Arts & Science 2D06

Quiz #2 **2019 Oct 9** **Name:** *Solutions*

NB: Mark values are given in brackets [ ] beside each problem. Write all your answers on the quiz paper. No books or notes allowed. Time to write quiz: 50 minutes.

Solution for quadratic equation: $x = (-b \pm \sqrt{b^2 - 4ac})/2a$

Uniform acceleration: $x = x_0 + v_0 t + \frac{1}{2}at^2$ $\quad v^2 = v_0^2 + 2a(x - x_0)$ $\quad v = v_0 + at$

$g = 9.8\ \text{m/s}^2$ $\quad$ centripetal $a_c = v^2/r$ $\quad$ linear K.E. $= (1/2)mv^2$

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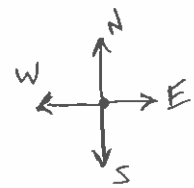
1. [3] You are trying to paddle a canoe across a river that flows southward with a strong current. Starting from the west bank, suppose you want to reach the other side directly east from your starting point. In what direction should you point your canoe?

(You can explain your answer if you would like, but it is not required.)

(a) Due east.
(b) Due north.
(c) Due south.
(d) In a southeasterly direction.
→ (e) In a northeasterly direction.



Let $\vec{V}_{CB}$ = canoe's velocity w.r.t. bank (east)

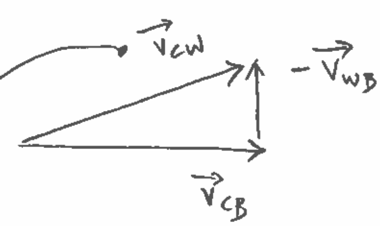
$\vec{V}_{CW}$ = canoe's velocity w.r.t. water

$\vec{V}_{WB}$ = water's velocity w.r.t. bank (south)

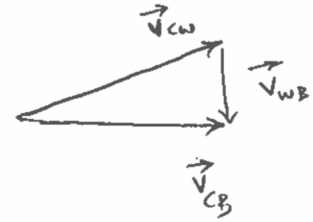
We want $\vec{V}_{CW}$: $\vec{V}_{CW} = \vec{V}_{CB} - \vec{V}_{WB}$ ("Rule of thumb" for relative velocities)

$\therefore \quad \vec{V}_{CW} = \vec{V}_{CB} + (-\vec{V}_{WB})$

towards North-east



(Note: also OK:

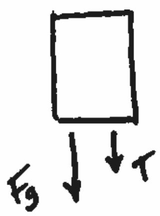


)

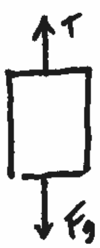
2. [3] Which of the following free-body/force diagrams best represents a person in an elevator that is traveling upward with a constant velocity? T and F_g are the magnitudes of the tension in the cable, and of the force of gravity on the whole system, respectively.

(Choose one; explain/derive your answer in the space below.)

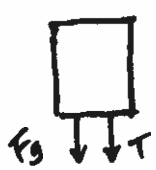
(a)



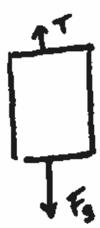
→ (b)



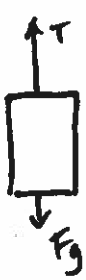
(c)



(d)



(e)



$\sum_i \vec{F}_i = m\vec{a}$, tension is <u>up</u>, F_g is <u>down</u>

constant velocity $\Rightarrow a = 0$

$\therefore \sum_i F_i = 0$

for (+), up : $\vec{T} + \vec{F}_g = 0$

$T - F_g = 0$

$T = F_g$

3. [4] Consider a 4.0-kg object that is moving along with a speed of 2.0 m/s, and a 1.0-kg object that is moving with a speed of 4.0 m/s. Suppose that both objects suddenly encounter the same constant braking force, and are brought to rest. Which object will have travelled the greater distance before stopping?

(Explain/derive your answer in the space below.)

Use the Work-Energy theorem : $W = K_f - K_i$

let $m_1 = 4$ kg, $V_1 = 2$ m/s

$m_2 = 1$ kg, $V_2 = 4$ m/s

f = braking force

$W_1 = K_{1f} - K_{1i} \Rightarrow -f \cdot \Delta X_1 = -\frac{1}{2} m_1 V_1^2$ —— (1)

$W_2 = K_{2f} - K_{2i} \Rightarrow -f \cdot \Delta X_2 = -\frac{1}{2} m_2 V_2^2$ —— (2)

divide (1) by (2) :

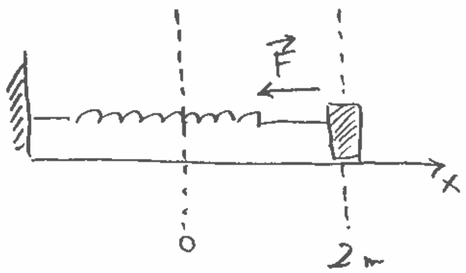
$$\frac{\Delta X_1}{\Delta X_2} = \frac{m_1 V_1^2}{m_2 V_2^2} = \frac{4 \cdot 4}{1 \cdot 4} = 1$$

$\therefore \Delta X_1 = \Delta X_2$

4. [5] An unusual spring has a restoring force of magnitude $F(x) = (3.0\ N/m^2)x^2 + (2.0\ N/m)x$, where x is the distance the spring is stretched from its equilibrium length. Suppose that a 3.00-kg mass is attached to this spring, and released from rest after stretching the spring by 2.00 m.

What is the speed of the mass when the spring returns to its equilibrium length?

(Assume the spring and mass are on a horizontal surface, and ignore any energy dissipation due to frictional forces.)



Work done by spring in pulling mass from $X = 2m \to X = 0$:

$$W = \int_2^0 (-F)dx$$

$$= \int_2^0 (-3x^2 - 2x)\,dx$$

$$= (-x^3 - x^2)\Big|_2^0$$

$$= 0 - (-8-4) = 12\,J$$

$\therefore \quad \frac{1}{2}mv^2 = 12J$ (Work-energy theorem)

$$v = \sqrt{\frac{2 \cdot 12}{3}} = 2.8\ m/s$$

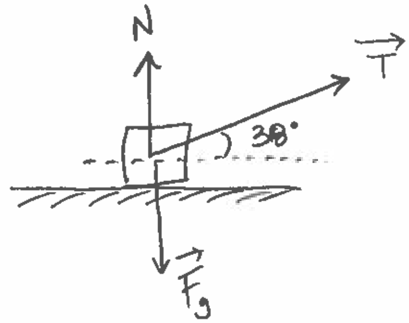
5. [5] A student pulls a box on a smooth (*i.e.*, frictionless) horizontal floor, with a force of 100 N at an angle of 38.0° above the horizontal. The box's mass is 40.0 kg.

(a) Draw the free-body/force diagram for the box.

(b) Calculate the acceleration of the box.

(c) Calculate the normal force acting on the box.

(a)



(b) Choose

Then, $\sum_i F_y = ma_y \Rightarrow N - F_g + T\sin 38° = 0$ —— (1)

$\sum_i F_x = ma_x \Rightarrow T\cos 38° = ma_x$ —— (2)

from (2) : $a_x = \frac{T\cos 38°}{m} = \frac{(100)\cos 38°}{40} = \underline{1.97\ m/s^2}$

(c) from (1) : $N = F_g - T\sin 38°$

$= (40)(9.81) - 100\sin 38°$

$= \underline{331\ N}$

20

[20] total marks