Arts & Science 2D06

Quiz #3 **2019 Nov 6** **Name:** Solutions

NB: Mark values are given in brackets [ ] beside each problem. Write all your answers on the quiz paper. No books or notes allowed. Time to write quiz: 50 minutes.

centripetal $a_c = v^2/r$ linear K.E. $= (1/2)mv^2$ Rotational K.E. $= (1/2)I\omega^2$

Energy conservation $E = K + U$ Gravitational force: $F_g = GMm/r^2$

Moment of inertia of disk: $I = (1/2)MR^2$ Sphere: $I = (2/5)MR^2$

---

1. [3] Two children are riding on a spinning merry-go-round. Child A is twice as far as Child B from the merry-go-round's centre. If the children have the same mass – they're twins, say – which of the following statements about Child A's moment of inertia relative to the axis of rotation, is TRUE?

(You can explain your answer if you would like, but you do not have to.)

→ a) Child A's moment of inertia is 4 times greater than child B's.
b) Child A's moment of inertia is 2 times greater than child B's.
c) Child A's moment of inertia is the same as child B's.
d) Child A has a greater moment of inertia, but it is impossible to say by how much.
e) Child A has a smaller moment of inertia, but it is impossible to say by how much.

$$I = mr^2 \Rightarrow I_A = m_A r_A^2$$

$$I_B = m_B r_B^2 = m_B \left(\frac{r_A}{2}\right)^2 = m_A \frac{r_A^2}{4}$$

$$\therefore \frac{I_A}{I_B} = \frac{r_A^2}{r_A^2/4} \Rightarrow \underline{I_A = 4I_B}$$

2. [3] Consider the following statement: "Earth does not move toward an apple while the apple is falling toward Earth, because the force exerted by Earth on the apple is a lot bigger than the force exerted by the apple on Earth." Is this statement TRUE or FALSE?

(Explain/derive your answer in the space provided.)

The statement is FALSE.

The two forces are equal in magnitude, opposite in direction.

(Note, however, that $a_{Earth} \ll a_{apple}$ since $M_{Earth} \gg M_{apple}$.)

3. [4] A mass of 2.0 kg travels at 3.0 m/s along a smooth (frictionless), horizontal plane, and hits an uncompressed spring. The mass is slowed to zero velocity when the spring has been compressed by 0.15 m. What is the spring constant k of this spring?

Use conservation of energy:

$$E_0 = E_f$$

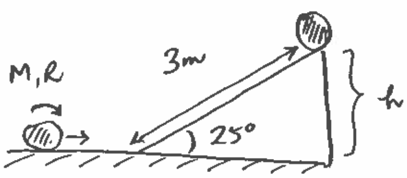
$$K_0 + \cancel{U_0}^{\,0} = \cancel{K_f}^{\,0} + U_f$$

$$\cancel{\tfrac{1}{2}} m v^2 = \cancel{\tfrac{1}{2}} k x^2$$

$$k = \frac{m v^2}{x^2} = \frac{2 \cdot 3^2}{0.15^2} = \underline{800\ N/m}.$$

4. [5] A solid sphere is rolling without slipping along a horizontal surface with a speed of 4.50 m/s when it starts up a ramp that makes an angle of 25.0° with the horizontal. What is the sphere's linear speed after it has rolled 3.00 m up the ramp?

(Explain/derive your answer in the space provided.)



Use conservation of energy, again:

$$E_0 = E_f$$

$$K_0 + U_0^{\,0} = K_f + U_f$$

$$\frac{1}{2}Mv_0^2 + \frac{1}{2}I\omega_0^2 = \frac{1}{2}Mv_f^2 + \frac{1}{2}I\omega_f^2 + Mgh$$

(v_f ← want this)

"rolling w/o slipping" $\Rightarrow$ $\omega = \frac{v}{R}$

Sphere : $I = \frac{2}{5}MR^2$

$h = 3\sin 25° = 1.3$ m

$$\therefore \quad \frac{1}{2}MV_0^2 + \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{V_0^2}{R^2}\right) = \frac{1}{2}MV_f^2 + \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{V_f^2}{R^2}\right) + Mgh$$

$$\left(\frac{1}{2}+\frac{1}{5}\right)V_0^2 = \left(\frac{1}{2}+\frac{1}{5}\right)V_f^2 + gh$$

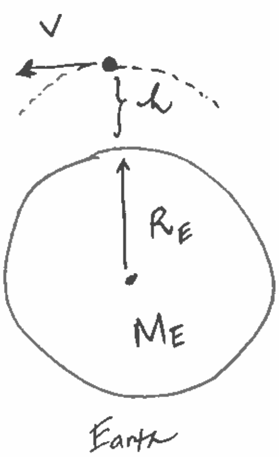
$$0.7\,V_f^2 = (0.7)(4.5)^2 - (9.81)(1.3)$$

$$V_f^2 = 2.03$$

$$V_f = 1.4 \text{ m/s}$$

5. [5] How high above Earth's surface should a GPS satellite be placed, such that its orbital period is 48.0 hours? You may assume the satellite's orbit is circular.

(The mass of the earth is 5.976×10^{24} kg and the radius of the earth is 6.378×10^{6} m.)



$$48 \text{ hours} = 48 \text{ hr} \cdot \left(\frac{3600 \text{ s}}{\text{hr}}\right) = 1.73 \times 10^{5} \text{ sec.}$$

let $r = h + R_E$

$$\rightarrow \quad T = \frac{2\pi r}{v} \quad \text{—— (1)}$$

$$\text{Now,} \quad a = \frac{v^2}{r} = \frac{GM_E}{r^2}$$

$$v = \sqrt{\frac{GM_E}{r}} \quad \text{—— (2)}$$

Sub (2) in (1) :

$$T = \frac{2\pi r}{\sqrt{\frac{GM_E}{r}}}$$

$$T^2 = \frac{4\pi^2 r^2}{\frac{GM_E}{r}} \rightarrow r = \left(\frac{GM_E}{4\pi^2} T^2\right)^{1/3}$$

$$r = \left[\frac{(6.67 \times 10^{-11})(5.976 \times 10^{24})(1.73 \times 10^{5})^2}{4\pi^2}\right]^{1/3} = 6.71 \times 10^{7} \text{ m}$$

$$\therefore \quad h = r - R_E = \underline{6.1 \times 10^{7} \text{ m}}$$

20

[20] total marks