Arts & Science 2D06

Quiz #4 **2018 Nov 28** **Name:** Solutions

NB: Mark values are given in brackets [ ] beside each problem. Write all your answers on the quiz paper. No books or notes allowed. Time to write quiz: 50 minutes.

Gamma factor: $\gamma = (1 - v^2/c^2)^{-1/2}$ Momentum: $p = \gamma m v$

Lorentz transformation: $x' = \gamma(x - vt)$, $t' = \gamma(t - \frac{v}{c^2}x)$.

Reverse Lorentz transformation: $x = \gamma(x' + vt')$, $t = \gamma(t' + \frac{v}{c^2}x')$

Velocity addition: $u' = \frac{(u-v)}{(1-uv/c^2)}$

Rest-mass energy: $E = mc^2$ Kinetic energy: $K = (\gamma - 1)mc^2$

Speed of light: 3×10^8 m/s (= 1 light-year per year)

(rest) Mass of proton: 1.67×10^{-27} kg

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1. [3] Suppose you are travelling to Mars on a spaceship. As the velocity of your spaceship increases, you would observe

(No explanation required.)

a) that the clock on your smartphone runs slower than normal.

b) that the length of your spaceship has decreased.

c) that your mass has increased.

d) all of the above.

→e) none of the above.

In your rest frame, all of these quantities stay the same.

2. [3] As a particle starts from rest and increases its speed, you, in the lab, would find that its ~~mass~~ Momentum is increasing. In the limit that the particle's speed approaches the speed of light, you would find that the momentum of the particle

(Explain/derive your answer in the space below.)

a) suddenly decreases.

b) oscillates up and down.

c) stays the same.

d) approaches zero.

→ e) approaches infinity.

$$p = \gamma m v = \frac{mv}{\sqrt{1 - v^2/c^2}}$$

as $v \to c$, $p \to \frac{mc}{\sqrt{1 - c^2/c^2}} = \frac{mc}{0} \to \infty$

3. [4] A high-speed spaceship is traveling with a speed of 0.80c. Relative to a stationary observer, how fast would light travel from the front headlights of the ship if the ship is moving away from him? What about the light from the rear lights of the ship?

- One solution : Einstein's 2nd postulate :
  - Front headlights : speed "c"
  - Rear headlights : speed "c"

- Another solution : $u = \dfrac{u' + v}{1 + u'v/c^2}$
  - Front headlights : $u' = c$

$$\therefore \quad u = \frac{c + 0.8c}{1 + \dfrac{0.8c^{\cancel{2}}}{\cancel{c^2}}} = c$$

  - Rear headlights : $u' = -c$

$$u = \frac{-c + 0.8c}{1 + \dfrac{(-c)(0.8c)}{c^2}} = \frac{(-c)(1 - 0.8c)}{1 - 0.8c} = -c$$

4. [5] A train approaches a train station with a speed of 0.50c. A high-performance athlete on the train measures her heart rate to be 45 beats per minute. What is her heart rate according to her coach who is waiting at the station?

$\Delta t_a = 60\ s$

$\Delta t_c = \gamma \Delta t_a$, $\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{1}{\sqrt{1 - 0.5^2}} = 1.15$

$\therefore \quad \Delta t_c = (1.15)60 = 69.3$ seconds $= 1.15$ minute

$\Rightarrow$ Heart rate according to coach

$= \frac{45 \text{ beats}}{69.3 \text{ sec}} = 0.65$ beats/second

or $\frac{45 \text{ beats}}{1.15 \text{ min}} = 39.1$ beats/minute

5. [5] Consider a proton that has been accelerated to a high, relativistic speed. Its kinetic energy is found to be 80% of its total energy. What is the speed of the proton? What is the mass of the proton (in the lab frame) as it flies by at this speed?

$$K = 0.8E$$

$$(\gamma - 1)mc^2 = 0.8\gamma mc^2$$

$$\gamma - 1 = 0.8\gamma$$

$$0.2\gamma = 1$$

$$\gamma = 5$$

$$\therefore \quad \frac{1}{\sqrt{1 - v^2/c^2}} = 5 \implies 25\left(1 - \frac{v^2}{c^2}\right) = 1$$

$$25\frac{v^2}{c^2} = 24$$

$$v^2 = 0.96c^2$$

$$\underline{v = 0.98c}$$

and $m = \gamma m_0 = 5(1.67 \times 10^{-27}\,kg) = \underline{8.4 \times 10^{-27}\,kg}$

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[20] total marks