Arts & Science 2D06

Quiz #4 2012 Nov 27 **Name:** Solutions

NB: Mark values are given in brackets [ ] beside each problem. Write all your answers on the quiz paper. No books or notes allowed. Time to write quiz: 50 minutes.

Taylor series: $(1+x)^a \simeq 1+ax$ for small x

Gamma factor: $\gamma = (1-v^2/c^2)^{-1/2}$ Momentum: $p = \gamma mv$

Lorentz transformation: $x' = \gamma(x-vt)$, $t' = \gamma(t-\frac{v}{c^2}x)$.

Velocity addition: $u' = \frac{(u-v)}{(1-uv/c^2)}$

Rest-mass Energy: $E = mc^2$ Kinetic energy: $K = (\gamma-1)mc^2$

Speed of light: 3×10^8 m/s 1 light-year = distance traveled by light in one year

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1. [3] Which of the following are NOT true of Einstein's theory of special relativity (assume reference frames to be inertial):

a) Two events that are simultaneous to one observer need not be simultaneous to another.

(b) Momentum is conserved, but energy is not.

(c) The equations of physics change their form depending on the observer's reference frame.

d) The speed of light c has the same value in all reference frames.

e) An event's space and time coordinates in different frames are related through the Lorentz transformations.

2. [3] A spaceship is moving in a straight line at a speed of $0.9c$ relative to the earth. It launches a shuttle that flies away from the spaceship, in the same direction as the spaceship, at a speed of $0.7c$. Relative to Earth, what is the shuttle's speed?

(a) Greater than $0.9c$ and less than c.

b) $1.6c$.

c) c.

d) Greater than c and less than $1.6c$.

e) $0.2c$.

3. [4] The kinetic energy of an object moving at 10% of the speed of light is: (Explain/derive your answer.)

(a) Slightly higher than $(1/2)mv^2$.

b) Slightly less than $(1/2)mv^2$.

c) Equal to $(1/2)mv^2$.

d) Equal to 10% of $(1/2)mc^2$.

e) Equal to zero.

$$K = (\gamma - 1)mc^2 = \left[\left(1 - v^2/c^2\right)^{-1/2} - 1\right]mc^2$$

$$\approx \frac{1}{2}\frac{v^2}{c^2} \cdot mc^2 + \text{higher order terms in } \frac{v^2}{c^2}$$

$$\approx \frac{1}{2}mv^2 + \text{higher order terms in } v^2/c^2.$$

4. [5] Two stars are 85 light-years apart, as measured in their common rest frame. If they appear to be 39 light-years apart to a spaceship, how fast is the spaceship moving? (Express your answer is terms of c.)

- proper length : $L_0 = 85$ lyrs
- spaceship : $L = 39$ lyrs

length contraction : $\frac{L_0}{\gamma} = L$

$$\therefore \gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{85}{39}$$

$$1 - v^2/c^2 = 0.21$$

$$v^2 = 0.79c^2$$

$$\therefore v = 0.89c$$

5. [5] A particle has speed 0.90c and total energy of 8.3×10^{-10} J. What is the particle's momentum?

$V = 0.9c$

$E = 8.3 \times 10^{-10} J$

$\therefore \gamma mc^2 = 8.3 \times 10^{-10} J$

$$m = \frac{8.3 \times 10^{-10} \cdot \sqrt{1 - \frac{(0.9c)^2}{c^2}}}{c^2} = \frac{(8.3 \times 10^{-10})(0.44)}{(3 \times 10^8)^2} = 4.1 \times 10^{-27} kg.$$

Momentum: $p = \gamma m v = \frac{1}{\sqrt{1-(0.9)^2}} (4.1 \times 10^{-27} kg)(0.9 \times 3 \times 10^8)$

$= 2.5 \times 10^{-18} kg \cdot m/s$

[20] total marks