Arts & Science 2D06

Quiz #3 2013 Oct 22 Name: Solutions

NB: Mark values are given in brackets [ ] beside each problem. Write all your answers on the quiz paper. No books or notes allowed. Time to write quiz: 50 minutes.

Solution for quadratic equation: $x = (-b \pm \sqrt{b^2 - 4ac})/2a$

centripetal $a_c = v^2/r$ linear K.E. $= (1/2)mv^2$

Energy conservation $E = K + U$ Gravitational force: $F_g = GMm/r^2$

$G = 6.67 \times 10^{-11}$ N m^2/kg^2 (Newton's law of gravity constant)

$M_E = 5.98 \times 10^{24}$ kg (mass of Earth) $R_E = 6.37 \times 10^6$ m (radius of Earth)

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1. [3] The potential energy of a spring is: No explanation required

a) proportional to the square root of the distance stretched.

b) proportional to the distance stretched.

→c) proportional to the square of the distance stretched.

d) independent of the distance stretched.

2. [3] Consider two satellites A and B circling around the earth in concentric orbits. A and B have the same mass. The distance of satellite A to the centre of the earth is half that of satellite B. Ignoring air resistance, the ratio of the centripetal force acting on B to that acting on A is:

(Explain/derive your answer in the space provided.)

a) 1

b) $\sqrt{1/2}$

c) 1/2

→d) 1/4

e) none of the above.

$F_c \rightarrow F_g = \frac{GM_E m}{r^2}$

$\therefore \quad \frac{F_{gB}}{F_{gA}} = \frac{GM_E m/r_B^2}{GM_E m/r_A^2} = \frac{r_A^2}{r_B^2}$

Since $r_A = \frac{r_B}{2}$, $\quad \frac{F_{gB}}{F_{gA}} = \frac{r_B^2}{4r_B^2} = \frac{1}{4}$

3. [5] Suppose that a rotating disk has a non-uniform angular acceleration given by $\alpha = 12t - 3t^2$ rad/s^2. If the angular speed at 2 seconds is 10 rad/s and the angular displacement at 3 seconds is 6 radians, find equations for the angular displacement and angular speed as functions of time.

$\alpha = 12t - 3t^2$ rad/s^2

$\omega(t=2) = 10$ rad/sec

$\theta(t=3) = 6$ radians

- $\alpha = \frac{d\omega}{dt} \quad \therefore \quad d\omega = \alpha\, dt$

$$\omega = \int \alpha\, dt = \int (12t - 3t^2)\, dt$$

$$\therefore \quad \omega(t) = \frac{12t^2}{2} - \frac{3t^3}{3} + C \quad ; \; C = \text{constant}$$

$$= 6t^2 - t^3 + C$$

Now $\omega(t=2) = 6(4) - 8 + C = 10$ rad/sec

$$\therefore C = 18 - 24 = -6$$

and $\omega(t) = 6t^2 - t^3 - 6$

- $\frac{d\theta}{dt} = \omega \implies \theta(t) = \int \omega\, dt = \int (6t^2 - t^3 - 6)\, dt$

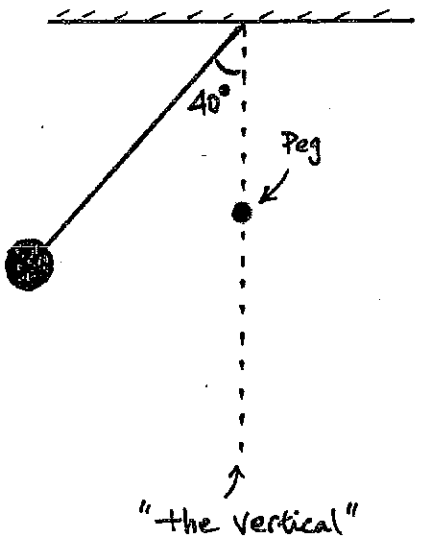
$$= 2t^3 - \frac{t^4}{4} - 6t + D \quad ; \quad D = \text{constant}$$

Since $\theta(t=3) = 2(27) - \frac{81}{4} - 18 + D = 6$ radians

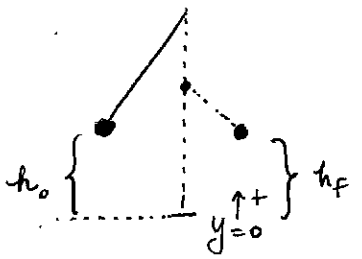
$$\therefore \quad D = -9.8$$

So $\theta(t) = 2t^3 - \frac{t^4}{4} - 6t - 9.8$.

4. [5] A ball of mass 0.8 kg is suspended by a (massless) string of length 1.6 m. The ball is released from rest with the string at 40° with respect to the vertical. The ball's motion is then interrupted by a peg located at 1 m below the top, as shown in the figure. Find the largest angle, with respect to the vertical, reached by the string after it hits the peg.



• Set $y = 0$ at lowest point of path



(not to scale)

• initial energy

$E_0 = K_0 + U_0$

$= 0 + mgh_0$

$= mgh_0$

• final energy : $E_f = K_f + U_f$

$= 0 + mgh_f$

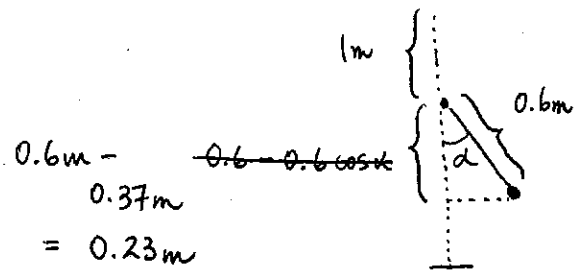
• by conservation of energy : $E_0 = E_f$

$mgh_0 = mgh_f$

$h_0 = h_f \quad (= h, \text{ say})$

• from the geometry, $h_0 = 1.6\text{m} - 1.6\cos 40° = \sout{0.83\text{m}}$ 0.37m

• Now determine largest angle :



$0.6\text{m} - \sout{0.6 - 0.6\cos\alpha}$

0.37m

$= 0.23\text{m}$

$\cos\alpha = \dfrac{0.23\text{m}}{0.6\text{m}}$

$\alpha = 67.5°$

5. [4] Calculate the acceleration due to gravity at a height that is a distance R_E above Earth's surface, where R_E is the radius of the earth.

$$F_g = \frac{GM_E m}{r^2} = ma \qquad \text{(Newton's 2nd law + universal gravitation)}$$

$$\therefore \quad a = \frac{GM_E}{r^2}$$

for $r = 2R_E$, $\quad a = \frac{GM_E}{(2R_E)^2} = \frac{GM_E}{4R_E^2}$

$$= \frac{(6.67 \times 10^{-11})(5.98 \times 10^{24})}{4\,(6.37 \times 10^{6}\,m)^2}$$

$$= 2.5\ m/s^2.$$

[20] total marks