Arts & Science 2D06

Quiz #4 2016 Nov 30 Name: Solutions

NB: Mark values are given in brackets [ ] beside each problem. Write all your answers on the quiz paper. No books or notes allowed. Time to write quiz: 50 minutes.

Gamma factor: $\gamma = (1 - v^2/c^2)^{-1/2}$ Momentum: $p = \gamma m v$

Lorentz transformation: $x' = \gamma(x - vt)$, $t' = \gamma(t - \frac{v}{c^2}x)$.

Reverse Lorentz tranformation: $x = \gamma(x' + vt')$, $t = \gamma(t' + \frac{v}{c^2}x')$

Velocity addition: $u' = \frac{(u-v)}{(1-uv/c^2)}$

Rest-mass energy: $E = mc^2$ Kinetic energy: $K = (\gamma - 1)mc^2$

Speed of light: 3×10^8 m/s (= 1 light-year per year)

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1. [3] According to special relativity, the following statement is true (choose one; no need to explain unless you would like to):

a) The speed of light is variable.

b) Light moves faster if it is emitted by a source moving in the direction of emission.

c) Light moves slower if it is emitted by a source moving in the direction of emission.

d) The laws of mechanics are only valid in one specially chosen inertial frame.

→ e) Light moves at the same speed regardless of the motion of the source.

2. [3] Suppose you are a passenger on a spaceship. As the spaceship's speed increases, you would observe:

(Explain/derive your answer in the space below.)

a) the length of your spaceship getting shorter.

b) the length of your spaceship getting longer.

c) the length of your spaceship varying erratically.

→ d) nothing unusual about the length of your spaceship.

e) none of the above.

Since you're in the rest frame of the spaceship, you will not observe any changes in the spaceship's length.

3. [4] What is the speed of a proton, if its mass (while it's moving) is found to be twice its rest mass?

$$m_{rel} = 2m_0$$

$$\gamma \cancel{m_0} = 2\cancel{m_0}$$

$$\frac{1}{\sqrt{1-v^2/c^2}} = 2$$

$$1 - \frac{v^2}{c^2} = \frac{1}{4}$$

$$\frac{v^2}{c^2} = 0.75$$

$$\underline{v = 0.87c}$$

4. [5] The closest star to our solar system is Alpha Centauri, which is 4.40 light-years away. A spaceship with a constant speed of 0.80c relative to Earth, leaves Earth and travels toward Alpha Centauri. How long does the journey take, according to a passenger on the spaceship?

Use time dilation:

let Δt_E = trip duration in Earth's frame

Δt_s = " " " spaceship's frame

then, $\Delta t_E = \dfrac{4.4 \text{ lyrs}}{0.8c} = \dfrac{4.4 \text{ lyrs}}{0.8 \frac{\text{lyr}}{\text{yr}}} = 5.5 \text{ lyr}$

Spaceship measures the proper time, so

$$\Delta t_E = \gamma \Delta t_s$$

$$\Delta t_s = \frac{\Delta t_E}{\gamma} = \sqrt{1 - v^2/c^2}\, \Delta t_E$$

$$= \sqrt{1 - 0.64}\, (5.5 \text{ lyr})$$

$$= \underline{3.3 \text{ lyr.}}$$

(NOTE: 2 other valid approaches:

- Use Lorentz transformations

OR

- Use Length contraction)

5. [5] Consider two reference frames, S and S', such that S' moves with $v = 0.85c$ with respect to S. A stationary car is located at a position of 75 m with respect to S'. Assuming that the origins of the two frames overlap when $t' = t = 0$, find the position of the object in frame S when the clock in S reads 1.5 microseconds.

(note: 1 microsecond = 10^{-6} second)

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{1}{\sqrt{1 - 0.72}} = 1.89$$

We're given: $t = 1.5\ \mu\text{sec}$, $x' = 75\text{ m}$; Want x

find t': $t = \gamma(t' + \frac{v}{c^2}x')$

$$1.5 \times 10^{-6} = (1.89)\left(t' + \frac{0.85 \cdot 75}{3 \times 10^8}\right)$$

$$1.89\,t' = 1.1 \times 10^{-6}$$

$$t' = 5.8 \times 10^{-7} \text{ seconds}$$

find x: $x = \gamma(x' + vt')$

$$= 1.89\left[75 + (0.85)(3 \times 10^8)(5.8 \times 10^{-7})\right]$$

$$= \underline{421\text{ m}}.$$

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[20] total marks