Arts & Science 2D06

Quiz #1 2016 Sept 23 Name: Solutions

NB: Mark values are given in brackets [ ] beside each problem. Write all your answers on the quiz paper. No books or notes allowed. Time to write quiz: 50 minutes.

Solution for quadratic equation: $x = (-b \pm \sqrt{b^2 - 4ac})/2a$

Uniform acceleration: $x = x_0 + v_0 t + \frac{1}{2}at^2$ $v^2 = v_0^2 + 2a(x - x_0)$

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1. [3] If the graph of velocity *vs.* time for an object is a straight line making an angle of 30° with the time axis, the object is

(no need to explain/derive; ignore air resistance.)

(a) moving with constant non-zero speed.

→ (b) moving with constant non-zero acceleration.

(c) at rest.

(d) moving with infinite speed.

(e) none of the above.

Uniformly accelerated motion: $V(t) = V_0 + at$

→ V vs. t is a line with slope = "a".

2. [4] For projectile motion in 2D, which one of the statements below is true when the projectile is at the highest point of its path?

(Choose one statement; *explain/derive* your choice in the space below; ignore air resistance.)

(a) Its acceleration is zero.

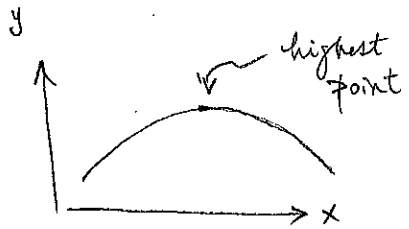
→ (b) Its velocity is perpendicular to the acceleration.

(c) Its velocity and acceleration are both zero.

(d) The horizontal component of its velocity is zero.

(e) The horizontal and vertical components of its velocity are zero.

Projectile motion in 2D:



At highest point : $v_y = 0$, $v_x = v_{0x}$

$a_y = -g$, $a_x = 0$

∴ the projectile's velocity is perpendicular to the acceleration.

3. [4] A airplane that is flying horizontally needs to accelerate from a speed of 200 m/s to a speed of 240 m/s while it flies a distance of 1200 m. What should the acceleration of the plane be?

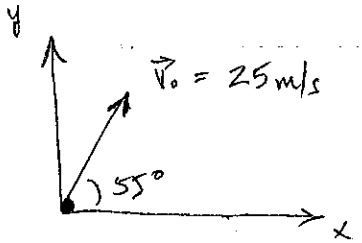
- assume a = constant
- then use $V^2 = V_0^2 + 2a(x - x_0)$

let $x - x_0 = 1200\ m$
$V_0 = 200\ m/s$
$V = 240\ m/s$

then, $(240)^2 = (200)^2 + 2 \cdot a \cdot (1200)$

$\Rightarrow \underline{a = 7.3\ m/s^2}$

4. [4] A projectile is launched from the (horizontal) ground at an angle of 55.0° with respect to the ground, with a speed of 25.0 m/s. How long will the projectile stay in the air before it makes contact with the ground again?



We have $y_0 = 0$

and $y = 0$, when in contact w/ ground again.

In the y-direction: constant $a = -g$

$\therefore \quad y = y_0 + V_{0y}t - \frac{g}{2}t^2$

$0 = 0 + (V_0 \sin 55°)t - \frac{9.81}{2}t^2$

$4.9t = 25 \sin 55°$

$\underline{t = 4.2 \text{ seconds}}$

5. [5] A car has its position given by

$$\vec{r}(t) = [2.5 \text{ m} + (3.0 \text{ m/s})\, t]\, \hat{i} + [3.0 \text{ m} - (2.0 \text{ m/s}^2)\, t^2]\, \hat{j}.$$

Find the car's speed and acceleration when $t = 3.5$ seconds.

· Velocity : $\vec{V} = \frac{d\vec{r}}{dt} = 3\hat{i} - 4t\hat{j}$

· acceleration : $\vec{a} = \frac{d\vec{V}}{dt} = -4\hat{j}$

$\therefore$ at $t = 3.5$ seconds,

$$\vec{V} = 3\hat{i} - 4(3.5)\hat{j}$$

$$= 3\hat{i} - 14\hat{j}$$

$\Rightarrow$ Speed $V = \sqrt{V_x^2 + V_y^2} = \sqrt{3^2 + 14^2} = \underline{14.3 \text{ m/s}}$

and acceleration $\underline{\vec{a} = -4 \text{ m/s}^2\, \hat{j}}$

[20] total marks