Arts & Science 2D06

Quiz #4 2019 Nov 27 **Name:** Solutions

NB: Mark values are given in brackets [ ] beside each problem. Write all your answers on the quiz paper. No books or notes allowed. Time to write quiz: 50 minutes.

Elastic collisions (1D), target m_2 stationary: $v_1 = \frac{(m_1-m_2)}{(m_1+m_2)}u_1$, $v_2 = \frac{2m_1}{(m_1+m_2)}u_1$

Gamma factor: $\gamma = (1 - v^2/c^2)^{-1/2}$ Momentum: $p = \gamma mv$

Lorentz transformation: $x' = \gamma(x - vt)$, $t' = \gamma(t - \frac{v}{c^2}x)$.

Reverse Lorentz transformation: $x = \gamma(x' + vt')$, $t = \gamma(t' + \frac{v}{c^2}x')$

Velocity addition: $u' = \frac{(u-v)}{(1-uv/c^2)}$

Rest-mass energy: $E = mc^2$ Kinetic energy: $K = (\gamma - 1)mc^2$

Speed of light: 3×10^8 m/s = 300,000 km/s = 1 light-year per year

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1. [3] Which one of the following statements best characterizes an *inelastic* collision between two objects?

(Choose one option; no explanation required.)

a) The total momentum of the system is conserved.

b) The total energy of the system remains constant.

c) The total kinetic energy of the system remains constant.

→ d) Only statements (a) and (b) are true.

e) Statements (a), (b), and (c) are all true.

Inelastic collision : $\vec{P}$ conserved

K not conserved, but E conserved.

(cf. Giancoli, p. 225)

∴ (a) definitely correct

(c) definitely wrong

(b) Somewhat ambiguous, depending on "system" ⇒ grading was lenient!

2. [3] An ArtsSci 2D06 student runs on horizontal ground with a velocity that approaches that of light. What happens to the student's height and width, as measured by an observer at rest with respect to the ground?

(Explain/derive your answer in the space below.)

a) The height is unchanged; the width approaches infinity.

b) The height approaches zero; the width is unchanged.

→c) The height is unchanged; the width approaches zero.

d) The height decreases slightly; the width increases slightly.

e) Neither the height nor the width changes.

Length contraction happens in the direction of motion

∴ height is unchanged.

$$L = \frac{L_0}{\gamma} = \sqrt{1 - v^2/c^2}\, L_0 \longrightarrow 0 \quad \text{as} \quad v \to c$$

∴ width approaches zero.

3. [4] A 0.3-kg tennis ball collides head-on and elastically with another ball that is initially at rest. This second tennis ball takes off with a speed that is half the original incoming speed of the first ball. What is the mass of the second ball?

From front page: $V_2 = \frac{2m_1}{m_1+m_2} u_1$

given: $V_2 = \frac{u_1}{2}$

$\therefore \quad \frac{u_1}{2} = \frac{2m_1}{m_1+m_2} u_1$

$\frac{1}{2} = \frac{2(0.3)}{0.3+m_2}$

$0.3 + m_2 = 1.2$

$m_2 = 0.9 \text{ kg}$

4. [5] A recent, and somewhat ambiguous, news headline in the Hamilton Gazette stated that, "Canadian spaceship returns from a 6-year interstellar voyage, while travelling at 0.70 c."

(a) If the reporter meant 6 years of Earth time, how much time passed on the ship during the trip?

(b) If, on the other hand, the reporter meant 6 years of spaceship time, how much time elapsed on Earth during the journey?

- Time dilation : $\Delta t = \gamma \underbrace{\Delta t_0}$ ↳ proper time (ship)

$\gamma > 1 \implies$ less time goes by on ship than on Earth.

- $V = 0.70\,c \rightarrow \gamma = \dfrac{1}{\sqrt{1 - \frac{0.7^2 c^2}{c^2}}} = 1.4$

(a) $\Delta t = \gamma \Delta t_0 \rightarrow 6 = 1.4 \Delta t_0$

$\Delta t_0 = 4.3$ years.

(b) $\Delta t = \gamma \Delta t_0 \rightarrow \Delta t = (1.4)(6) =$ 8.4 years.

5. [5] Suppose that in the earth's reference frame, a tree is located at the origin and a stop-sign is at $x = 3.9$ km. Observers in this frame see lightning bolts strike both the tree and the stop-sign at $t = 8$ microseconds (1 microsecond $= 10^{-6}$ second). These same lightning strikes are also seen by a rocket ship flying in the positive x-direction with a speed of 0.6 c.

(a) What are the space and time coordinates for these two events in the rocket ship's reference frame?

(b) Do these events happen simultaneously in the rocket ship's frame? If not, which event happens first?

event 1 (tree): $x_1 = 0$, $t_1 = 8 \times 10^{-6}\,s$

event 2 (sign): $x_2 = 3.9$ km, $t_2 = 8 \times 10^{-6}\,s$

$$\gamma = \frac{1}{\sqrt{1 - 0.6^2}} = 1.25$$

(a) $x_1' = \gamma(x_1 - vt_1) = 1.25(0 - 0.6 \cdot 300000\ km/s \cdot 8 \cdot 10^{-6}\,s) = -1.8$ km

$$t_1' = \gamma\left(t_1 - \frac{v}{c^2}x_1\right) = 1.25\left[8 \times 10^{-6} - \frac{0.6}{300{,}000} \cdot 0\right] = 10^{-5}\,s$$

$x_2' = \gamma(x_2 - vt_2) = 1.25(3.9 - 0.6 \cdot 300{,}000 \cdot 8 \cdot 10^{-6}) = 3.1$ km

$$t_2' = \gamma\left(t_2 - \frac{v}{c^2}x_2\right) = 1.25\left(8 \times 10^{-6} - \frac{0.6}{300\,000} \cdot 3.9\right) = 2.5 \times 10^{-7}\,s$$

(b) Since $t_2' < t_1'$, event 2 (sign) happens first in ship's frame.

20/20 [20] total marks