Arts & Science 2D06

Quiz #4 2014 Nov 20 Name: Solutions

NB: Mark values are given in brackets [ ] beside each problem. Write all your answers on the quiz paper. No books or notes allowed. Time to write quiz: 50 minutes.

Elastic collisions, target m_2 stationary: $v_1 = \frac{(m_1 - m_2)}{(m_1 + m_2)} u_1$, $v_2 = \frac{2m_1}{(m_1+m_2)} u_1$

Gamma factor: $\gamma = (1 - v^2/c^2)^{-1/2}$ Momentum: $p = \gamma m v$

Lorentz transformation: $x' = \gamma(x - vt)$, $t' = \gamma(t - \frac{v}{c^2}x)$.

Velocity addition: $u' = \frac{(u-v)}{(1-uv/c^2)}$

Rest-mass Energy: $E = mc^2$ Kinetic energy: $K = (\gamma - 1)mc^2$

Speed of light: 3×10^8 m/s

---

1. [3] Consider two objects that collide inelastically. If all of the initial kinetic energy is converted into other forms of energy (e.g., heat, sound, etc.), then what can we conclude about the collision? (Choose one option; explain your answer.)

a) The final speed of the heavier object is lower than that of the lighter one.

→ b) The total initial momentum of the system is zero.

c) The objects were initially moving in the same direction.

d) Nothing, since such a collision is physically impossible.

After collision : system has no K.E.

$\Rightarrow \vec{P}_f = 0$ (total final momentum)

$\therefore$ by conservation of momentum :

$\vec{P}_i = 0$ (total initial momentum)

2. [2] The Michelson-Morley experiment was designed to measure:

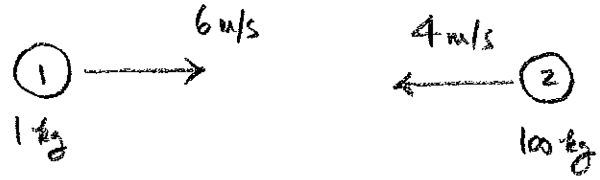
a) the relativistic momentum of the electron.

b) the acceleration of gravity on the Earth's surface.

c) the relativistic kinetic energy of a moving clock.

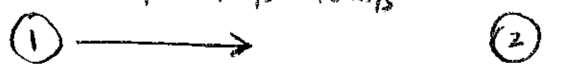
→ d) the velocity of the Earth relative to the luminiferous aether.

3. [5] A 1-kg object traveling at 6 m/s collides head-on with a 100-kg object traveling in the opposite direction at 4 m/s. If the collision is perfectly elastic, what is the final speed of the 1-kg object?



easiest approach: Switch to rest frame of one of objects; I'll choose "2":

$u_1' = 6\,m/s + 4\,m/s = 10\,m/s$



$u_2' = 0$

find V_1 :
$$V_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) u_1'$$
$$= \left(\frac{1 - 100}{1 + 100}\right)(10\,m/s)$$
$$= -9.8\,m/s$$

Now, switch back to original frame:

$$V_1 = V_1' - 4\,m/s = \underline{-13.8\,m/s}$$

Another way...

3. [5] A 1-kg object traveling at 6 m/s collides head-on with a 100-kg object traveling in the opposite direction at 4 m/s. If the collision is perfectly elastic, what is the final speed of the 1-kg object?

$$\underset{6\,m/s}{① \longrightarrow} \quad \underset{4\,m/s}{\longleftarrow ②} \quad \longrightarrow x$$

- elastic collision : $\vec{p}$, K are conserved ; "1" → 1 kg ; "2" → 100 kg

$$\therefore \quad m_1 V_1 + m_2 V_2 = m_1 u_1 + m_2 u_2 \quad \text{—— (1)}$$

$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 \quad \text{—— (2)}$$

- rewrite (1): $m_1 (V_1 - u_1) = m_2 (u_2 - V_2)$ —— (3)
- rewrite (2): $m_1 (V_1^2 - u_1^2) = m_2 (u_2^2 - V_2^2)$

$$m_1 (V_1 + u_1)(V_1 - u_1) = m_2 (u_2 + V_2)(u_2 - V_2) \quad \text{—— (4)}$$

- divide (4) by (3) : $V_1 + u_1 = u_2 + V_2$

$$u_2 = V_1 + u_1 - V_2$$
$$= 6 + u_1 - (-4)$$
$$u_2 = 10 + u_1$$

(1) : $(1)(6) + (100)(-4) = u_1 + 100 u_2$

$$6 - 400 = u_1 + 100(10 + u_1)$$
$$-394 = u_1 + 1000 + 100 u_1$$
$$101 u_1 = -1394$$
$$u_1 = -13.8 \text{ m/s}$$

4. [5] If the velocity of your spaceship increases from $0.4c$ to $0.7c$, then by what percentage will your mass have increased?

relativistic momentum : $p = m_{rel} \cdot V$

where $m_{rel} = \gamma m_0$ ← rest mass (constant)

1) $V = 0.4c$:

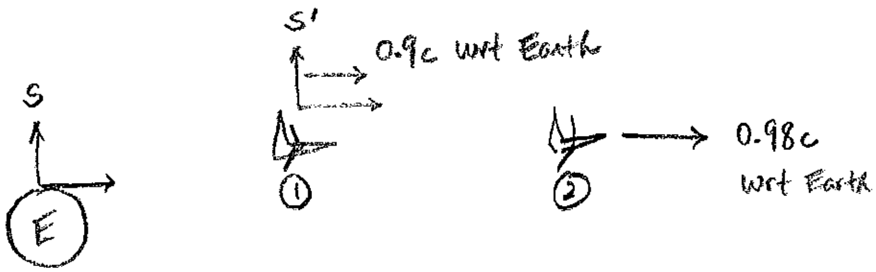
$$M_{rel,1} = \frac{m_0}{\sqrt{1 - \frac{(0.4c)^2}{c^2}}} = 1.09\, m_0$$

2) $V = 0.7c$:

$$m_{rel,2} = \frac{m_0}{\sqrt{1 - \frac{(0.7c)^2}{c^2}}} = 1.40\, m_0$$

∴ % increase : $\frac{1.40 m_0 - 1.09 m_0}{1.09 m_0} \times 100 = \underline{28.4\%}$

5. [5] Suppose that with respect to the earth, Rocket 1 is moving at a speed of $0.90c$ to the right. Rocket 2, in turn, is also moving to the right with respect to Earth, but with a speed of $0.98c$. How fast is Rocket 2 moving with respect to Rocket 1? Express your answer in terms of the speed of light, c.



We'll choose S' to be Rocket 1's frame; find u'.

Then, $V = 0.9c$

$u = 0.98c$

and
$$u' = \frac{u - V}{(1 - uV/c^2)}$$

$$= \frac{0.98c - 0.9c}{1 - \frac{(0.98c)(0.9c)}{c^2}}$$

$$= \frac{0.08c}{1 - 0.88}$$

$$= \underline{0.67c}$$

[20] total marks