Arts & Science 2D06

**Quiz #2** **2016 Oct 19** **Name:** Solutions

NB: Mark values are given in brackets [ ] beside each problem. Write all your answers on the quiz paper. No books or notes allowed. Time to write quiz: 50 minutes.

Solution for quadratic equation: $x = (-b \pm \sqrt{b^2 - 4ac})/2a$

Equations of motion for uniform acceleration: $x = x_0 + v_0 t + \frac{1}{2}at^2$, $v^2 = v_0^2 + 2ax$

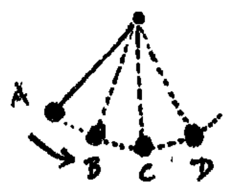
$g = 9.8$ m/s$^2$ centripetal $a_c = v^2/r$ linear K.E. $= (1/2)mv^2$

Energy conservation $E = K + U$

1. [3] A ball is attached to one end of a string. The string's other end is attached to a rigid support. The ball is released at location A and swings in a vertical arc to points B, C, and D, as shown in the picture. At which of these points does the ball have the most potential energy?

(You can explain your answer if you would like, but it is not required.)

→ (a) A
(b) B
(c) C
(d) D
(e) none of the above

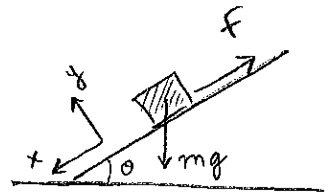



Point A is the highest position from Earth's surface (or centre).

2. [3] A block of mass $m$ is at rest on an inclined plane that makes an angle $\theta$ with the horizontal. The force of static friction $f$ must be such that

(Explain/derive your answer in the space below.)

(a) $f >$ mg
(b) $f >$ mg $\cos\theta$
(c) $f >$ mg $\sin\theta$
(d) $f =$ mg $\cos\theta$
→(e) $f =$ mg $\sin\theta$


$\sum F_x = m a_x$ , $a_x = 0$ since block is at rest.

$mg\sin\theta - f = 0$

$\therefore$ $\underline{f = mg\sin\theta}$

( Note : the force of static friction is always equal to the applied force. )

3. [4] A ball is tied to the end of a cable of negligible mass. The ball is spun in a circle with a radius of 2.00 m, and makes 0.70 revolutions per second. What is the centripetal acceleration of the ball?

$a_c = \frac{v^2}{r}$ , where $r = 2$ m

. distance covered in 0.7 revolution : $0.7 \cdot 2\pi r$
$= 8.8$ m

$\therefore$ $V = 8.8$ m/s

$\Rightarrow$ $a_c = \frac{(8.8)^2}{2} = \underline{38.7 \text{ m/s}^2}$

4. [5] A 0.50-kg object rests on a horizontal frictionless surface, and compresses a (horizontal) spring by a distance of 12.0 cm from the spring's equilibrium position. When the object is released, it leaves the spring with a speed of 20.0 cm/s (*i.e.*, this is its speed when the spring is no longer compressed). What is the spring constant of this spring?

Use conservation of energy:

$$E_0 = K_0 + U_0 = \frac{1}{2}kx^2 \quad (K_0 \to 0), \quad \text{where } x = 12\text{ cm}$$

$$E_f = K_f + U_f = \frac{1}{2}mv^2 \quad (U_f \to 0), \quad \text{where } m = 0.50\text{ kg}, \; v = 20.0\text{ cm/s}$$

$$\Rightarrow \quad E_0 = E_f$$

$$\frac{1}{2}kx^2 = \frac{1}{2}mv^2$$

$$k = \frac{mv^2}{x^2} = \frac{(0.5)(0.2)^2}{(0.12)^2} = \underline{1.39\text{ N/m}}$$

5. [5] Suppose that the force pushing a 0.5-kg book depends on the book's position, such that $F(x) = (1.0\ \mathrm{N/m^2})x^2 + (4.0\ \mathrm{N/m})x$. If the book starts from rest at $x = 0.0$, what will be its speed when it reaches the position $x = 4.0$ m? (Assume that the book is constrained to move along the $x$-axis and that the surface is frictionless.)

<u>Work-Energy theorem:</u>

$$W = E_f - E_0$$

$$\int_0^4 F\,dx = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2 \quad (\to 0)$$

$$\therefore \quad v_f^2 = \frac{2}{m}\int_0^4 (x^2 + 4x)\,dx$$

$$= \frac{2}{m}\left(\frac{x^3}{3} + \frac{4x^2}{2}\right)\Bigg|_0^4$$

$$= \frac{2}{m}\left(21.3 + 32\right)$$

$$= 213.2\ \mathrm{m^2/s^2}$$

$$\Rightarrow \quad v_F = 14.6\ \mathrm{m/s}$$

20/ [20] total marks