mg + F_B = 0$$

$$T = mg - F_B = mg - \rho_{H_2O} V g$$

$$= (1)(9.8) - (1000)(3.7 \times 10^{-4})(9.8)$$

$$= 6.2\ N$$

5. [5] A mass of 1.53 kg is attached to a spring and the system is undergoing simple harmonic oscillations with a frequency of 1.95 Hz and an amplitude of 7.50 cm. What is the total mechanical energy of the system?

SHM : $x = A\cos(\omega t + \phi)$

2 ways to solve : (a) $E_{total} = \frac{1}{2}kx_{max}^2 = \frac{1}{2}kA^2$

$A = 0.075\ m$

$\omega = 2\pi f = \sqrt{\frac{k}{m}}$

$\Rightarrow k = (2\pi f)^2 \cdot m$

$= [2\pi(1.95)]^2 \cdot 1.53$

$= 229.7\ N/m$

$\therefore E_{total} = \frac{1}{2}(229.7)(0.075)^2 = 0.65\ J$

(b) $E_{total} = \frac{1}{2}mv_{max}^2$

$v = \frac{dx}{dt} = -\omega A\sin(\omega t + \phi) \Rightarrow v_{max} = \omega A$ (magnitude)

$\therefore E_{total} = \frac{1}{2}m\omega^2A^2 = \frac{1}{2}(1.53)(2\pi(1.95))^2(0.075)^2$

$= 0.65\ J$ as before

[20] total marks