Arts & Science 2D06

Quiz #2 **2018 Oct 17** **Name:** Solutions

NB: Mark values are given in brackets [ ] beside each problem. Write all your answers on the quiz paper. No books or notes allowed. Time to write quiz: 50 minutes.

Solution for quadratic equation: $x = (-b \pm \sqrt{b^2 - 4ac})/2a$

Uniform acceleration: $x = x_0 + v_0 t + \frac{1}{2}at^2$ $v^2 = v_0^2 + 2a(x - x_0)$ $v = v_0 + at$

$g = 9.8$ m/s^2 centripetal $a_c = v^2/r$ linear K.E. $= (1/2)mv^2$

Energy conservation $E = K + U$

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1. [3] Suppose a bird is flying with constant speed, and with a constant acceleration that is always perpendicular to its velocity. What can we conclude about the bird's flight?

(You can explain your answer if you would like, but it is not required.)

(a) It is flying in a straight line.

→ (b) It is flying in a circle.

(c) It is flying in a parabola.

(d) It is flying in an ellipse.

(e) It is flying up and down.

In <u>circular motion</u>, the velocity $\vec{v}$ is always perpendicular to $\vec{a}_c$. $\vec{v}$ is tangent to the path; $\vec{a}_c$ is directed toward the centre of the path.

2. [3] An airplane is headed eastward at a speed of 156 m/s. A 20.0 m/s wind is blowing southward while the plane is in flight. What is the speed of the plane relative to the ground?

(Explain/derive your answer in the space below.)

→(a) 157 m/s
(b) 155 m/s
(c) 136 m/s
(d) 176 m/s

$$\vec{V} = \vec{V}_P + \vec{V}_W$$

$$\rightarrow \text{Speed} = \sqrt{V_P^2 + V_W^2}$$

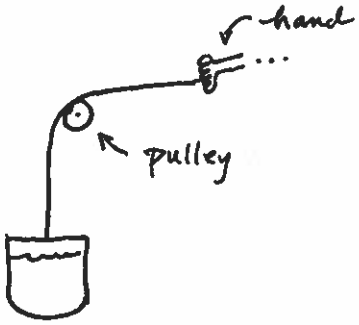
$$= \sqrt{156^2 + 20^2}$$

$$= \underline{157 \text{ m/s}}$$

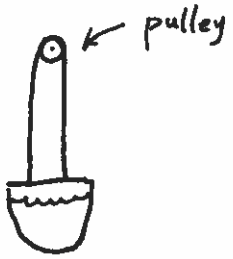
3. [4] Compare the scenarios in the picture below. In both cases, the rope and bucket are the same, the buckets are not moving, and they contain the same amount of water. In which scenario is the rope's tension lower?

(Explain/derive your answer in the space below.)

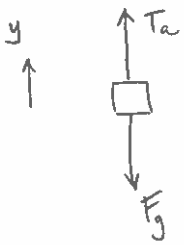
(a)



(b)



(a)

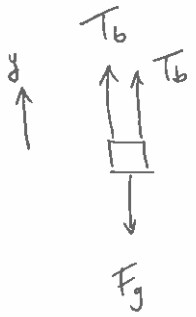


$$\sum F = ma = 0$$

$$T_a - F_g = 0$$

$$T_a = F_g$$

(b)



$$\sum F = ma$$

$$T_b + T_b - F_g = 0$$

$$2T_b = F_g$$

$$T_b = \frac{F_g}{2}$$

$$\therefore \quad T_a = 2T_b$$

$$\Rightarrow \quad \underline{T_b < T_a}$$

4. [5] A force on a particle depends on position, such that $F(x) = (3.0\ N/m^2)x^2 + (2.0\ N/m)x$. (The particle is constrained to move along the x-axis.) Find the work done by this force as the particle moves from $x = 0.0$ m to $x = 2.0$ m.

$$\text{Work } W = \int_{x_i}^{x_f} F(x)\,dx$$

$$= \int_0^2 (3x^2 + 2)\,dx$$

$$= \int_0^2 3x^2\,dx + \int_0^2 2\,dx$$

$$= 3\int_0^2 x^2\,dx + 2\int_0^2 dx$$

$$= 3 \cdot \frac{x^3}{3}\Big|_0^2 + 2 \cdot x\Big|_0^2$$

$$= \cancel{3} \cdot \frac{8}{\cancel{3}} + 4$$

$$= 12\ \text{J}$$

5. [5] A mass of 2.0 kg traveling at 3.0 m/s along a smooth (*i.e.*, frictionless), horizontal plane hits an uncompressed spring. The mass is slowed until it stops, at which point the spring has been compressed by 0.15 m. What is the spring constant of this spring?

$$E_0 = E_f$$

$$K_0 + U_0^{\nearrow 0} = K_f^{\nearrow 0} + U_f$$

$$\tfrac{1}{2} m v_0^2 = \tfrac{1}{2} k x^2$$

$$k = \frac{m v_0^2}{x^2} = \frac{(2)(3)^2}{(0.15)^2}$$

$$\Rightarrow \underline{k = 800 \text{ N/m}}$$

20/[20] total marks