**Arts & Science 2D06**

**Quiz #3 2018 Nov 7** **Name:** Solutions

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NB: Mark values are given in brackets [ ] beside each problem. Write all your answers on the quiz paper. No books or notes allowed. Time to write quiz: 50 minutes.

centripetal $a_c = v^2/r$ linear K.E. $= (1/2)mv^2$ Rotational K.E. $= (1/2)I\omega^2$

Energy conservation $E = K + U$ Gravitational force: $F_g = GMm/r^2$

Impulse $\mathbf{J} = \Delta\mathbf{p} = \mathbf{F}_{ave}\, dt$

Moment of inertia of disk: $I = (1/2)MR^2$ Ring: $I = MR^2$

Elastic collisions (1D), target $m_2$ stationary: $v_1 = \frac{(m_1 - m_2)}{(m_1 + m_2)} u_1$, $v_2 = \frac{2m_1}{(m_1 + m_2)} u_1$

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1. [3] Two children are riding on a merry-go-round. Child A is farther from the axis of rotation than child B. Over a fixed time interval, which child has a larger *angular* displacement?

(You can explain your answer if you would like, but you do not have to.)

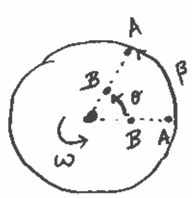
a) Child A

b) Child B

c) Neither child has an angular displacement.

→ d) They have the same non-zero angular displacement.

e) There is not enough information given to answer the question.




$\theta$ = angular displacement of B

$\beta$ = 〃 〃 〃 A

$\Rightarrow \theta = \beta \neq 0$,

by inspection.

2. [3] Planet $Z$ is located between two stars $X$ and $Y$ at a point where the total gravitational force on Planet $Z$ (due to the two stars) is zero. If Planet $Z$ is moved slightly toward one of the stars, then the total gravitational force on Planet $Z$

(Explain/derive your answer in the space provided.)

a) is still zero.

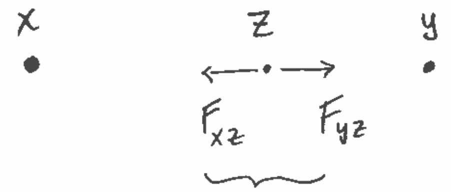
b) points toward the star that is more massive.

→ c) points in the same direction in which $Z$ is moved.

d) points in the opposite direction in which $Z$ is moved.

e) is perpendicular to the displacement of $Z$.

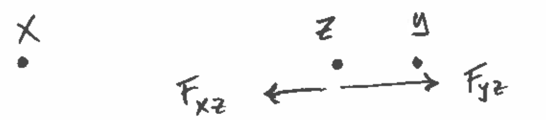
At first :




$F_{xz} = F_{yz}$ $\left( F = \dfrac{G M_{star} M_z}{r^2} \right)$

$\Rightarrow$ total grav. force : $\vec{F}_{xz} + \vec{F}_{yz} = 0$

Suppose $Z$ is moved toward Star $y$ :




Now $F_{yz} > F_{xz}$

$\Rightarrow$ total grav. force $= \vec{F}_{xz} + \vec{F}_{yz} \neq 0$

and points toward Star $y$.

( Similar argument if $Z$ is moved toward $x$ )

3. [4] A batter hits a 0.140-kg baseball that was approaching him at 40.0 m/s and, as a result, the ball leaves the bat at 30.0 m/s toward the pitcher. What is the magnitude of the impulse delivered to the baseball by the batter?

Impulse $\vec{I} = \Delta\vec{p} = \vec{p}_f - \vec{p}_i = m\vec{v}_f - m\vec{v}_i$

- let (+) direction be toward pitcher

$\Rightarrow \quad \vec{I} = (0.140)(-40) - (0.140)(+30)$

$= -9.8 \text{ kg} \cdot \text{m/s}$

magnitude: $\underline{I = 9.8 \text{ kg} \cdot \text{m/s}}$

4. [5] Consider a solid disk of radius $R$ and mass $M$ that is rolling on a surface without slipping. Which form of kinetic energy is larger – translational or rotational? What if the rolling object is a thin, solid ring?

(Explain/derive your answer in the space provided.)

Disk :

$$K_T = \frac{1}{2}Mv^2$$

$$K_R = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{2}M\cancel{R^2}\right)\left(\frac{v}{\cancel{R}}\right)^2$$

$$= \frac{1}{2}\left(\frac{1}{2}Mv^2\right)$$

$$= \frac{1}{2}K_T$$

$$\therefore \quad \underline{K_T > K_R}$$

Ring :

$$K_T = \frac{1}{2}Mv^2$$

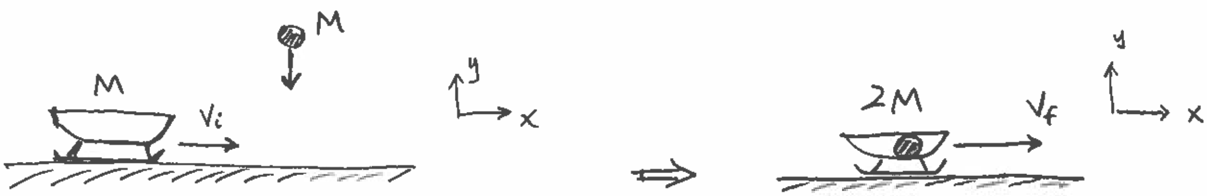
$$K_R = \frac{1}{2}I\omega^2 = \frac{1}{2}(M\cancel{R^2})\left(\frac{v}{\cancel{R}}\right)^2$$

$$= \frac{1}{2}Mv^2$$

$$\therefore \quad \underline{K_T = K_R}$$

5. [5] A sled of mass $M$ slides with constant speed on a frozen river (ignore friction). As the sled passes under a bridge, a package, also of mass $M$, is dropped from the bridge straight down onto the sled. The sled and package then slide together along the sled's original direction.

Derive an equation relating the kinetic energy of the sled-package system ($K_f$, say) to the sled's original kinetic energy ($K_i$, say). After the package landed, was kinetic energy lost or gained, or did it stay the same?




Use conservation of momentum in x-direction:

$$P_{ix} = P_{fx}$$

$$MV_i = 2MV_f$$

$$\Rightarrow V_f = \frac{V_i}{2}$$

$$\therefore K_f = \frac{1}{2}(2M)V_f^2 = M \cdot \frac{V_i^2}{4} = \frac{1}{2}\left(\frac{1}{2}MV_i^2\right)$$

$$\Rightarrow \underline{K_f = \frac{K_i}{2}}$$

$\therefore$ kinetic energy was <u>lost</u>.

20

[20] total marks