Arts & Science 2D06

Quiz #3 2014 Oct 27 Name: Solutions

NB: Mark values are given in brackets [ ] beside each problem. Write all your answers on the quiz paper. No books or notes allowed. Time to write quiz: 50 minutes.

centripetal $a_c = v^2/r$ linear K.E. $= (1/2)mv^2$ Rotational K.E. $= (1/2)I\omega^2$

Energy conservation $E = K + U$ Gravitational force: $F_g = GMm/r^2$

$G = 6.67 \times 10^{-11}$ N m^2/kg^2 (Newton's law of gravity constant)

Moment of Inertia (discrete) $= \sum m_i r_i^2$

$M_E = 5.98 \times 10^{24}$ kg (mass of Earth) $R_E = 6.37 \times 10^6$ m (radius of Earth)

$M_M = 7.35 \times 10^{22}$ kg (mass of Moon) $R_M = 1.74 \times 10^6$ m (radius of Moon)

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1. [3] A very light object and a very heavy object are sliding with equal speeds along a level, frictionless surface. They then both slide up the same frictionless hill. Which object rises to a greater height?

(Explain/derive your answer in the space provided.)

a) The heavy object, because it has greater kinetic energy.

b) The light object, because it has smaller kinetic energy.

c) The light object, because it weighs less.

d) The heavy object, because it weighs more.

→ e) They both slide to the same height.

$\frac{1}{2}mv^2 = mgh$

$\Rightarrow h = \frac{v^2}{2g}$, which is independent of mass.

2. [4] A clock has a seconds-hand of length 10 cm.

(a) Find the angular speed of the hand.

$T = 60$ sec

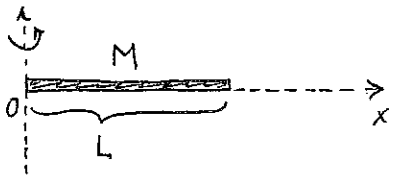
$\Rightarrow \omega = \frac{2\pi}{T} = \frac{2\pi}{60} = 0.15\ s^{-1}$

(b) What is the linear speed of the hand's tip?

$V = \omega R$, $R = 0.1\ m$

$\therefore V = (0.15)(0.1) = 0.015\ m/s$

3. [5] Consider a uniform thin rod of length L and mass M, as seen in the figure. Treat the rod as a one-dimensional object.



(a) Show that its moment of inertia for rotation about an axis through the origin (see figure) is $(1/3)ML^2$.

$$I = \int r^2 dm = \int_0^L x^2 \left(\frac{M}{L}\right) dx = \frac{M}{L}\,\frac{x^3}{3}\Big|_0^L = \frac{1}{3}ML^2,$$

as required.

(b) Find the moment of inertia again, now assuming that the rod's mass density (i.e., mass per unit length) is given by λx, where λ is a constant with units of kg/m^2.

$$I = \int r^2 dm = \int_0^L x^2 (\lambda x)\, dx = \lambda \int_0^L x^3 dx$$

$$= \frac{\lambda L^4}{4}$$

4. [5] The period of a pendulum is given by $T = 2\pi(L/g)^{1/2}$, where g is the local acceleration due to gravity and L is the pendulum's length. Suppose that such a pendulum has a period of 3 seconds on Earth. What would be its period on the surface of moon?

In general,

$$F = \frac{GMm}{r^2} = mg \Rightarrow g = \frac{GM}{r^2}$$

- On Earth: $3 = 2\pi\left(\frac{L}{9.8}\right)^{1/2}$

$$L = \left(\frac{3}{2\pi}\right)^2 \cdot 9.8 = 2.2 \text{ m}$$

- On the Moon: $g_{moon} = \frac{GM_{moon}}{R^2_{moon}}$

$$= \frac{(6.67\times10^{-11})(7.35\times10^{22})}{(1.74\times10^{6})^2}$$

$$= 1.62 \text{ m/s}^2$$

$$\therefore T_{moon} = 2\pi\left(\frac{2.2}{1.62}\right)^{1/2} = 7.3 \text{ seconds.}$$

5. [3] A ball thrown vertically upward reaches a maximum height H. Where along the way is the kinetic energy 50% of the potential energy? (Set $U = 0$ at $y = 0$.)

$E_i = E_f$

$\Rightarrow K_i = mgH$

for the question at hand, assume ball reaches height "h":

$E_i = E_f$

$K_i = K_f + U_f$

$mgH = 0.5U_f + U_f$

$mgH = 1.5U_f = 1.5mgh$

$\Rightarrow h = 0.67H.$ ($\frac{2}{3}$ of the way up.)

[20] total marks