Arts & Science 2D06

Quiz #1 2012 Sept 20 **Name:** Solutions

NB: Mark values are given in brackets [ ] beside each problem. Write all your answers on the quiz paper. No books or notes allowed. Time to write quiz: 40 minutes.

Solution for quadratic equation: $x = (-b \pm \sqrt{b^2 - 4ac})/2a$

Uniform acceleration: $x = x_0 + v_0 t + \frac{1}{2}at^2 \qquad v^2 = v_0^2 + 2a(x - x_0)$

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1. [3] Two identical objects (A and B) are released from rest from different heights. If object B takes twice as long as A to reach the ground, how much higher was B's starting height than A's? (Ignore air resistance.) *Explain/derive* your choice in the space below.

(a) $\sqrt{2}$ times higher.
(b) 2 times higher.
(c) 4 times higher.
(d) 8 times higher.

$y_A = y_{0_A} + v_{0_A} t - \frac{1}{2} g t_A^2$

$y_B = y_{0_B} + v_{0_B} t - \frac{1}{2} g t_B^2$

given: $v_{0_A} = v_{0_B} = 0$; $t_B = 2t_A$ / let $y_A = y_B = 0$

$\therefore \quad y_{0_A} = \frac{1}{2} g t_A^2$, $y_{0_B} = \frac{1}{2} g t_B^2$ $\Rightarrow$ $y_{0_B} = \frac{t_B^2}{t_A^2} y_{0_A} = \frac{4t_A^2}{t_A^2} y_{0_A} = 4 y_{0_A}$

2. [3] A stone is thrown at some angle relative to the ground. When the stone has fallen back to the height from which it was thrown, which one of the following statements is most nearly correct? (Ignore air resistance.)

(a) Its acceleration is $2g$.
(b) Its velocity is the same as when it was thrown.
(c) Its speed is the same as when it was thrown.
(d) Both its speed and velocity are zero.

- acceleration is "g"
- Speed and velocity aren't zero
- Velocity is not the same b/c directions are different
- Speed is magnitude of $\vec{v}$: the same.

3. [4] A cyclist is riding on a flat road with a constant speed of 20 km/h. How long will it take for a car driver to pass the cyclist, if the driver starts from rest when the cyclist is 220 meters ahead, and drives with an acceleration of 4.0 m/s^2?

driver : $V_0 = 0$, $a = 4.0\ m/s^2$, let $x_0 = 0$ } $x_{driver} = \frac{1}{2}at^2 = 2.0t^2$

cyclist : $V = 20\ km/hr = 5.6\ m/s$, $x_0 = 220\ m$ } $x_{cyclist} = 220 + 5.6t$

∴ by the time the car catches the bike :

$$x_{driver} = x_{cyclist}$$

$$2.0t^2 = 220 + 5.6t$$

$$t^2 - 2.8t - 110 = 0$$

∴ $t = 12$ seconds

(ignore negative solution)

4. [4] A ball is thrown at an angle of 44.0 degrees relative to the horizontal ground, with an initial speed of 17.2 m/s and from an unknown initial height.

(a) If the horizontal distance traveled by the ball was 32.1 m, for how long was the ball in the air?

$x = x_0 + v_x t$, where $\begin{cases} v_x = (17.2)\cos 44^\circ \\ = 12.4 \text{ m/s} \end{cases}$

let $x_0 = 0$, then

$$32.1 = 12.4 \cdot t$$

$$t = 2.59 \text{ seconds}$$

(b) From what height was the ball thrown?

$y = y_0 + v_{0y} t - \frac{1}{2} g t^2$ (choose $y=0$ on the ground)

since $v_{0y} = (17.2)\sin 44^\circ = 11.9$ m/s ,

$$y_0 = \frac{1}{2}(9.81)(2.59)^2 - (11.9)(2.59)$$

$$= 2.08 \text{ m}$$

5. [6] Consider a plane flying with the following equation of motion (in meters):

$$\mathbf{r}(t) = (3.2\,t^2)\,\mathbf{i} + (0.6 - 93.0\,t)\,\mathbf{j}$$

(a) Find the plane's instantaneous velocity, written in terms of the unit vectors **i**, **j**.

$\vec{v} = \frac{d\vec{r}}{dt} = 6.4t\,\hat{i} - 93.0\,\hat{j}$

(b) Is the plane moving at constant velocity? If not, calculate the plane's speed when $t = 35$ seconds.

$\vec{v}$ from (a) depends on t $\Rightarrow$ not constant

$\vec{v}(t=35s) = 6.4(35)\,\hat{i} - 93.0\,\hat{j}$

$= 224\,\hat{i} - 93.0\,\hat{j}$ (m/s)

Speed $v = \sqrt{v_x^2 + v_y^2} = \sqrt{(224)^2 + (-93.0)^2} = 240$ m/s

(c) Is the plane moving at constant acceleration? Explain or derive your answer.

$\vec{a} = \frac{d\vec{v}}{dt} = 6.4\,\hat{i}$, which is constant.

[20] total marks