Arts & Science 2D06

Quiz #3 **2017 Nov 8** **Name:** Solutions

NB: Mark values are given in brackets [ ] beside each problem. Write all your answers on the quiz paper. No books or notes allowed. Time to write quiz: 50 minutes.

Solution for quadratic equation: $x = (-b \pm \sqrt{b^2 - 4ac})/2a$

centripetal $a_c = v^2/r$ linear K.E. $= (1/2)mv^2$ Rotational K.E. $= (1/2)I\omega^2$

Energy conservation $E = K + U$ Gravitational force: $F_g = GMm/r^2$

$G = 6.67 \times 10^{-11}$ N m^2/kg^2 $R_E = 6.37 \times 10^6$ m (radius of Earth)

Moment of inertia of sphere: $I = (2/5)MR^2$

Elastic collisions, target m_2 stationary: $v_1 = \frac{(m_1 - m_2)}{(m_1 + m_2)} u_1$, $v_2 = \frac{2m_1}{(m_1 + m_2)} u_1$

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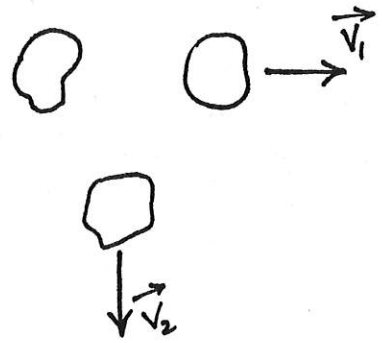
1. [4] A radioactive atom at rest suddenly disintegrates into three parts of equal mass, as shown in the picture. Which of the options below best represents the direction of the velocity of the third piece?

(Explain/derive your answer in the space provided.)

a)
b)
c)
d)
e)



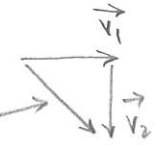
@ rest, then moving : $\vec{p}$ conserved, K.E. not conserved

$\therefore \quad \vec{P}_0 = \vec{P}_f \Rightarrow \vec{P}_f = 0$

$m\vec{V}_1 + m\vec{V}_2 + m\vec{V}_3 = 0$

$\vec{V}_3 = -(\vec{V}_1 + \vec{V}_2)$

$\vec{V}_1 + \vec{V}_2$:



$\Rightarrow$ $\vec{V}_3$ "=" (a)

2. [3] What is the quantity used to measure an object's resistance to changes in rotational motion?

(You can explain your answer if you would like, but you do not have to.)

a) mass
→ b) moment of inertia
c) momentum
d) angular velocity
e) angular acceleration

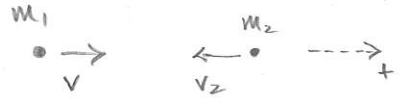
Compare $K_{rot} = \frac{1}{2}I\omega^2$ and $K = \frac{1}{2}mv^2$:

"I" and "m" are analogous.

3. [4] Two objects of the same mass move toward each other. The first mass (m_1) is moving with speed v. The objects collide in a perfectly inelastic collision and move with speed $0.1v$ in the direction of the m_1's velocity before the collision. What was the speed of the second mass (m_2) before the collision? (Express your answer in terms of v.)

- "perfectly inelastic collision": objects stick together after colliding; momentum is conserved

$\therefore \quad \vec{P}_0 = \vec{P}_f$



$$m_1 v - m_2 v_2 = (m_1 + m_2) v_f$$

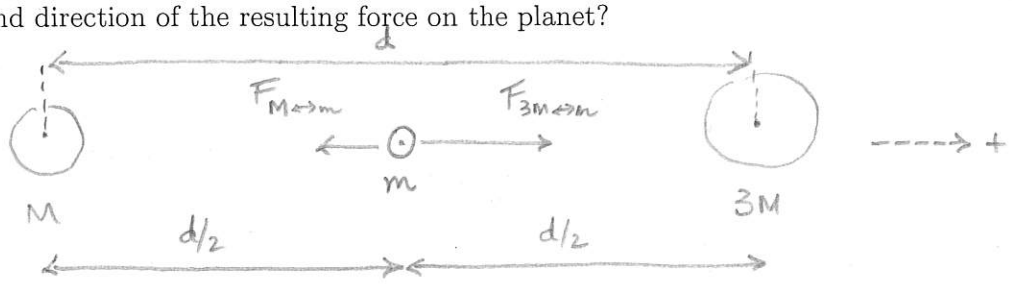
Now $v_f = +0.1v$, so: $m_1 v - m_2 v_2 = 0.1(m_1 + m_2)v$

$$m_2 v_2 = m_1 v - 0.1(m_1 + m_2)v$$

$$m_2 v_2 = 0.9 m_1 v - 0.1 m_2 v$$

$$\Rightarrow \quad v_2 = 0.9\left(\frac{m_1}{m_2}\right)v - 0.1v \qquad \text{(magnitude of } \vec{v}_2\text{)}$$

4. [4] A planet with mass m is located halfway between a star of mass M and a star of mass $3M$. The stars are themselves separated by a distance d. What are the magnitude and direction of the resulting force on the planet?



$$\vec{F}_{M\leftrightarrow m} = -\frac{GMm}{r^2} = -\frac{GMm}{(d/2)^2} = -\frac{4GMm}{d^2}$$

$$\vec{F}_{3M\leftrightarrow m} = +\frac{G(3M)m}{r^2} = \frac{12GMm}{d^2}$$

$$\therefore \quad \Sigma\vec{F} = \vec{F}_{M\leftrightarrow m} + \vec{F}_{3M\leftrightarrow m} = -\frac{4GMm}{d^2} + \frac{12GMm}{d^2}$$

$$\Sigma\vec{F} = +\frac{8GMm}{d^2}$$

to the right

5. [5] A ball (sphere) is rolling without slipping along a frictionless, horizontal surface with a speed of 4.5 m/s. It then starts moving down a ramp that is inclined at an angle of 25° from the horizontal. What is the speed of the ball after it has rolled 3.0 m down the ramp?

- "rolling w/o slipping": the "v" in "$V = \omega R$" is the same as the translational V (i.e., 4.5 m/s)
- Sphere: $I = \frac{2}{5}MR^2$
- Energy conservation: $E_0 = E_f$

$$E_0 = K_{tr,0} + K_{rot,0} + U_0$$

$$= \frac{1}{2}mV_0^2 + \frac{1}{2}I\omega_0^2 + mgh_0$$

$$= \frac{1}{2}mV_0^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\left(\frac{V_0^2}{R^2}\right) + mgh_0$$

$$= \frac{1}{2}mV_0^2 + \frac{1}{5}mV_0^2 + mgh_0 = \frac{7}{10}mV_0^2 + mgh_0$$

likewise, $E_f = K_{tr,f} + K_{rot,f} + U_f = \frac{7}{10}mV_f^2 + mgh_f$

$$\therefore \quad \frac{7}{10}\cancel{m}V_0^2 + \cancel{m}gh_0 = \frac{7}{10}\cancel{m}V_f^2 + \cancel{m}gh_f$$

$$V_f^2 = \frac{10}{7}\left(\frac{7}{10}V_0^2 + g\underbrace{(h_0 - h_f)}\right)$$

$$= (3.0)\sin 25°$$

$$V_f = \sqrt{(4.5)^2 + \frac{10}{7}(9.8)(3.0)\sin 25°}$$

$$\Rightarrow \underline{V_f = 6.2 \text{ m/s}}$$

20

[20] total marks