Arts & Science 2D06

Quiz #1 2018 Sept 21 **Name:** Solutions

NB: Mark values are given in brackets [ ] beside each problem. Write all your answers on the quiz paper. No books or notes allowed. Time to write quiz: 50 minutes.

Constant velocity: $x = x_0 + vt$

Uniform acceleration: $x = x_0 + v_0 t + \frac{1}{2}at^2 \qquad v^2 = v_0^2 + 2a(x - x_0) \qquad v = v_0 + at$

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1. [3] On a position *vs.* time graph, the slope of a tangent line at a particular time is the

(An explanation is not required, but you can provide one if you would like.)

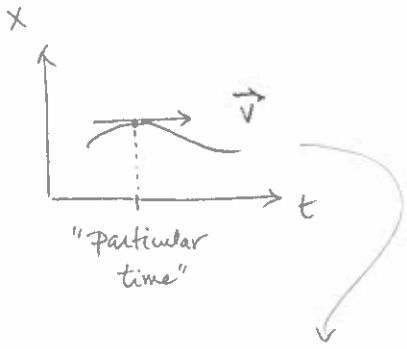
(a) displacement.

→ (b) instantaneous velocity.

(c) average velocity.

(d) instantaneous acceleration.

(e) average acceleration.



$\vec{V} = \frac{d\vec{x}}{dt}$ → slope of x vs. t curve.

2. [3] A student kicks a soccer ball in a high arc toward the opponent's goal. At the highest point in its trajectory, which of the following statements about its *speed* is true?

(Choose one statement; *explain/derive* your choice in the space below; ignore air resistance.)

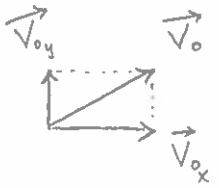
(a) It is zero.

→ (b) It is less than its initial speed.

(c) It is equal to its initial speed.

(d) It is greater than its initial speed.

(e) More information is needed to answer this question.

• Initially, 

• At the highest point : 

• Projectile motion : $\vec{V}_x = \vec{V}_{0x}$ $\therefore$ $|\vec{V}| < |\vec{V}_0|$

3. [4] A runner maintains a constant acceleration after starting from rest as she runs a distance of 60.0 m. Her speed at the end of the 60.0 m is 9 m/s. How long did the runner take to complete the 60.0 m distance?

a = constant

$x - x_0 = \Delta x = 60.0\ m$

$V_f = 9\ m/s$, $V_0 = 0$

$t = ?$

Use $V_f^2 = V_0^2 + 2a\Delta x$

$81 = 2(60)a$

$\therefore \quad a = 0.68\ m/s^2$

$\Rightarrow \quad V_f = V_0 + at$

$9 = 0 + 0.68t$

$\therefore \quad \underline{t = 13.2\ m/s^2}$

4. [5] The instantaneous velocity of an object is given by the equation:

$$\mathbf{v}(t) = (2.4\ t)\ \mathbf{i} + (0.8)\ \mathbf{j}$$

(a) What is the object's equation of motion, $\mathbf{r}(t)$, written in terms of the unit vectors $\mathbf{i}$ and $\mathbf{j}$? Assume that when $t = 0$, $\mathbf{r}(0) = 0$; *i.e.*, that the object starts moving from the origin.

$$\vec{v} = \frac{d\vec{r}}{dt}$$

$$\vec{r} = \int \vec{v}\, dt$$

$$= \int_0^t \left[(2.4t)\,\hat{i} + (0.8)\,\hat{j}\right] dt$$

$$= 2.4\,\frac{t^2}{2}\hat{i} + 0.8t\,\hat{j} + \text{Constant}$$

$$= 1.2t^2\hat{i} + 0.8t\,\hat{j} + \text{Constant}$$

Now $\vec{r}(0) = 0 \Rightarrow$ Constant $= 0$

$$\therefore \quad \underline{\vec{r} = 1.2\,t^2\,\hat{i} + 0.8\,t\,\hat{j}} \quad [m]$$

(b) What is the object's acceleration?

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left[(2.4t)\,\hat{i} + (0.8)\,\hat{j}\right]$$

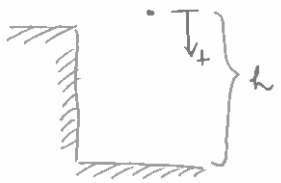
$$= \frac{d}{dt}(2.4t)\,\hat{i} + \frac{d}{dt}(0.8)\,\hat{j}$$

$$= 2.4\,\hat{i} + 0\,\hat{j}$$

$$\Rightarrow \quad \underline{\vec{a} = 2.4\,\hat{i}} \quad [m/s^2]$$

5. [5] Suppose that you drop a rock from a vertical cliff and you find that it takes 3 seconds to reach the ground straight below. Next, you throw a second rock vertically from the cliff; this rock takes 2 seconds to reach the ground from the time it is released. With what velocity was the second rock released?

Rock 1 : $a = g = 9.81\ m/s^2$



$V_0 = 0$, $x_0 = 0$, $x_f = h$

$t = 3\ s$

Use : $x = x_0 + V_0 t + \frac{1}{2} a t^2$

$h = 0 + 0 + \frac{9.81}{2}(3)^2 = 44.1\ m$

Rock 2 : $a = g = 9.81\ m/s^2$

$t = 2$ seconds

$x = x_0 + V_0 t + \frac{1}{2} g t^2$

$44.1 = 2V_0 + \frac{9.81}{2}(2)^2$

$2V_0 = 24.5$

$\underline{V_0 = 12.2\ m/s\ (down)}$

20/

[20] total marks