Arts & Science 2D06

Quiz #3 2016 Nov 9 **Name:** Solutions

NB: Mark values are given in brackets [ ] beside each problem. Write all your answers on the quiz paper. No books or notes allowed. Time to write quiz: 50 minutes.

Solution for quadratic equation: $x = (-b \pm \sqrt{b^2 - 4ac})/2a$

centripetal $a_c = v^2/r$ linear K.E. $= (1/2)mv^2$ Rotational K.E. $= (1/2)I\omega^2$

Energy conservation $E = K + U$ Gravitational force: $F_g = GMm/r^2$

$G = 6.67 \times 10^{-11}$ N m^2/kg^2 $R_E = 6.37 \times 10^6$ m (radius of Earth)

Moment of inertia of cylinder: $I = (1/2)MR^2$

Moment of inertia of sphere: $I = (2/5)MR^2$

Elastic collisions, target m_2 stationary: $v_1 = \frac{(m_1 - m_2)}{(m_1 + m_2)} u_1$, $v_2 = \frac{2m_1}{(m_1 + m_2)} u_1$

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1. [3] Two children are riding on a merry-go-round. Child A is at a greater distance from the axis of rotation than child B. Which child has the larger angular speed?

(You can explain your answer if you would like, but it is not required.)

a) Child A
b) Child B
c) They both have no angular speed.
→ d) They have the same non-zero angular speed.
e) There is not enough information given to answer the question.

The angular speed ω is independent of the distance from the rotation axis.

2. [3] A new planet is found to have a mass and radius both equal to one-third those of the earth. With g representing, as usual, the acceleration due to gravity on the surface of earth, what is the acceleration due to gravity on the surface of this planet?

(Explain/derive your answer in the space provided.)

a) $g/3$
→ b) $3g$
c) $6g$
d) $g/9$
e) $9g$

$$F_g = ma \Rightarrow \frac{GM\cancel{m}}{r^2} = \cancel{m}g$$

$$\therefore \quad g = \frac{GM_E}{R_E^2}$$

$$\Rightarrow g_{new} = \frac{GM_{new}}{R_{new}^2} = \frac{GM_E/3}{(R_E/3)^2} = 3\frac{GM_E}{R_E^2} = 3g$$

3. [4] A uniform solid cylinder of radius R and mass M, and a uniform solid sphere, also of radius R and mass M, roll without slipping. If both objects have the same kinetic energy, what is the ratio of the speed of the cylinder to the speed of the sphere?

$$I_c = \frac{1}{2}MR^2 \quad , \quad I_s = \frac{2}{5}MR^2$$

$$K_{cyl} = K_{sph}$$

$$\frac{1}{2}MV_c^2 + \frac{1}{2}I_c\omega_c^2 = \frac{1}{2}MV_s^2 + \frac{1}{2}I_s\omega_s^2$$

$$\frac{1}{2}MV_c^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\frac{V_c^2}{R^2} = \frac{1}{2}MV_s^2 + \frac{1}{2}\left(\frac{2}{5}MR^2\right)\frac{V_s^2}{R^2}$$

$$\left(\frac{1}{2}+\frac{1}{4}\right)V_c^2 = \left(\frac{1}{2}+\frac{1}{5}\right)V_s^2$$

$$\frac{3}{4}V_c^2 = \frac{7}{10}V_s^2$$

$$\frac{V_c}{V_s} = \sqrt{\frac{7/10}{3/4}} = \sqrt{\frac{28}{30}} = 0.97.$$

4. [5] A satellite of mass 400 kg orbits the Earth with a period of 6000 seconds. The earth has a mass of 5.98×10^{24} kg. Determine the altitude of the satellite above the Earth's surface.

$$F_g = \frac{mv^2}{r} \quad \text{where} \quad r = R_E + h$$

$$v = \frac{2\pi r}{T}$$

$$\therefore \quad \frac{GM_E m}{r^2} = \frac{mv^2}{r}$$

$$r = \frac{GM_E}{v^2} = \frac{GM_E T^2}{4\pi^2 r^2}$$

$$\Rightarrow \quad r^3 = \frac{GM_E T^2}{4\pi^2}$$

$$r = \sqrt[3]{\frac{GM_E T^2}{4\pi^2}} = \sqrt[3]{\frac{(6.67\times10^{-11})(5.98\times10^{24})(6000)^2}{4\pi^2}}$$

$$r = 6.32 \times 10^6 \text{ m}$$

$$\therefore \quad h = r - R_E = 7.14\times10^6 \text{ m} - 6.37\times10^6 \text{ m} = \underline{7.7\times10^5 \text{ m}}$$

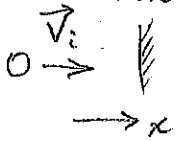
5. [5] A 0.300-kg ball approaches a wall, with a velocity of 15.0 m/s directed perpendicular to the wall. The ball then rebounds with 60.0% of its initial kinetic energy. What is the magnitude of the change in momentum of the ball? (Assume the rebound direction is also perpendicular to the wall.)

- Kinetic energy: $K_f = 0.6\,K_i$

$$\cancel{\tfrac{1}{2}}mV_f^2 = 0.6\left(\cancel{\tfrac{1}{2}}mV_i^2\right)$$

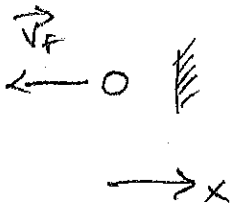
$$V_f = 0.77\,V_i$$

- initial momentum: $p_i = mV_i = 0.3 \times 15 = 4.5\ kg \cdot m/s$



- final momentum: $p_f = -mV_f = -0.3(0.77\,V_i)$

$$= -3.5\ kg \cdot m/s$$



- change in momentum: $|\Delta p| = 4.5 - (-3.5) = 8.0\ kg \cdot m/s$

20/
[20] total marks