Arts & Science 2D06

Quiz #4 **2017 Nov 29** **Name:** Solutions

NB: Mark values are given in brackets [ ] beside each problem. Write all your answers on the quiz paper. No books or notes allowed. Time to write quiz: 50 minutes.

Gamma factor: $\gamma = (1 - v^2/c^2)^{-1/2}$ Momentum: $p = \gamma m v$

Lorentz transformation: $x' = \gamma(x - vt)$, $t' = \gamma(t - \frac{v}{c^2}x)$.

Reverse Lorentz tranformation: $x = \gamma(x' + vt')$, $t = \gamma(t' + \frac{v}{c^2}x')$

Velocity addition: $u' = \frac{(u-v)}{(1-uv/c^2)}$

Rest-mass energy: $E = mc^2$ Kinetic energy: $K = (\gamma - 1)mc^2$

Speed of light: 3×10^8 m/s (= 1 light-year per year)

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1. [4] If you were to measure your pulse rate while in a spaceship moving with a speed close to the speed of light, you would find that it was

(Explain/derive your answer in the space below; ignore the fact that you might be excited/scared about the trip.)

a) much faster than on Earth.

b) much slower than on Earth.

c) faster than on Earth at first, and then slower.

→ d) the same as it was here on Earth.

Both measurements – on the ship, on Earth – are made "by you", i.e., in your rest frame.

∴ no change in measured pulse rate.

2. [3] From the perspective of a stationary observer, the mass of an object that is accelerating

(No explanation required.)

a) fluctuates according to a sinusoidal function.

a) decreases.

b) stays constant.

→ c) increases.

d) decreases or increases depending on the object's composition.

"accelerating" ⇒ Speed is increasing

$$m_{rel} = \gamma m_0 = \frac{m_0}{\sqrt{1-v^2/c^2}}$$, which increases with increasing v.

3. [4] The sun shines energy in the form of light at a rate of 4×10^{26} J/s. (The Sun's energy source is nuclear fusion, a reaction that converts mass into energy.) Calculate how much mass the sun loses every year.

Mass ~~Energy~~ corresponding to 4×10^{26} J :

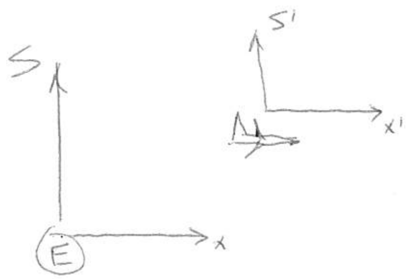
$$E = mc^2 \rightarrow m = \frac{E}{c^2} = \frac{4\times10^{26}}{(3\times10^8)^2} = 4.4\times10^9 \text{ kg}$$

$$\therefore \text{ Mass loss} = \frac{4.4\times10^9 \text{ kg}}{\text{s}} \times \frac{3600 \text{ s}}{\text{hr}} \times \frac{24 \text{ hr}}{\text{day}} \times \frac{365 \text{ days}}{\text{year}}$$

$$= 1.4\times10^{17} \text{ kg/year}$$

4. [5] In the distant future, a spaceship flies in the positive x-direction with a speed of $0.5c$ with respect to Earth. Along the way, a scientist on the rocket observes a collision between two asteroids, and records the space and time coordinates of the collision as 2.5×10^{10} m and 180 seconds, respectively. What are the space and time coordinates of the collision from the earth's point of view? (You can assume that the spaceship started its stopwatch when it left Earth.)

Treat collision as an event observed in two frames:



$*$

Collision @ $\underbrace{(x', t')}_{\text{given}}$, (x, t)

Use Lorentz transformations:
(reverse)

$$x = \gamma(x' + vt'), \quad t = \gamma\left(t' + \frac{v}{c^2}x'\right) \quad \text{with } \gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{1}{\sqrt{1 - 0.25}}$$

$$= 1.15$$

So $x = (1.15)\left(2.5\times10^{10} + (0.5)(3\times10^8)(180)\right) = 6.0 \times 10^{10}$ m

$$t = (1.15)\left(180 + \frac{(0.5)(2.5\times10^{10})}{3\times10^8}\right) = 255 \text{ s}$$

5. [4] Yet another spacecraft, this time searching far and wide for alien life, is flying toward a star at a speed of 1.5×10^8 m/s. To the spacecraft's pilots, at what speed is the starlight approaching?

2 ways :

(1) 2^{nd} postulate of S.R. : Speed of light is the same for all inertial frames

$\therefore$ Starlight approaches with speed $= c$

(2) Use "velocity transformation" equation :

given $\begin{cases} V = 1.5 \times 10^8 \text{ m/s} = \frac{c}{2} \\ u = -c \end{cases}$

$$u' = \frac{u - V}{1 - \frac{uV}{c^2}} = \frac{-c - c/2}{1 - \frac{(-c)(c/2)}{c^2}} = \frac{-1.5c}{1.5} = -c$$

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[20] total marks